MATH 1007G **Term test #3 – solution** **6 March 2017**

This test was written in room: ________________ Total marks: **28** marks

This test is closed book. No calculators or electronic aids are permitted. **Please supply your answers on this sheet.**

PLEASE PRINT ________________________________ ______________

First name Last name Student number

Please show your work where appropriate! TA's have extra paper if you need it. Test duration: 50 minutes.

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1. Let $f(x) = \begin{cases} \sqrt{4-x} + 1, & \text{if } x \le 4 \\ 9 - ax, & \text{if } x > 4 \end{cases}$

a. [3] Determine $\lim\limits_{x\to4^-} f(x)$ and $\lim\limits_{x\to4^+} f(x)$

$\lim\limits_{x\to4^-} f(x) = \lim\limits_{x\to4^-}\left[\sqrt{4-x}+1\right] \xrightarrow{D.S.} \boxed{1} \left(= f(4)\right)$

$\lim\limits_{x\to4^+} f(x) = \lim\limits_{x\to4^+}\left[9-ax\right] \xrightarrow{D.S.} \boxed{9-4a}$

b. [2] For which value of a is f continuous at $x = 4$?

Set $\lim\limits_{x\to4^+} f(x) = \lim\limits_{x\to4^-} f(x) = f(4)$

$9-4a = 1$

$-4a = 1-9 = -8$

$\boxed{a=2}$

2. [4] Fill in the blanks

a. $\lim\limits_{x\to0^+}(\log_5 x) = \underline{-\infty}$

b. $\lim\limits_{x\to\infty}\left(\dfrac{2x^3-3x^2+7}{12-5x-6x^3}\right) = \underline{-1/3}$

c. $\lim\limits_{x\to+\infty}\left(e^{1/x}\right) = \underline{1}$

d. $\lim\limits_{x\to-\infty}\tan^{-1} x = \underline{-\pi/2}$

3. [3] Using the fundamental definition of the derivative (DEF. 1 or DEF. 2), determine the derivative of $f(x) = x^2 - x$.

DEF 1 : $f'(x) = \lim\limits_{h\to0}\left(\dfrac{f(x+h)-f(x)}{h}\right)$

DEF 2 : $f'(x) = \lim\limits_{z\to x}\left(\dfrac{f(z)-f(x)}{z-x}\right)$

$f'(x) = \lim\limits_{h\to0}\left[\dfrac{f(x+h)-f(x)}{h}\right]$

$= \lim\limits_{h\to0}\left[\dfrac{(x+h)^2-(x+h)-x^2+x}{h}\right]$

$= \lim\limits_{h\to0}\left[\dfrac{\cancel{x^2}+2xh+h^2-\cancel{x}-h-\cancel{x^2}+\cancel{x}}{h}\right]$

$\to \lim\limits_{h\to0}\left[\dfrac{2xh+h^2-h}{h}\right] = \lim\limits_{h\to0}\left[\dfrac{\cancel{h}(2x+h-1)}{\cancel{h}}\right]$

$= \lim\limits_{h\to0}\left[2x+h-1\right] \xrightarrow{D.S.} \boxed{2x-1}$

Same result using DEF. 2 : $f'(x) = \lim\limits_{z\to x}\left[\dfrac{f(z)-f(x)}{z-x}\right]$

4. Determine the following limits if they exist. If a limit does not exist, then determine whether it is $+\infty$, $-\infty$, or neither.

a. [3] $\lim\limits_{x\to-2^+}\dfrac{2x}{x+2} \xrightarrow{D.S.} \dfrac{2(-2^+)}{-2^++2} = \dfrac{-4^+}{0^+} \to$ infinite limit... $+\infty$ OR $-\infty$? ...

$\dfrac{-4^+}{0^+} \Rightarrow \dfrac{(-)}{(+)} = (-) \Rightarrow \boxed{-\infty}$

... Question 4 continues on other side...

b. [3] $\lim_{x\to -4^+} \frac{2x}{x^2-16} \xrightarrow{D.S.} \frac{2(-4^+)}{(-4^+)^2-16} = \frac{-8^+}{16^- - 16} = \frac{-8^+}{0^-} \longrightarrow$

$\ldots \longrightarrow$ infinite limit...

$+\infty$ OR $-\infty$? ...

$$\frac{-8^+}{0^-} \Rightarrow \frac{(-)}{(-)} = (+) \Longrightarrow \boxed{+\infty}$$

c. [4] $\lim_{x\to +\infty} 6x - \sqrt{36x^2+8x} \xrightarrow{D.S.} \infty - \infty$

$$\ldots = \lim_{x\to +\infty}\left[\left(6x - \sqrt{36x^2+8x}\right)\cdot \frac{6x+\sqrt{36x^2+8x}}{6x+\sqrt{36x^2+8x}}\right]$$

$$\ldots = \lim_{x\to +\infty}\left[\frac{36x^2-36x^2-8x}{6x+\sqrt{36x^2+8x}}\right] = \lim_{x\to +\infty}\left[\frac{-8x}{6x+\sqrt{x^2}\left(\sqrt{36+8/x}\right)}\right]$$

$$\ldots = \lim_{x\to +\infty}\left[\frac{-8\cancel{x}}{\cancel{x}\left(6+\sqrt{36+8/x}\right)}\right] = \lim_{x\to +\infty}\left[\frac{-8}{6+\sqrt{36+8/x}}\right]$$

$$\xrightarrow{D.S.} \frac{-8}{6+\sqrt{36+\frac{8}{\infty}}} = \frac{-8}{6+\sqrt{36}} = -\frac{8}{12} = \boxed{-2/3}$$

d. [3] $\lim_{x\to +\infty}\left(\frac{-4x^3+7x-21}{2-4x-6x^2}\right) \xrightarrow{D.S.} \frac{-\infty}{-\infty}$

$$= \lim_{x\to +\infty}\left(\frac{x^3(-4+7/x^2-21/x^3)}{x^2(2/x^2-4/x-6)}\right) = \lim_{x\to +\infty}\left(\frac{-4+7/x^2-21/x^3}{-6-4/x+2/x^2}\cdot x\right) \longrightarrow$$

$\ldots \longrightarrow$ infinite limit ...

$\ldots +\infty$ OR $-\infty$?

$$= \lim_{x\to +\infty}\left(\frac{-4+7/x^2-21/x^3}{-6-4/x+2/x^2}\cdot x\right) = \lim_{x\to +\infty}\left(2/3\cdot x\right) \longrightarrow \boxed{+\infty}$$

e. [3] $\lim_{x\to 3}\left(\frac{\sin(x-3)}{x^2-2x-3}\right) \xrightarrow{D.S.} \frac{\sin(3-3)}{3^2-2(3)-3} = \frac{0}{0}$

$$= \lim_{x\to 3}\left[\frac{\sin(x-3)}{(x+1)(x-3)}\right] = \lim_{x\to 3}\left[\frac{1}{x+1}\cdot\frac{\sin(x-3)}{x-3}\right] =$$

$$\ldots = \lim_{x\to 3}\left[\frac{1}{x+1}\right]\cdot \lim_{x\to 3}\left[\frac{\sin(x-3)}{x-3}\right].$$

but $(x-3)\to 0$ as $x\to 3$, so:

$$\ldots = \lim_{x\to 3}\left[\frac{1}{x+1}\right]\cdot \underbrace{\lim_{(x-3)\to 0}\left[\frac{\sin(x-3)}{x-3}\right]}_{\to 1} = \lim_{x\to 3}\left[\frac{1}{x+1}\right] \xrightarrow{D.S.} \frac{1}{3+1} = \boxed{\frac{1}{4}}$$