CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	205	All
Examination	**Date**	**Duration**
Midterm	23 October 2016	1 h 30 min
Special	Only approved calculators are allowed	
Instructions:	**Show all your work for full marks**	

[*Marks*]

[11] **1.** **(a)** Write the sigma notation formula for the right Riemann sum R_n of the function $f(x) = 4 - x^2$ on the interval [0,2] using n subintervals of equal length, and calculate the definite integral $\int_0^2 f(x)\,dx$ as the limit of R_n at $n \to \infty$.

(Reminder: $\sum_{k=1}^{n} k = n(n+1)/2, \qquad \sum_{k=1}^{n} k^2 = n\,(n+1)\,(2n+1)/6$)

(b) Use the Fundamental Theorem of Calculus to calculate the derivative of $F(x) = \int_{e^{-x}}^{x} \ln(t^2+1)\,dt$

[15] **2.** Calculate the following indefinite integrals

(**a**) $\displaystyle\int \frac{3\,x^3}{\sqrt{16-x^2}}\,\mathrm{d}x$ (**b**) $\displaystyle\int \frac{x^2+3}{x^2-2x+5}\,\mathrm{d}x$ (**c**) $\displaystyle\int \frac{2^x}{2^{2x}-4}\,\mathrm{d}x$

[6] **3.** Find the antiderivative $F(x)$ of $f(x) = \dfrac{\sec^2(x)}{\sec^2(x)+3}$ such that $F(0) = 0$.

[12] **4.** Evaluate the following definite integrals (give the exact values, **do not approximate**):

(**a**) $\displaystyle\int_0^4 \frac{x^2}{\sqrt{2x+1}}\,\mathrm{d}x$ (**b**) $\displaystyle\int_0^1 x^2\,\cos(\pi x)\,\mathrm{d}x$

[6] **5.** Find the area enclosed by the graphs of the functions $f(x) = x^2 - 3\,x - 5$ and $g(x) = 3 - x$.

[3] **Bonus question.** Given that

$$\int_0^{\pi} [f(x) + f''(x)] \sin x \mathrm{d}x = 2$$

and $f(\pi) = 1$, find $f(0)$