ASSIGNMENT 3:,
First Law of Thermodynamics,
Heat and Work in Gas Processes
Kinetic Theory of Gases

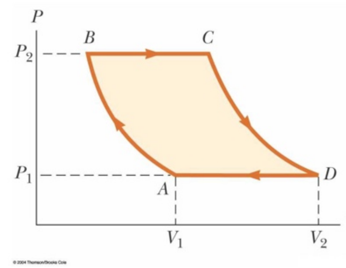
UNIVERSITY OF OTTAWA
Principles of Physics
PHY1321/31 Fall 2017
Dr. A. Czajkowski

STUDENT #: ____________________

NAME: ______________________________

Released: Sept 29, **Due: Oct 6** **6PM**

1 An ideal gas is carried through a thermodynamic cycle consisting of two isobaric and two isothermal processes as shown. Show that the net work done on the gas in the entire cycle is given by:



$$W_{\text{net}} = -P_1(V_2 - V_1)\ln\frac{P_2}{P_1}$$

SOLUTION: W = W_{AB}+ W_{BC}+ W_{CD}+ W_{DA}

$$W = \text{-}nRT_A ln\frac{V_B}{V_A}\text{-}p_B(V_C\text{-}V_B)\text{-}nRT_C ln\frac{V_D}{V_C}\text{-}p_A(V_A\text{-}V_D)$$

$$W = \text{-}nRT_A ln\frac{V_B}{V_A}\text{-}nRT_C ln\frac{V_D}{V_C}\text{-}p_B V_C + p_B V_B\text{-}p_A V_A + p_A V_D \Rightarrow W = \text{-}nRT_A ln\frac{V_B}{V_A}\text{-}nRT_C ln\frac{V_D}{V_C}\text{-}nRT_C + nRT_B\text{-}nRT_B + nRT_C$$

$$W = -nRT_A ln\frac{V_B}{V_A} - nRT_C ln\frac{V_D}{V_C} - nRT_C + nRT_B - nRT_B + nRT_C \Rightarrow W = -nRT_A ln\frac{V_B}{V_A} - nRT_C ln\frac{V_D}{V_C}$$

$$W = -nRT_A ln\frac{V_B}{V_A} - nRT_C ln\frac{V_D}{V_C} = -nRT_A ln\frac{\frac{nRT_A}{p_B}}{\frac{nRT_A}{p_A}} - nRT_C ln\frac{\frac{nRT_C}{p_D}}{\frac{nRT_C}{p_C}} = -nRT_A ln\frac{p_A}{p_B} - nRT_C ln\frac{p_C}{p_D}$$

$$W = -nRT_A ln\frac{p_A}{p_B} - nRT_C ln\frac{p_C}{p_D} = -nRT_A ln\frac{p_1}{p_2} - nRT_C ln\frac{p_2}{p_1} = (nRT_A - nRT_C)ln\frac{p_2}{p_1} = (p_1V_1 - p_1V_2)ln\frac{p_2}{p_1}$$ Q.E.D

2 **A)** One mole of an ideal gas does 3 000 J of work on its surroundings as it expands isothermally to a final pressure of 1.00 atm and volume of 25.0 L. Determine (i) the initial volume and (ii) the temperature of the gas.

(i) $W = -nRT\ln\left(\frac{V_f}{V_i}\right) = -P_fV_f\ln\left(\frac{V_f}{V_i}\right)$ s $V_i = V_f\exp\left(+\frac{W}{P_fV_f}\right) = (0.025\,0)\exp\left[\frac{-3\,000}{0.025\,0(1.013\times10^5)}\right] = \boxed{0.007\,65\text{ m}^3}$

(ii) $T_f = \frac{P_fV_f}{nR} = \frac{1.013\times10^5\text{ Pa}(0.025\,0\text{ m}^3)}{1.00\text{ mol}(8.314\text{ J/K}\cdot\text{mol})} = \boxed{305\text{ K}}$

B)As a 1.00-mol sample of a monatomic ideal gas expands adiabatically, the work done on it is –2 500 J. The initial temperature and pressure of the gas are 500 K and 3.60 atm. Calculate (iii) the final temperature, and (iv) the final pressure.

(iii) $W = nC_V(T_f - T_i)$ so that $-2\,500\text{ J} = 1\text{ mol}\frac{3}{2}8.314\text{ J/mol}\cdot\text{K}(T_f - 500\text{ K})$ and $T_f = \boxed{300\text{ K}}$

(iv) $P_iV_i^\gamma = P_fV_f^\gamma$ and thus $P_i\left(\frac{nRT_i}{P_i}\right)^\gamma = P_f\left(\frac{nRT_f}{P_f}\right)^\gamma$ $T_i^\gamma P_i^{1-\gamma} = T_f^\gamma P_f^{1-\gamma}$

$\frac{T_i^{\gamma/(\gamma-1)}}{P_i} = \frac{T_f^{\gamma/(\gamma-1)}}{P_f}$ and $P_f = P_i\left(\frac{T_f}{T_i}\right)^{\gamma/(\gamma-1)}$ $P_f = P_i\left(\frac{T_f}{T_i}\right)^{(5/3)(3/2)} = 3.60\text{ atm}\left(\frac{300}{500}\right)^{5/2} = \boxed{1.00\text{ atm}}$

3 Fill the table below:

	Degrees of Freedom	AVG Energy of single molecule	Cv	Cp	Gamma
A	1	1/2kT	½ R	3/2 R	3
B	3	3/2 kT	3/2 R	5/2 R	5/3
C	7	7/2kT	7/2R	9/2 R	9/7
D	13	13/2 kT	13/2R	15/2 R	15/13
E	6	3kT	3R	4R	4/3

4 Using the approach demonstrated during the lecture show that for pV^γ =const. for adiabatic gas process. (Present your derivation on the opposite site of this page). DETAILS OF THIS CALCULATION WERE GIVEN IN LECTURE.

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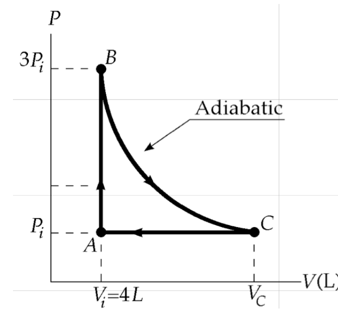
5 A 4 liter sample of a diatomic gas with γ =1.4 confined to a cylinder, is carried through a closed cycle. The gas is initially at 1.00atm. and 300K. First, its pressure is tripled under constant volume. Then it expands adiabatically to its original pressure. Finally the gas is compressed isobarically to its original volume.

a) draw pV diagram of this cycle
b) determine the volume of the end of the adiabatic expansion
c) find the temperature of the gas at the start of the adiabatic expansion
d) find the temperature at the end of the cycle
e) what was the net work done on the gas for this cycle



(a) See the diagram at the right.

(b) $P_B V_B^{\gamma} = P_C V_C^{\gamma}$ so that $3P_i V_i^{\gamma} = P_i V_C^{\gamma}$

so that $V_C = \left(3^{1/\gamma}\right)V_i = \left(3^{5/7}\right)V_i = 2.19V_i$ $V_C = 2.19(4.00\text{ L}) = \boxed{8.77\text{ L}}$

(c) $P_B V_B = nRT_B = 3P_i V_i = 3nRT_i$ which gives $T_B = 3T_i = 3(300\text{ K}) = \boxed{900\text{ K}}$

(d) After one whole cycle, $T_A = T_i = \boxed{300\text{ K}}$

(e) In AB, $Q_{AB} = nC_V \Delta V = n\left(\frac{5}{2}R\right)(3T_i - T_i) = (5.00)nRT_i$, $Q_{BC} = 0$ as this process is adiabatic

$P_C V_C = nRT_C = P_i(2.19V_i) = (2.19)nRT_i$ so $T_C = 2.19T_i$,

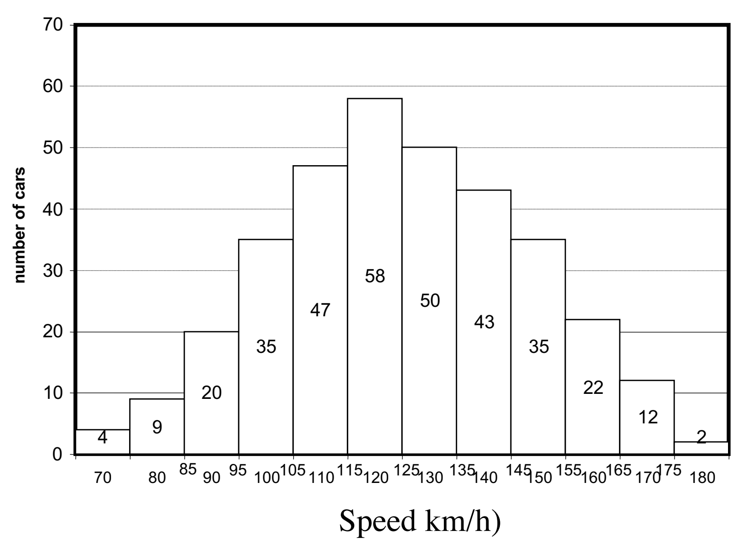
$Q_{CA} = nC_P \Delta T = n\left(\frac{7}{2}R\right)(T_i - 2.19T_i) = (-4.17)nRT_i$

For the whole cycle

$$Q_{ABCA} = Q_{AB} + Q_{BC} + Q_{CA} = (5.00 - 4.17)nRT_i = (0.829)nRT_i$$
$$(\Delta E_{\text{int}})_{ABCA} = 0 = Q_{ABCA} + W_{ABCA}$$
$$W_{ABCA} = -Q_{ABCA} = -(0.829)nRT_i = -(0.829)P_i V_i$$
$$W_{ABCA} = -(0.829)\left(1.013\times10^5\text{ Pa}\right)\left(4.00\times10^{-3}\text{ m}^3\right) = \boxed{-336\text{ J}}$$

6 Given is distribution of speeds of cars at 417 Highway as measured by OPP.

a) Is this a discrete or continuous distribution?
b) Find the Vmp, Vrms, Vavg.
c) Find the probability that a randomly picked car will have speed larger than 115km/h.
d) Find the probability that a randomly picked car will have speed larger than 95km/h and less then 125km/h.



Answers:

a) discrete
b) V_{mp}=120km/h; V_{rms} =127km/h; V_{avg}=125km/h
c) P(v>115) = 0.525
d) P(95<v<125) = 0.42