uOttawa
L'Université canadienne
Canada's university

ELG6380 Theory of Semiconductor Devices

Assignment 2 Solutions

Due: 16 February 2017

From Pierret, 2nd ed., Advanced Semiconductor Fundamentals

3.4 A certain hypothetical material with cubic crystal symmetry is characterized by the E-k plot sketched in Fig. P3.4.

(a) Which set of holes, band A holes or band B holes, will exhibit the greater [100]-direction (m_{xx}) effective mass? Explain.

(b) Sketch the expected form of the valence-band constant-energy surfaces for the represented cubic material. Assume that the E-k relationship is parabolic (i.e., an ellipsoid of revolution).

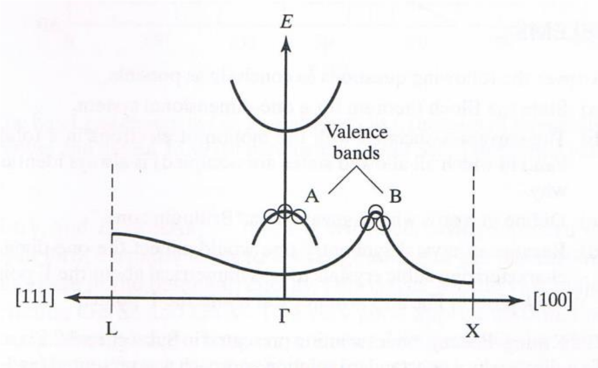


Figure P3.4

3.7 (a) The E-k relationship about the GaAs conduction-band minimum becomes non-parabolic at energies only slightly removed from Ec and is more accurately described by

$$E - E_c = ak^2 - bk^4 \qquad (a > 0, b > 0)$$

What effect will the cited fact have on the effective mass of electrons in the GaAs conduction band? Substantiate your conclusion. (Is your answer here in agreement with the Table 3.1 footnote?)

(b) Electrons in GaAs can transfer from the Γ minimum to the L minima at sufficiently high electric fields. If electrons were to transfer from the Γ minimum to the L minimum shown in Fig. 3.13(d) of the notes/text, would their effective mass increase or decrease? Explain. (The constant-energy surfaces about the L minima are actually ellipsoidal, but for simplicity assume the surfaces to be spherical in answering this question.)

3.8 Like GaAs, GaP crystallizes in the zincblende lattice and the valence band maxima occur at the Γ- point in the first Brillouin zone. Unlike GaAs, the conduction band minima in GaP occur at the X-points in the Brillouin zone.

(a) Where are the X-points located in *k*-space?

(b) Is GaP a direct or indirect material? Explain.

(c) Given that the constant energy surfaces at the X-points are ellipsoidal with $m_l^*/m_0 = 1.12$ and $m_t^*/m_0 = 0.22$, what is the ratio of the longitudinal length to the maximum transverse width of the surfaces?

(d) Picturing only that portion of the constant energy surfaces within the first Brillouin zone, construct a constant-energy surface diagram characterizing the conduction-band structure in GaP.

4.11. Six different silicon samples maintained at 300 K are characterized by the energy band diagrams given in the figure below. Answer the following questions for ANY THREE of the diagrams:

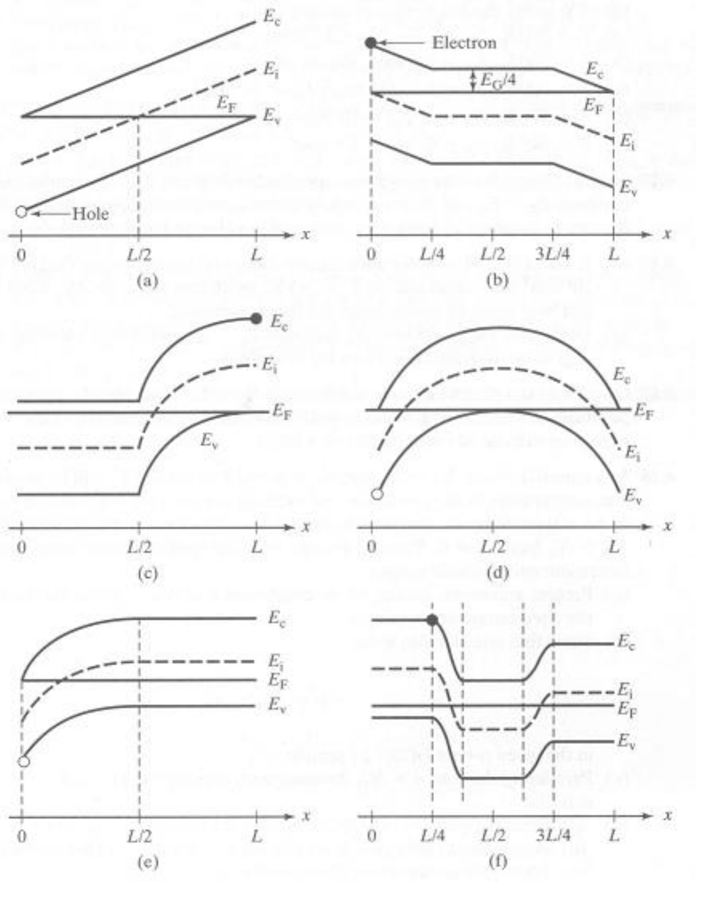
(a) Sketch the electrostatic potential *V* as a function of *x*.

(b) Sketch the electric field $\mathscr{E}$ inside the semiconductor as a function of *x*.

(c) The carrier pictured on the diagram moves back and forth without changing its total energy. Sketch the kinetic energy and potential energy of the carrier as a function of position inside the semi-conductor. Let E_F be the energy reference level.

(d) Roughly sketch *n* and *p* versus *x*.

(e) Is the semiconductor degenerate at any point? If so, where?



4.6. In Si, where $m_{hh}^{*}/m_0 = 0.537$, $m_{lh}^{*}/m_0 = 0.153$, what fraction of the holes are heavy holes? Assume that the 4 K effective masses can be used at any temperature.

4.20. In InSb at 300 K, E_G = 0.18 eV (the smallest band gap of all semiconductor compounds), $m_n^{*}/m_0 = 0.0116$, $m_p^{*}/m_0 = 0.40$ and $n_i = 1.6 \times 10^{16}$ /cm^3.

(a) Would you expect the intrinsic Fermi energy (E_i) in InSb to lie closer to E_c or E_v? Present a qualitative argument that supports your answer – the text relationship for E_i is NOT to be used.

(b) Assuming nondegenerate statistics, determine the positioning of E_i in the InSb band gap at 300 K.

(c) Draw a dimensioned energy band diagram showing the positioning of E_i determined in part (b). (Numerical values for relevant energy differences are noted on a "dimensioned" diagram.) Do you see anything wrong with the part (b) result? Explain.

(d) If something is wrong with the part (b) result, determine the correct positioning of E_i in the InSb band gap.

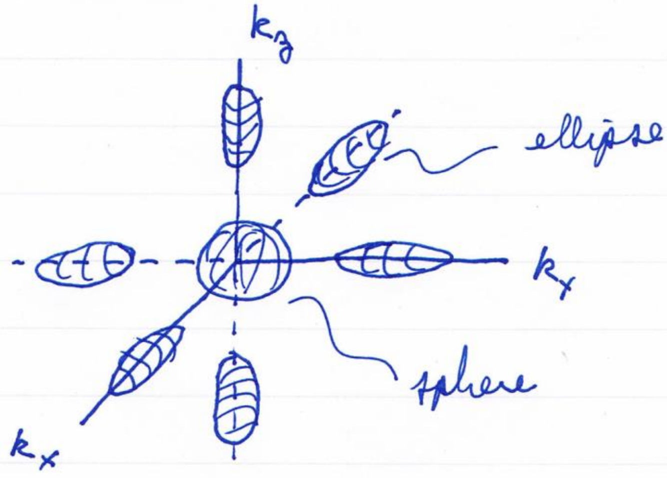
(e) Given an InSb sample doped with 10^{14}/cm^3 donors, what is the approximate positioning of E_F in the sample at 300 K? Please note how you deduced your answer.

(3.4) (a) Since $$m_{xx} = \frac{\hbar^2}{\partial^2 E/\partial k_x^2}$$

$$\propto \frac{1}{\text{curvature of [100] direction plots at maximum}}$$

then band A holes have the greater mass, since the curvature of the B-band at the extremum point is greater than that of the A-band.

(b)



NB. The A band is centred at $k=0$ and was taken to be spherical because the [100] and [111] E-k plots have the same curvature.

NB. The B band extrema locations lie along <100> directions. There are six equivalent <100> directions.

(3.7) The effective mass is given by

(a)

$$m_e^* = \frac{\hbar^2}{\partial^2 E/\partial k^2}$$

Since $E - E_c = a k^2 - b k^4$, we have

$$\partial E/\partial k = 2ak - 4bk^3$$

$$\hookrightarrow \partial^2 E/\partial k^2 = 2a - 12bk^2$$

Thus: $$m_e^* = \frac{\hbar^2}{2a - 12bk^2}$$

Since $a, b > 0$, m_e^* increases as one moves away from the $k=0$ CB minimum, in agreement with the footnote of Table 3.1

(b) The curvature of Γ_6 is clearly greater than the curvature of L_6, therefore their effective masses would increase going from the Γ minimum to the L minimum, in accordance with $m^* \propto \text{curvature}^{-1}$.

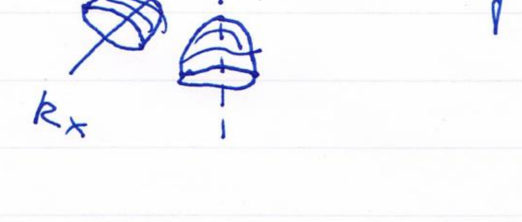
(3.8) (a) The 1ST Brillouin zone for GaP is shown in Fig. 3.12. The X-points are located along <100> directions at the Brillouin zone boundary.

(b) Given that the VB maxima occur at the Γ point, while CB minima occur at the X-points, GaP must be an indirect semiconductor.

(c) Given $m_l^*/m_0 = 1.12$ and $m_t^*/m_0 = 0.22$, equation (3.48) may be written as

$$\frac{\text{longitudinal length}}{\text{transverse width}} = \sqrt{\frac{m_l^*}{m_t^*}} = \sqrt{\frac{1.12}{0.22}} = 2.26$$

(d) From the above, it is clear that the constant energy surface has six ellipsoids, of 2.26 aspect ratio, but that only ½ of each ellipsoid is in the 1ST Brillouin zone.



(4.11) Using $E_{Ref} = E_F$ and assuming the indicated carrier has constant energy E, the detailed plots are on the following pages. The relevant equations are

(a) $V = -\frac{1}{q}(E_c - E_{Ref})$

(b) $\mathcal{E} = -\frac{dV}{dx}$

(c) Electron:

$PE = E_c - E_{Ref}$

$KE = E - E_c$

Hole:

$PE = E_{Ref} - E_v$

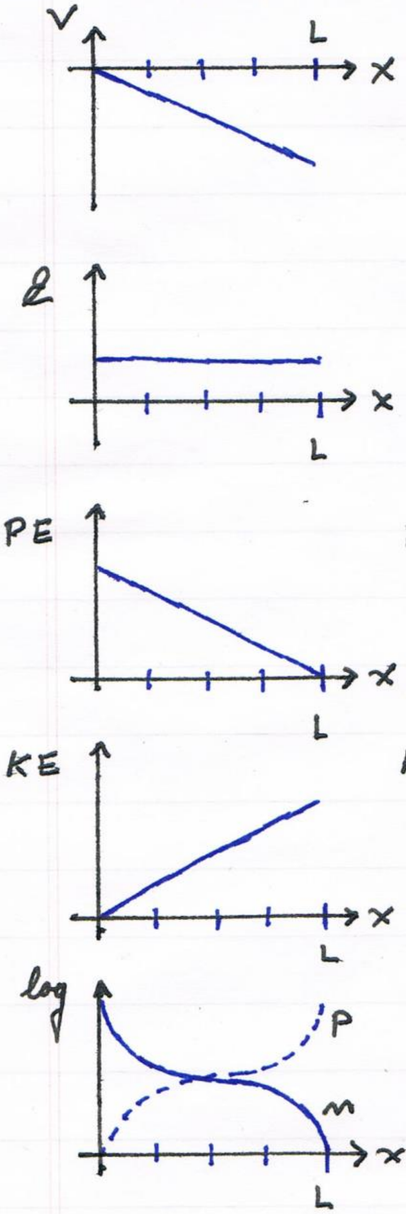
$KE = E_v - E$

(d) $n = n_i e^{(E_F - E_i)/kT}$

$p = n_i e^{(E_i - E_F)/kT}$

(e) Degenerate if E_F is within $3kT$ of a band edge, or is within a band.

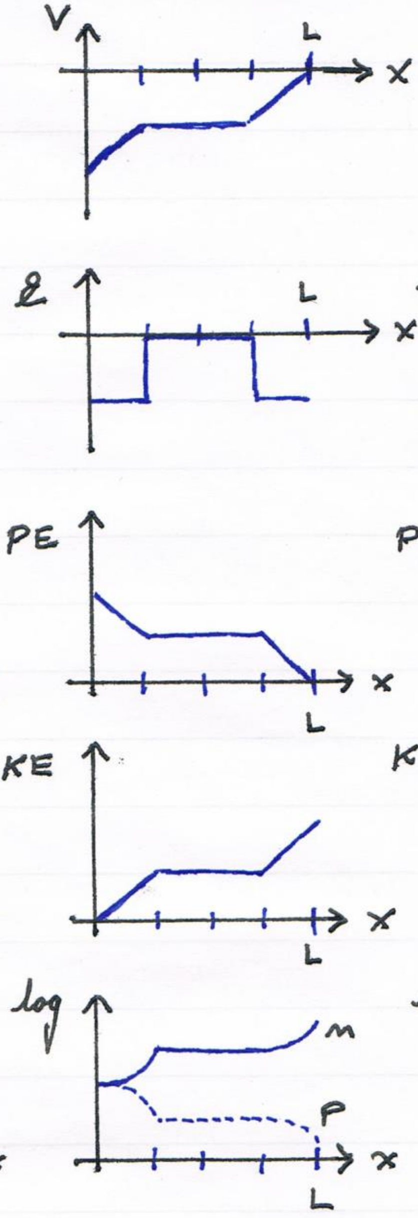
Diagram (a)



Degenerate near

$x = 0, L$

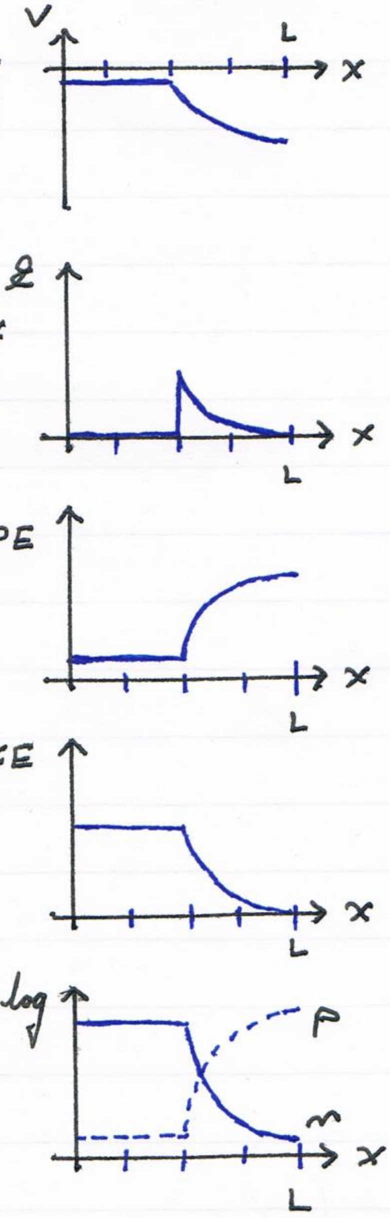
Diagram (b)



Degenerate near

$x = L$

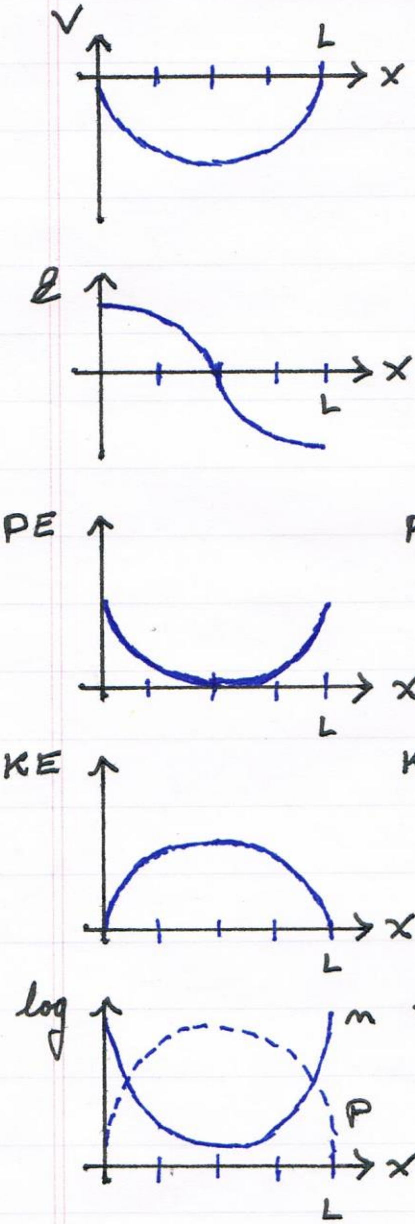
Diagram (c)



Degenerate near

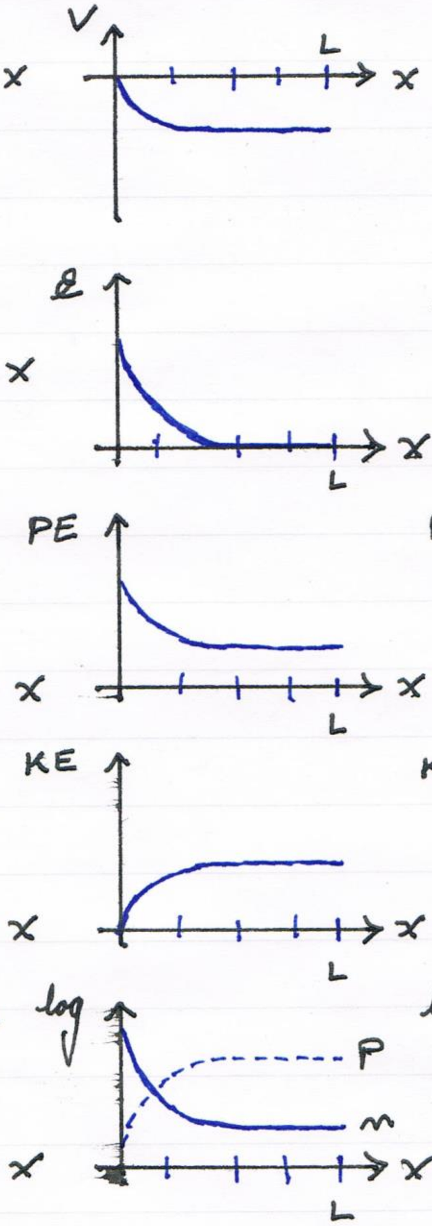
$x = L$

Diagram (d)



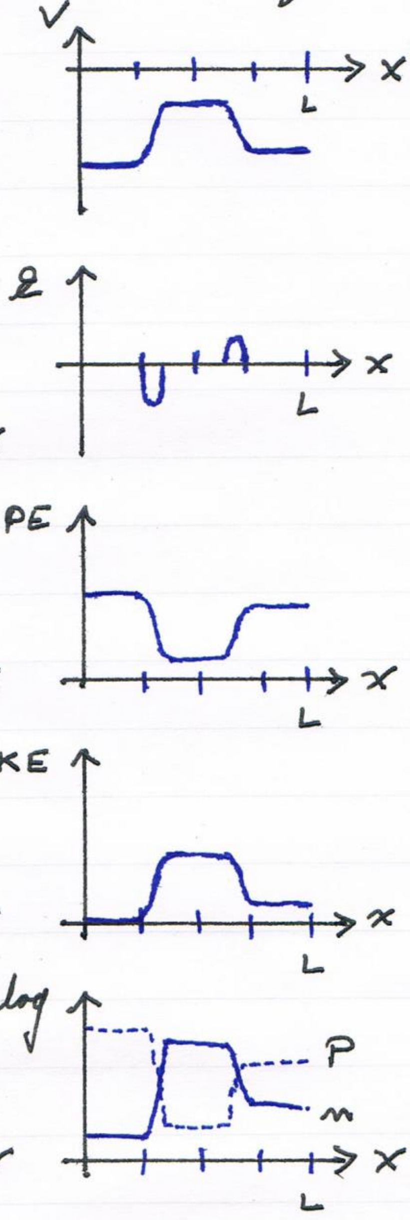
Degenerate near $x = 0, \frac{1}{2}L, L$

Diagram (e)



Degenerate near $x = 0$

Diagram (f)



not degenerate

(4.6) Si: $m^*_{hh}/m_0 = 0.537$, $m^*_{lh}/m_0 = 0.153$

Recall:

$$p = \int_{E_{bottom}}^{E_v} g_v(E)[1-f(E)]\,dE$$

$$g_v(E) = g_v^{hh}(E) + g_v^{lh}(E)$$

Thus:

$$\frac{p_{heavy}}{p} = \frac{\int_{E_{Bottom}}^{E_v} g_v^{hh}(E)[1-f(E)]\,dE}{\int_{E_{Bottom}}^{E_v} g_v(E)[1-f(E)]\,dE}$$

$$= \frac{(m^*_{hh})^{3/2}}{(m^*_{hh})^{3/2} + (m^*_{lh})^{3/2}}$$

– using (4.26) & (4.27)

$$= \frac{(0.537)^{3/2}}{(0.537)^{3/2} + (0.153)^{3/2}} = 0.868$$

$$\frac{p_{light}}{p} = \frac{(m^*_{lh})^{3/2}}{(m^*_{hh})^{3/2} + (m^*_{lh})^{3/2}} = 0.132$$

(4.20) InSb @ 300 K , $E_G = 0.18\,\mathrm{eV}$, $n_i = 1.6 \times 10^{16}\ \mathrm{cm}^{-3}$

$m_n^* / m_0 = 0.0116$, $m_p^* / m_0 = 0.40$

(a) $m_n^* \ll m_p^*$ means that the conduction band density of states is smaller than the valence band density of states. The intrinsic Fermi level is the energy the Fermi level would assume if there were an equal concentration of electrons & holes in the material. Thus, to counter the low CB DOS, E_i should be nearer the CB edge

(b) assuming non-degenerate statistics, E_i is given by (4.86) as

$$E_i = \tfrac{1}{2}(E_c + E_v) + \tfrac{3}{4} kT \ln \left(m_p^* / m_n^* \right)$$

assuming $E_v = 0$, we find

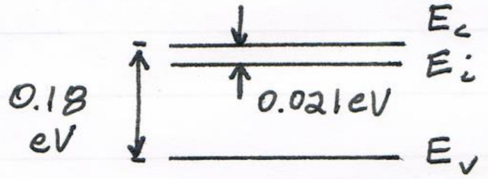
$$E_i = \tfrac{1}{2}(0.18) + (0.75)(8.617 \times 10^{-5})(300) \times \ln(0.4/0.0116)$$

$$= 0.09 + 0.0686$$

$= 0.1586\ \mathrm{eV}$ above the VB edge

(or: $0.0214\ \mathrm{eV}$ below the CB edge)

(c) Based on (b), the energy band diagram is



$$E_c - E_i = 0.021 \text{ eV}$$

$$3kT = 3(8.617\times10^{-5})(300)$$
$$= 0.0776 \text{ eV}$$

$$\therefore \quad 3kT > E_c - E_i$$

$\Rightarrow$ a degenerate semiconductor
But, we assumed it was non-degenerate!

(d) For an intrinsic material, even if degenerate, $n = p$. Since $E_i - E_v > 3kT$, we can use the non-degenerate relation for p...

$$p = N_v \, e^{(E_v - E_i)/kT}$$

... but we need to use the general relation for n:

$$n = N_c \, \mathcal{F}_{1/2}(\eta_c)$$

where $\eta_c = (E_F - E_c)/kT = (E_i - E_c)/kT$

Equating...

$$N_c \mathcal{F}_{1/2}(\eta_c) = N_v e^{(E_v - E_i)/kT}$$

$$= N_v e^{-E_G/kT} e^{(E_c - E_i)/kT}$$

$$\Rightarrow \mathcal{F}_{1/2}(\eta_c) = \underbrace{(N_v/N_c)}_{\left(m_p^*/m_n^*\right)^{3/2}} e^{-E_G/kT} e^{-\eta_c}$$

Thus:

$$\mathcal{F}_{1/2}(\eta_c) = \left(0.40/0.0116\right)^{3/2} e^{-0.18/0.0259} e^{-\eta_c}$$

$$= 0.194\, e^{-\eta_c}$$

$$\Rightarrow \mathcal{F}_{1/2}(\eta_c)\, e^{\eta_c} = 0.194$$

Consulting the tabulated values of Fig. 4.15...

η	$\mathcal{F}_{1/2}(\eta)\, e^{\eta}$	$\ln[\mathcal{F}_{1/2}(\eta)\, e^{\eta}]$
0	0.7652	-0.2676
-1	0.1206	-2.115

Computing the equation of a straight line...

$$\ln[\mathcal{F}_{1/2}(\eta)e^{\eta}] = m\eta + b$$

… we find: $m = \dfrac{-0.2676 - (-2.115)}{0 - (-1)} = 1.848$

and: $-0.2676 = 0 + b$

Hence: $\ln(0.194) = 1.848\eta_c - 0.2676$

$$\Rightarrow \eta_c = \frac{\ln(0.194) + 0.2676}{1.848} = -0.743$$

Finally, $\eta_c = (E_i - E_c)/kT$

$$\Rightarrow E_i = E_c - 0.743\,kT$$

$$= 0.18 - (0.743)(8.617\times10^{-5})(300)$$

$$= 0.18 - 0.019$$

$= 0.161$ eV above the VB edge

or: 0.019 eV below the CB edge

(e) $n_i = N_c \mathcal{F}_{1/2}(\eta_c)$

where $N_c = 2\left(\dfrac{2\pi m_n^* kT}{h^2}\right)^{3/2}$

$$N_c = 2\left[\frac{(2\pi)(0.0166)(9.11\times10^{-31})(8.617\times10^{-5})(300)}{(6.63\times10^{-34})^2}\ \underline{\times(1.6\times10^{-19})}\right]^{3/2}$$

$$= 3.123\times10^{22}\ \text{cm}^{-3}$$

Thus: $n_i = (3.123\times10^{22})(0.194)e^{+0.743}$

$$= 1.27\times10^{22}\ \text{cm}^{-3}$$

Since $1.27\times10^{22} \gg 10^{14}$

$\therefore\quad E_F \simeq E_i$