MAT1320E

Dr. Fink - practise sheet for the final exam

1. Compute the following limits.

1. $\lim_{x\to 4} \frac{\ln(x)-\ln(4)}{x-4}$
2. $\lim_{x\to 5^+} \frac{x+1}{x-5}$
3. $\lim_{x\to -2} \frac{x+2}{x^3+8}$
4. $\lim_{x\to 0} \sqrt{9+h} - 3h$
5. $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3}$
6. $\lim_{t\to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t}$
7. $\lim_{x\to\infty}(\sqrt{9x^2+x} - 3x)$
8. $\lim_{t\to\infty} \frac{\sqrt{x+3x^2}}{4x-1}$
9. $\lim_{x\to\infty} \frac{1+x^6}{x^4+1}$
10. $\lim_{x\to -\infty} \frac{2x^2+x-1}{x^2+x-2}$
11. $\lim_{x\to -\infty} \frac{2x^3+x-1}{x^2+x-2}$
12. $\lim_{x\to\infty} \frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}$
13. $\lim_{x\to 0} \frac{\ln(x)}{x}$
14. $\lim_{x\to 0} \frac{\sinh(x)-x}{x^3}$
15. $\lim_{x\to\infty} \frac{e^{u/10}}{u^3}$
16. $\lim_{x\to\infty} x^3 e^{-x^2}$
17. $\lim_{x\to\infty}(x - \ln(x))$
18. $\lim_{x\to 0} \frac{e^x-e^{-x}-2x}{x-\sin(x)}$
19. $\lim_{x\to 1^+} x^{1/(1-x)}$
20. $\lim_{x\to\infty} x^{1/x}$
21. $\lim_{x\to 0^+}(4x+1)^{\cot(x)}$

2. Find the first and second derivatives of the following functions.

By using the definition of the derivative:

1. $f(x) = x + 3$
2. $f(x) = \frac{2}{x+4}$

By using derivative rules:

1. $f(x) = x^3 + 3x + 1$
2. $y(t) = \sqrt{3t^2 + \cos(t)}$
3. $f(z) = e^{z^2}\sqrt{z+3}$
4. $f(x) = \frac{3+x^2}{5x^2+1}$

5. $f(x) = \frac{\sqrt{3x+1}}{6x^2+1}$
6. $f(x) = \sin(3x^2 e^x)$
7. $f(x) = \frac{\sin(3x)}{\cos(3x)}$
8. $f(x) = \frac{\sin(2x)+1}{\cos(4x)}$
9. $f(t) = \sqrt{\frac{3}{t} + 2t}$
10. $f(y) = y^{500} \cdot e^{y^2}$
11. $f(x) = e^{\cos(5x^2)}$
12. $f(t) = \sqrt[4]{\arctan(t^3) \cdot \sin(t)}$
13. $g(x) = \frac{\sin(x)+3x^3}{\arcsin(x)+7\cos(x)}$
14. $u(y) = \ln(y + e^y)$
15. $s(v) = \cos(v^3 + \tan(v))$
16. $p(x) = \sec(x) \cdot e^{x^3+4\cos(x)}$

3. Linearization.

Approximate the following functions using the linearization.

1. $f(x) = \ln(x)+1$, compute $f(0.1)$ using the linearization at $a = 0$.
2. $f(x) = \sqrt{x}+1$, compute $f(1.01)$ using the linearization at $a = 1$.
3. $f(x) = \frac{1}{1+\sin x}$, compute $f(\Pi + 0.1)$ using the linearization at $a = \Pi$.

4. Find the derivate of the following functions:

1. $\int_0^{3x} \frac{u^2-1}{u^2+1}$
2. $\int_1^{x^2} e^{t^2} dt$
3. $\int_3^{\sqrt{x}} \frac{t^2+1}{2t^4+5t^2+1} dt$
4. $\int_{\sqrt{x}}^{2x} \arctan(t) dt$
5. $\int_0^{x^2} \sin(t^3 + 1) dt$
6. $\int_2^{t^2} \sqrt{4x + 7} dx$
7. $\int_{\sin(t)}^{t^2} \frac{x+3}{\cos(x)}$

5. Differentiate the following equations and find the equations of the tangent lines at the given points.

1. $x^2-4xy+y^2 = 4$, tangent line at $(1, 1)$
2. $\frac{x^2}{x+y} = y^2 + 1$
3. $x^y + y = x$, tangent line at $(a, 0)$, where you have to determine the value for a first such that it lies on the curve.
4. $x \cdot e^y = x - y$, slopes at the point $(1, y)$ for appropriate y.
5. $2x^2+xy-y^2 = 2$, tangent line at $(1, 1)$
6. $y^2 = x^3+x+7$, slopes at all points with $x = 1$. Remark: this type of curve is called an **elliptic curve** and is important in number theory and cryptography.

6. Related Rates.

Textbook Chapter 3.9 exercises 2-7, 14-16.

7. Approximate Integration

Set up the Riemann Sums for the following areas. Do the same with the Trapezoidal rule for the given number of steps and Simpson's rule. Recall that the number of approximation values must be even in that case. Double the given value if needed.

1. $f(x) = \frac{2x}{2x^2+1}$ from 1 to 3, take 4 steps.
2. $f(x) = x^2 + \sqrt{1+2x}, \quad 4 \leq x \leq 7,$ take 9 steps.
3. $f(x) = \sqrt{sin(x)}, \quad -\pi/2 \leq x \leq \pi/2,$ take 8 steps.
4. $f(x) = e^{5x}, \quad 0 \leq x \leq 5,$ take 5 steps.
5. $f(x) = \tan(4x), \quad 0 \leq x \leq \pi/6,$ take 3 steps.

8. Compute the following integrals:

1. $\int_1^9 \sqrt{x}dx$
2. $\int \frac{1+x}{1+x^2}dx$
3. $\int \frac{z^2}{z^3+1}dz$
4. $\int \frac{x}{1+x^4}dx$
5. $\int (x-1)e^{(x-1)^2}dx$
6. $\int x^2\sqrt{2+x}dx$
7. $\int \frac{\sin(2x)}{1+\cos^2(x)}dx$
8. $\int \sec^2(\theta) \cdot \tan^3(\theta)d\theta$
9. $\int \frac{e^u}{(1-e^u)^2}du$
10. $\int \frac{\sin(x)}{1+\cos^2(x)}dx$
11. $\int_0^{\pi/6} \frac{\sin(t)}{\cos^2(t)}dt$
12. $\int \cos^3(\theta)\sin(\theta)d\theta$
13. $\int 5^t \cdot \sin(5^t)dt$
14. $\int \cos(1+5t)dt$
15. $\int_e^{e^4} \frac{dx}{x\cdot\sqrt{\ln(x)}}dx$
16. $\int_0^1 \frac{dx}{(1+\sqrt{x})^4}dx$
17. $\int_0^1 x \cdot e^{-x^2}dx$

9. More integrals.

1. $\int x \cdot \cos(5x)dx$
2. $\int x^2e^{-x}dx$
3. $\int (\ln(x))^2dx$
4. $\int t^4 \cdot \ln(t)dt$

5. $\int \ln(\sqrt{x})dx$
6. $\int_0^{2\pi} x^2 \cdot \sin(x)dx$
7. $\int_1^5 \frac{M}{e^M}dM$
8. $\int_1^2 x^4 \cdot (\ln(x))^2 dx$
9. $\int e^{\cos(t)} \cdot \sin(2t)dt$
10. $\int x \cdot \ln(1+x)dx$
11. $\int_e^{e^2} \frac{5(\ln(x))^{1/5}}{x}dx$
12. $\int \frac{x^3}{\sqrt{x^2+4}}dx$
13. $\int \frac{x^3}{\sqrt{x^2-4}}dx$
14. $\int \frac{4+x}{(1+2x)(3-x)}dx$
15. $\int \frac{x^3+1}{x^3-3x^2+2x}dx$
16. $\int \frac{4y^2-7y-12}{y(y+2)(y-3)}dy$
17. $\int \frac{4+x}{(1+2x)(3-x)}dx$
18. $\int \frac{4x}{x^3+x^2+x+1}dx$
19. $\int \frac{x^3+6x-2}{x^4+6x^2}dx$
20. $\int \frac{\sin(x)}{\cos^2(x)+\cos(x)+1}dx$

10. And more integrals.

1. $\int_0^8 \sin(x)dx$
2. $\int_1^9 \sqrt{x}dx$
3. $\int_0^1 (x^e + e^x)dx$
4. $\int_1^{\sqrt{x}} \frac{z^2}{z^4+1}dz$
5. $\int_1^8 x^{-2/3}dx$
6. $\int_0^4 2^s ds$
7. $\int_1^{18} \sqrt{\frac{3}{z}}dz$
8. $\int_0^1 (1+r)^3 dr$
9. $\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}}dx$
10. $\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{8}{1+x^2}dx$

11. Curve Sketching Sketch the following curves, using the guidelines discussed in class and in the textbook Chapter 4.5.

1. $y = \frac{x}{\sqrt{x^2+1}}$
2. $y = x - 3x^{1/3}$
3. $y = \sin^3(x)$
4. $y = x + \cos(x)$
5. $y = \frac{x^2+5x}{25-x^2}$
6. $y = \frac{x-x^2}{2-3x+x^2}$

12. Bonus topic: Optimization

Since we haven't done this in class yet, the optimization question on the exam will give bonus points.

Exercises from Chapter 4.7: 2-18