

## Outline (Chapter 3)

- Crystalline & Amorphous Structures
- Crystal Structures
  - ✓ Basic Concepts
    - Unit Cell
    - Single Cubic (SC)
    - Coordination number
    - Atomic packing factor
    - Crystal density
  - ✓ Body Centered Cubic (BCC)
  - ✓ Face Centered Cubic (FCC)
  - ✓ Hexagonal Close-Packed (HCP)

## Crystalline vs. Amorphous

### ➤ Crystalline Structure:

If atoms bonded together in a regular 3-D pattern they form a **CRYSTAL**

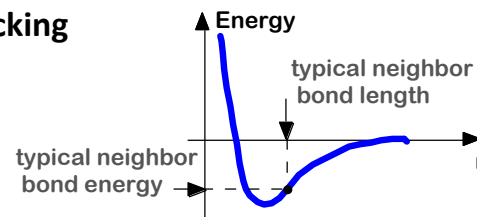
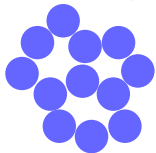
### ➤ Amorphous Structure:

If atoms bonded to each other but there is no repeating pattern (**short range order**) . e.g. water, glasses (**AMORPHOUS - non-crystalline**)

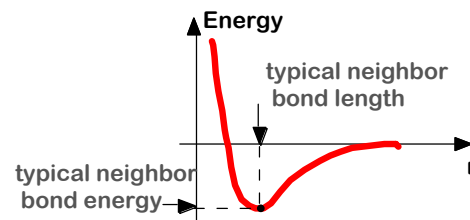
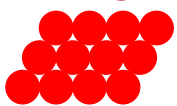
- Metals: (>99%) → Crystalline
- Ceramics: Crystalline (except glasses → Amorphous)
- Polymers: Semi-crystalline (mixed of crystalline and amorphous) or Amorphous
- In gases, atoms have no order (Amorphous).

## Crystalline vs. Amorphous

- Non dense, **random** packing



- Dense, **regular** packing

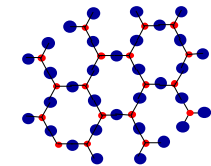


Dense, regular-packed structures tend to have lower energy.

## Crystalline vs. Amorphous

### Crystalline materials...

- atoms pack in periodic, 3D arrays
- typical of: -metals  
-many ceramics  
-some polymers

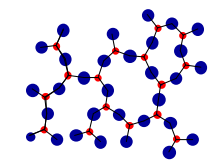


crystalline SiO<sub>2</sub>

### Noncrystalline materials...

- atoms have no periodic packing
- occurs for: -complex structures  
-rapid cooling

• Si • Oxygen



noncrystalline SiO<sub>2</sub>

"Amorphous" = Noncrystalline

# Crystal Structures

- **Motivation:** Many of the properties of materials (*especially mechanical*) are determined by the arrangement of the constituent atoms.

✓ *This arrangement is called the material's **crystal structure**.*

- **An important distinction...**

- **Atomic structure** relates to the number of protons and neutrons in the nucleus of an atom, as well as the number and probability distributions of the electrons.
- **Crystal structure** pertains to the arrangement of atoms in the crystalline solid material.

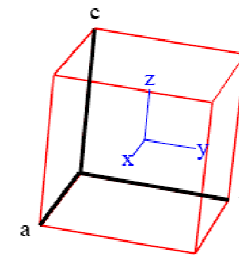
# What is Unit Cell?

- **Unit cell:** The smallest group of atoms which can generate the entire crystal by translation.

- **Lattice parameter:** The length of each unit cell axis

✓ In other words, **unit cell** is the smallest repetitive volume which contains the complete **lattice pattern** of a crystal.

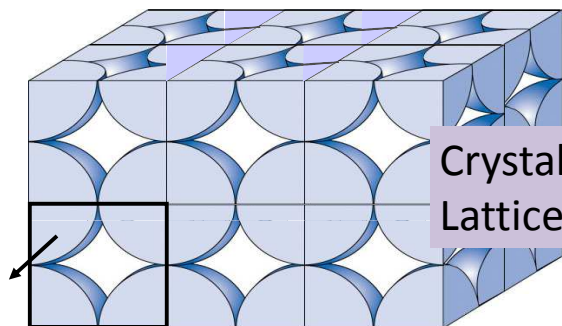
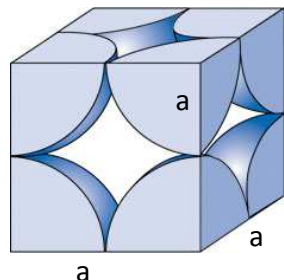
✓ *To illustrate the concept of crystal structure and lattice systems, we first identify a **coordinate system** (x, y, z)*



- **a, b, and c** are the **lattice parameters**.
- *In cubic systems, all three orthogonal lattice parameters are equal.*
- Lattice parameters are typically on the order of a few tenths of a nanometer ( $10^{-9}$  m) (or a few Angstroms -  $10^{-10}$  m)

# What is Unit Cell?

Unit cell of:  
**Simple cubic (SC)**



**In Simple Cubic Structure:**

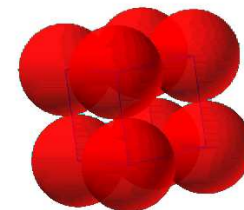
- Atoms "touch" along the crystal axes
- These directions are referred to as "close-packed" in the simple cubic system

# Simple Cubic Structure (SC)

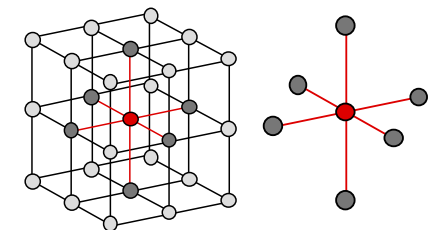
- Rare due to poor packing (only Po has this structure)
- **Close-packed directions** are cube edges.

**Coordination number (CN):** Number of touching atoms surrounding one atom

✓ Large atoms tend to have larger CN, small atoms usually have small CN. Because, it's easier to surround a big atom with lots of atoms than a smaller one.



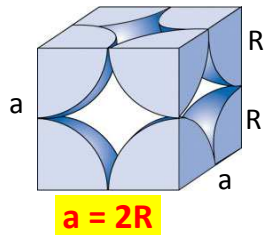
- **Coordination number = 6**  
(# nearest neighbors)



# Simple Cubic Structure (SC)

- **Number of Atoms in Unit Cell (n):** In SC, unit cell contains 1 full atom (8 x 1/8 atoms)
- **Atomic Packing Factor (APF):** Volume of unit cell occupied by atoms over total unit cell volume (the higher APF → the higher the density)

$$APF = \frac{\text{volume of unit cell occupied by atoms}}{\text{total unit cell volume}}$$



$$APF = \frac{\text{atoms unit cell} \cdot \frac{4}{3} \pi (0.5a)^3}{\text{volume unit cell}} = \frac{1 \cdot \frac{4}{3} \pi (0.5a)^3}{a^3}$$

- **APF for a SC structure = 0.52**

# Crystal Structures

- **Crystal Density:** The true density,  $\rho$ , of material (free from defects) can be calculated knowing its crystal structure.

$$\rho = \frac{\text{Mass of unit cell}}{\text{Volume of unit cell}} = \frac{\frac{\text{\# of atoms unit cell} \cdot \text{Atomic weight (g/mol)}}{N_A}}{V_c}$$

$\rho_{\text{metals}} > \rho_{\text{ceramics}} > \rho_{\text{polymers}}$

- n = number of atoms in unit cell
- A = Atomic Weight of element (g/mol)
- $V_c$  = volume of unit cell (=  $a^3$  for cubic system)
- $N_{av}$  = Avogadro's number ( $6.022 \times 10^{23}$  atoms/mol)

# Summary: Crystal Structures (Basic Concepts)

- **Unit cell:** The smallest repetitive volume which contains the complete lattice pattern of a crystal.
- **Coordination number (CN):** Number of touching atoms surrounding one atom
- **Atom/Unit Cell (n):** Number of Atoms in Unit Cell
- **Atomic Packing Factor (APF):** Volume of unit cell occupied by atoms over total unit cell volume
- **Crystal Density:** The true density,  $\rho$ , of material (free from defects) can be calculated knowing its crystal structure.

# Crystal Structures

- **Simple Cubic (SC):**

$$a = 2R, APF = 0.52, \text{coordination \#} = 6, n=1$$

Metallic Crystal Structures

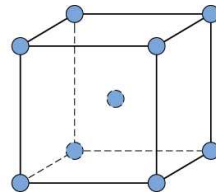
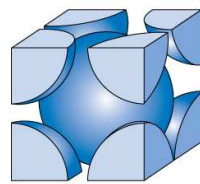
- Body Centered Cubic (BCC)
- Face Centered Cubic (FCC)
- Hexagonal Close-Packed (HCP)

- ✓ **Most** metals crystallize into one of these three densely packed structures

# Body Centered Cubic (BCC)

- Atoms touch each other along cube diagonals.
- **Note:** All atoms are identical; the center atom is shaded differently only for ease of viewing.

ex: Cr, W, Fe ( $\alpha$ ), Tantalum, Molybdenum



- **Coordination number = 8**  
(# nearest neighbors)

- **# of atoms/unit cell (n) = 2**  
(1 atom + 8 x 1/8 atoms)

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# Body Centered Cubic (BCC)

plane x, x', x'', x'''

$4R = a\sqrt{3}$   
so  $R = \frac{a\sqrt{3}}{4}$

atoms/unit cell  $\rightarrow 2$

volume/atom  $\leftarrow \frac{4}{3} \pi (\frac{\sqrt{3}a}{4})^3$

APF =  $\frac{2 \cdot \frac{4}{3} \pi (\frac{\sqrt{3}a}{4})^3}{a^3}$

volume/unit cell  $\leftarrow a^3$

**$4R = a\sqrt{3}$**

- **APF for a BCC structure = 0.68**

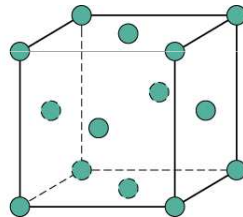
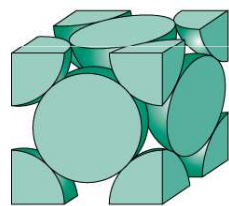
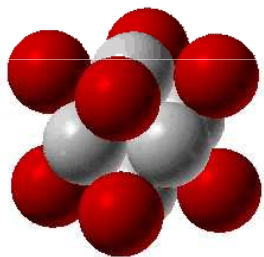
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# Face Centered Cubic (FCC)

- Atoms touch each other along face diagonals.
- **Note:** All atoms are identical; face-centered atoms are shaded differently only for ease of viewing.

ex: Al, Cu, Au, Pb, Ni, Pt, Ag



- **Coordination number = 12**  
(# nearest neighbors)

- **# of atoms/unit cell (n) = 4**  
(6 face x 1/2 + 8 corners x 1/8)

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# Face Centered Cubic (FCC)

$4R = a\sqrt{2}$   
so  $R = \frac{a\sqrt{2}}{4}$

atoms/unit cell  $\rightarrow 4$

volume/atom  $\leftarrow \frac{4}{3} \pi (\frac{\sqrt{2}a}{4})^3$

APF =  $\frac{4 \cdot \frac{4}{3} \pi (\frac{\sqrt{2}a}{4})^3}{a^3}$

volume/unit cell  $\leftarrow a^3$

**$4R = a\sqrt{2}$**

- **APF for a FCC structure = 0.74**

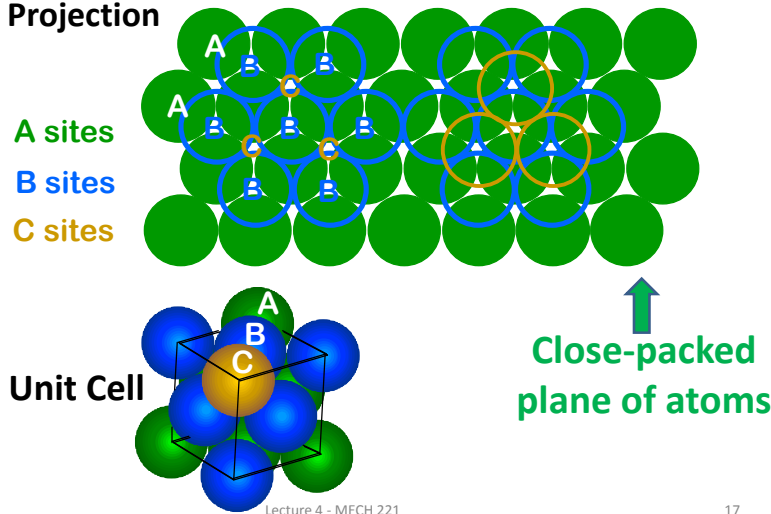
**maximum achievable APF**

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# Face Centered Cubic (FCC)

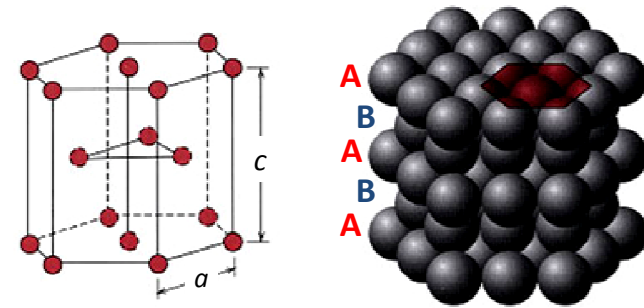
- **ABC-ABC...** Stacking Sequence
- 2D Projection



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# Hexagonal Close-Packed Structure (HCP)

➤ Hexagonal arrangement (not cubic), have 2 lattice parameters "a", & "c"



- **AB-AB...** Stacking Sequence
- examples: Cd, Mg, Ti, Zn

$$c/a = 1.633$$

- **APF for an HCP structure = 0.74**  
(Same as FCC)

• **Coordination number = 12**  
(# nearest neighbors)

• **# of atoms/unit cell (n) = 6**  
( $2 \times 1/2 + 12 \text{ corner} \times 1/6 + 3$ )

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# Crystal Structures

- **Polymorphism:** When some metals (or non-metals) have more than one crystal structure. (depending on **Pressure** and **Temperature** condition)
- **Allotropy:** When more than one crystal structure found in elemental solids

## Examples:

**Carbon:** Forms as Graphite (at ambient pressure) & Diamond (at high pressure)

**Iron:** Various structures at different temperatures

Can be **IMPORTANT** as some crystal structures more dense (better packing, higher APF) than others.

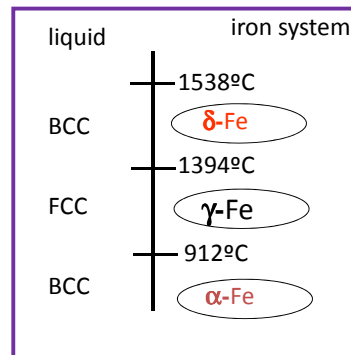
So a change in crystal structure can often result in volume change of material.

APF (FCC) = 0.74

APF (BCC) = 0.68

i.e. expands on transforming from FCC to BCC.

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# Crystal Structures (review problem)

- Calculate the Theoretical Density,  $\rho$ , of Copper (Cu). Cu has an atomic radius 0.128 nm an FCC crystal structure & an atomic weight of 63.5 g/mol.
- Note:** Use the data in Table 3.1 (Callister 9<sup>th</sup> edition)

$$\rho = \frac{\text{Mass of unit cell}}{\text{Volume of unit cell}} = \frac{\frac{\text{\# of atoms}}{\text{unit cell}} \cdot A}{N_A \cdot V_c}$$

Atomic weight (g/mol)

Avogadro's number  
( $6.023 \times 10^{23}$  atoms/mol)

volume unit cell ( $\text{cm}^3/\text{unit cell}$ )

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