1. Compute the following integrals.

[3] (a) $\int_0^1 y(1+2y)^2 \, dy$

---

[3] (b) $\int_e^{e^4} \frac{1}{x\sqrt{\ln x}} \, dx$

[3] (c) $\int_{-2}^{2} g(x)\, dx$ where $g(x) = \begin{cases} 2 & \text{if } -2 \leq x \leq 0 \\ 4 - x^2 & \text{if } 0 < x \leq 2 \end{cases}$

---

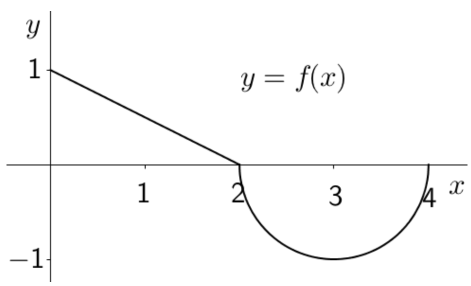
[3] (d) $\int x^3\sqrt{x^2+1}\, dx$

[6] **2.** **True or False**. For each statement indicate whether it is True or False by circling **T** for True or **F** for False. You do not need to justify your answers. Each question is worth 1 point.

T or **F** If $\int_0^9 g(x)\,dx = 4$ then $\int_0^3 xg(x^2)\,dx = 4$.

T or **F** $\int_1^5 x^2 - 1\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(1+\frac{4i}{n}\right)^2 - 1\right)\frac{4}{n}$

T or **F** For the function f whose graph is drawn below, $\int_0^4 f(x)\,dx = 1 + \frac{\pi}{2}$.

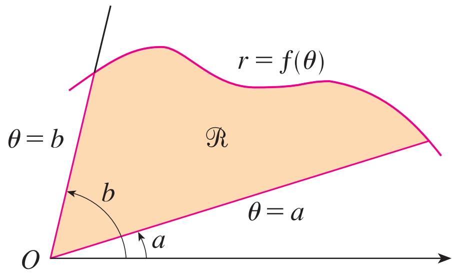


T or **F** If f is continuous on the interval $[a, b]$ then $\frac{d}{dx}\left(\int_a^b f(x)\,dx\right) = f(x)$.

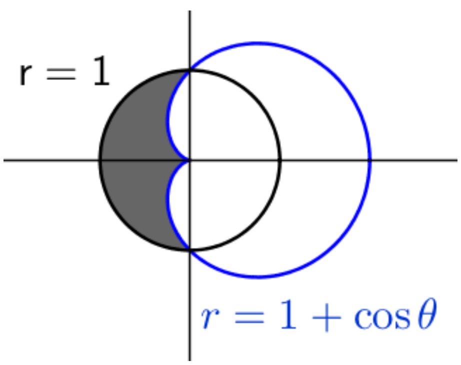
T or **F** $\int_{-1}^{2} x^{-2}\,dx = -x^{-1}\Big|_{-1}^{2} = -\frac{1}{2} - \left(-\frac{1}{-1}\right) = -\frac{3}{2}$.

T or **F** $\frac{2}{3} \leq \int_0^2 \frac{1}{\sqrt{1+x^3}}\,dx \leq 2$.

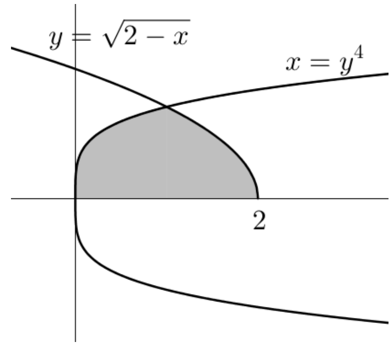
[2] **3.** (a) Write down the integral that gives the **area** of a polar region $\mathcal{R}$ bounded by the polar curve $r = f(\theta)$, for $\theta \in [a, b]$.



[4] (b) Find the area of the region that lies inside the circle $r = 1$ and outside the curve $r = 1 + \cos\theta$. The two polar curves are shown below and the region between them is shaded.
(Simplify your final answer to the form $a + b\pi$ where a and b are some real numbers.)



4. in this question we consider the region shaded in the diagram, which is enclosed by the curves $x = y^4$ and $y = \sqrt{2-x}$, around the x-axis.



[3] (a) **Set-up, but do not evaluate**, an integral that represents the **area** of the region.

[4] (b) **Set-up, but do not evaluate**, an integral that represents the **volume** of the solid obtained by rotating the region about the line $x = -1$.

(To receive full credit you must sketch a diagram, sketch a typical washer, and label all important quantities in your diagram. You may use the Shell method if you prefer, just make sure you provide the relevant sketch and label all important quantities.)

[5] **5.** Show $\int_a^b x\,dx = \frac{b^2 - a^2}{2}$ by **using the definition of the definite integral** as a limit of a Riemann sum.
(You may find some of the following formulae useful:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2.)$$

(Begin by finding explicit expressions for Δx and x_i in terms of a, b, n and i.)

[4] **6.** Are the following two areas equal? Explain your reasoning.