Lecture 1 — Functions

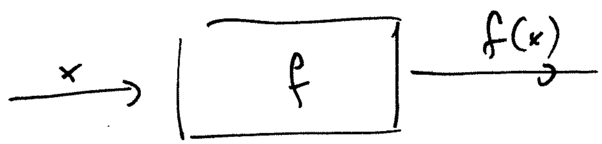
① What is a function?

Def^n A function f is a rule that assigns to each element x in a set A a unique element $f(x)$ in a set B.
Write $f: A \to B$.

The set A is called the domain of f, while B is called the codomain of f.

In this course, we will often have $f: \mathbb{R} \to \mathbb{R}$, or $f: A \to B$ where $A, B \subseteq \mathbb{R}$
↪ subsets.

Ex (i) $f(x) = x^2+1$ is a function $f: \mathbb{R} \to \mathbb{R}$



May also write
$$f: \mathbb{R} \to \mathbb{R}$$
$$x \longmapsto x^2+1$$

We have
$$f(-3) = (-3)^2+1 = 10$$
$$f(\sqrt{2}) = (\sqrt{2})^2+1 = 3$$
$$f(a) = a^2+1$$
$$f(2a-3) = (2a-3)^2+1 = 4a^2-6a+10$$

(ii) $g(x) = 1/x$

This is <u>not</u> a function $\mathbb{R} \to \mathbb{R}$, because $g(0)$ is not defined. We can regard g as a function $g : \mathbb{R} \setminus \{0\} \longrightarrow \mathbb{R}$.

In other words: $\text{dom}(g) = \mathbb{R} \setminus \{0\}$

(iii) $h(x) = (x-1)^2 + 2x$

We have $h = f$

<u>Note</u> the variable x in $f(x)$ is just a placeholder; $f(x) = x^2 + 1$ is the same function as $k(t) = t^2 + 1$.

We often write $y = f(x)$; in this case x is called the in<u>dependent</u> <u>variable</u> and y is called the de<u>pendent</u> <u>variable</u>

(y depends on x via f)

<u>Defⁿ</u> The <u>graph</u> of f is the set of points of the form $(x, f(x))$ where $x \in A$.

② Some standard functions

(i) Polynomials

$$f(x) = x^2+1$$
$$g(x) = 2$$
$$h(x) = x^4 - 7/2\, x + \sqrt{3}$$

These have domain $\mathbb{R}$.
The degree of a polynomial is the highest power of x occurring.

(ii) Rational functions: are quotients of polynomials.

- $f(x) = 3/x+2$ (domain: $\mathbb{R} \setminus \{-2\}$)
- $g(x) = \dfrac{x^2+1}{x^2-1}$ (domain: $\mathbb{R} \setminus \{-1, 1\}$)
- $h(x) = \dfrac{3x-5}{x^2+5}$ (domain: $\mathbb{R}$)

(iii) Radical functions $f(x) = x^{3/2}$ (domain: $[0, \infty)$).

(recall: $x^{n/m}$ means $\sqrt[m]{x^n}$)

Domain: $\begin{cases} [0,\infty) & \text{for } m \text{ even} \\ \mathbb{R} & \text{for } m \text{ odd} \end{cases}$

(iv) Exponential functions $\quad f(x) = e^x$

$g(x) = 2^x$

$h(x) = \left(-\frac{1}{4}\right)^{(x+3)}$

(domain: $\mathbb{R}$)

(v) Trigonometric functions

$\sin(x) \qquad \cos(x) \qquad \tan(x)$

$\csc(x) \qquad \sec(x) \qquad \cot(x)$

(vi) Logarithmic functions $\quad \log_{10}(x), \quad \ln(x)$

(domain: $(0, \infty)$)

③ Range of a function.

Def^n Let $f : A \to B$

$\text{Rng}(f) = \{ f(x) \mid x \in A \}$

set of all elements of the form $f(x)$.

Ex $\quad f(x) = 3x^2 - 5$. Solve $y = f(x)$:

$y = 3x^2 - 5 \Rightarrow y + 5 = 3x^2 \Rightarrow \frac{y+5}{3} = x^2$

$\Rightarrow x = \pm \sqrt{\frac{y+5}{3}}$. Hence for $y \geq -5$ we can find x with $f(x) = y$. So $\text{Rng}(f) = [-5, \infty)$.