Lecture 7

Rules for differentiation.

Using the definition of derivative, one can derive the following:

(i) $\frac{d}{dx} c = 0$ $\quad (c \in \mathbb{R})$

(ii) $\frac{d}{dx} cx = c$

(iii) $\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$

(iv) $\frac{d}{dx} c \cdot f(x) = c \cdot \frac{d}{dx} f(x)$

} linearity of the derivative.

Here's how you might use this:

$$\frac{d}{dx}\left(3x^3 - 2\sqrt{x} + \frac{2}{x}\right) =$$

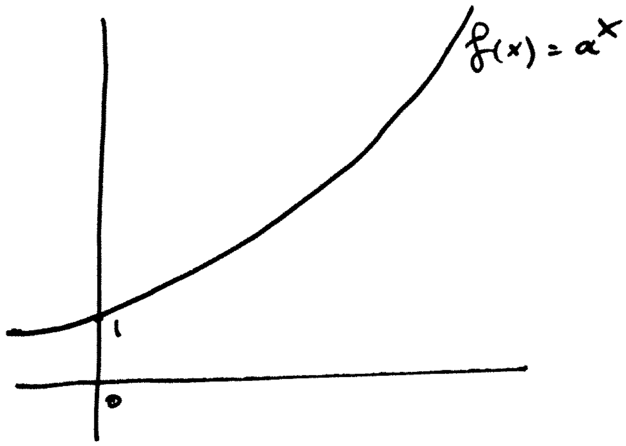
$$3\frac{d}{dx}x^3 - 2\frac{d}{dx}\sqrt{x} + 2\frac{d}{dx}\frac{1}{x} =$$

$$3 \cdot 3x^2 - 2 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x}} + 2 \cdot -1 \cdot \frac{1}{x^2} =$$

$$9x^2 - \frac{1}{\sqrt{x}} - \frac{2}{x^2}.$$

(Helps to break down complicated derivatives into simpler components.)

We know how to differentiate x^n, but not yet a^x.



Recall: e is the number for which the slope of e^x is 1 at $x=0$. So $\frac{d}{dx}e^x = e^x$:

$$\lim_{h\to 0}\frac{e^{x+h}-e^x}{h} = \lim_{h\to 0} e^x\left(\frac{e^h-1}{h}\right)$$

$$= e^x \cdot \lim_{h\to 0}\frac{e^h-1}{h}$$

$$= e^x \cdot \lim_{h\to 0}\frac{e^h-e^0}{h}$$

$$= e^x \cdot \frac{d}{dx}e^x(0)$$

$$= e^x \cdot 1$$

$$= e^x .$$

(Tangent: construction of e.

Suppose you have \$1 in a bank account that pays 100% interest per year.

Interest compounded annually: after 1 year you have \$1 + 100% · \$1 = \$2. Write as

$\$(1+1)$.

Interest compounded monthly: after 1 month you have $\$(1+\frac{1}{12})$, so after 12 months you have $\$(1+\frac{1}{12})^{12}$.

Compounded daily: $\$(1+\frac{1}{365})^{365}$.

<u>Continuous compounding</u>: after one year you have $\$\lim_{n\to\infty}(1+\frac{1}{n})^n = \$e$.)