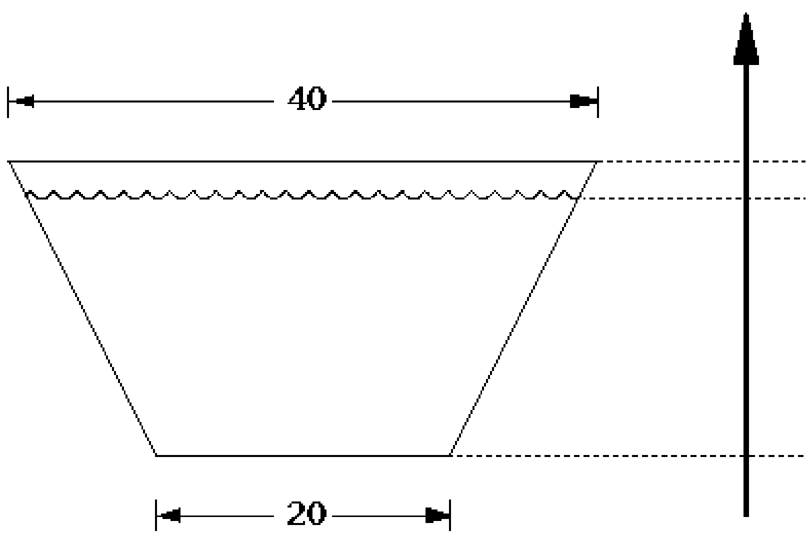
Assignment 4 - Winter 2018

[PRINT]

MAT1322 - Winter 2018, Assignment 4 - Winter 2018
Ahmed Magzoub, 2/2/18 at 11:31:20 AM EST

Question 1: *Score 0/1*

A vertical dam has the form of an isosceles trapezoid with horizontal sides parallel. The dam is 25 m high, 20 m in its lower part and 40 m in its upper part. Finally, the dam retains 15 m of water, as indicated in the figure below.



a) Let y denote the height in meters measured from the base of the dam. The hydrostatic force exerted by the water on the portion of the door comprised between y m and $y + \Delta y$ m is approximately $p(y)\,\Delta y\ \ N$. What is $p(y)$? Note that the density of water is $\rho = 1000\ \ Kg/m^3$ and the acceleration due to gravity on the earth's surface is $g = 9.8\ \ m/s^2$. Express your answer as a formula.
Answer:

Your response	Correct response
No answer	**9800*(15-y)*(20 + 2*(10)*y/(25))**

Grade: 0/1.0

b) In Newtons, what is the total hydrostatic force exerted on the dam? Give the answer correct to 3 significant digits.
Answer:

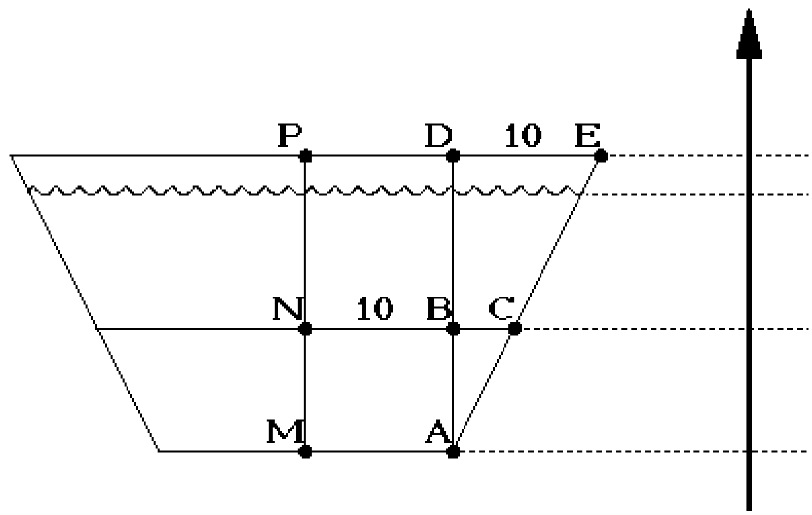
Your response	Correct response
	26,460,000 ± 1.0%

Grade: 0/1.0

Total grade: 0.0×1/2 + 0.0×1/2 = 0% + 0%

Comment:

The figure below shows the dam.



Given the dam has the form of an isosceles trapezoid, it is symmetric with respect to the line MP that joins the midpoint M of its lower horizontal side and the midpoint P of its upper horizontal side. Its width at the height y is $2\,|NC| = 2\,|NB| + 2\,|BC| = 20 + 2\,|BC|$. Trivially, we know the height of the dam is $|AD| = 25$ and we have $|DE| = \frac{40-20}{2} = 10$. Since the triangles $\triangle\ ABC$ and $\triangle\ ADE$are similar and $|AB| = y$, we deduce that

$$|BC| = \frac{|AB| \cdot |DE|}{|AD|} = \frac{10y}{25}.$$

Therefore, the width of the dam at height y is

$20 + 2\,|BC| = 20 + \frac{2\ (10y)}{25} = 20 + (4/5)\,y,$

and the portion of the dam comprised between y m and $y + \Delta y$ m has an area of

$(20 + (4/5)\,y)\Delta y \quad \mathrm{m}^2.$

At height y, the column of water is at $(15 - y)$ meters, thus it exerts a pressure of $1000g\,(15 - y) = 9800\,(15 - y)\ N\,/\,m^2$. The hydrostatic force on that portion of the dam is

$(\text{pressure}) \times (\text{area}) \cong 9{,}800\,(15 - y)\,(20 + (4/5)\,y)\Delta y.$

Therefore,

$$p\,(y) = 9{,}800\,(15 - y)\,(20 + (4/5)\,y).$$

As the submerged portion of the dam is comprised between $y = 0$and $y = 15$, the total hydrostatic force that is exerted on it is:

$$\int_0^{15} p(y)\,dy = 9{,}800\int_0^{15} (15 - y)\,(20 + (4/5)\,y)\;dy$$
$$= 9{,}800\left[300\,y - \frac{4}{15}\,y^3 - 4\,y^2\right]_0^{15} \cong 26{,}460{,}000.$$

Question 2: *Score 0/1*

For which value of r does the function $y = e^{rx}$ satisfy the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 5y\ ?$$

Answer:

Your response	Correct response
	5

Grade: 0/1.0

Total grade: 0.0×1/1 = 0%

Comment:

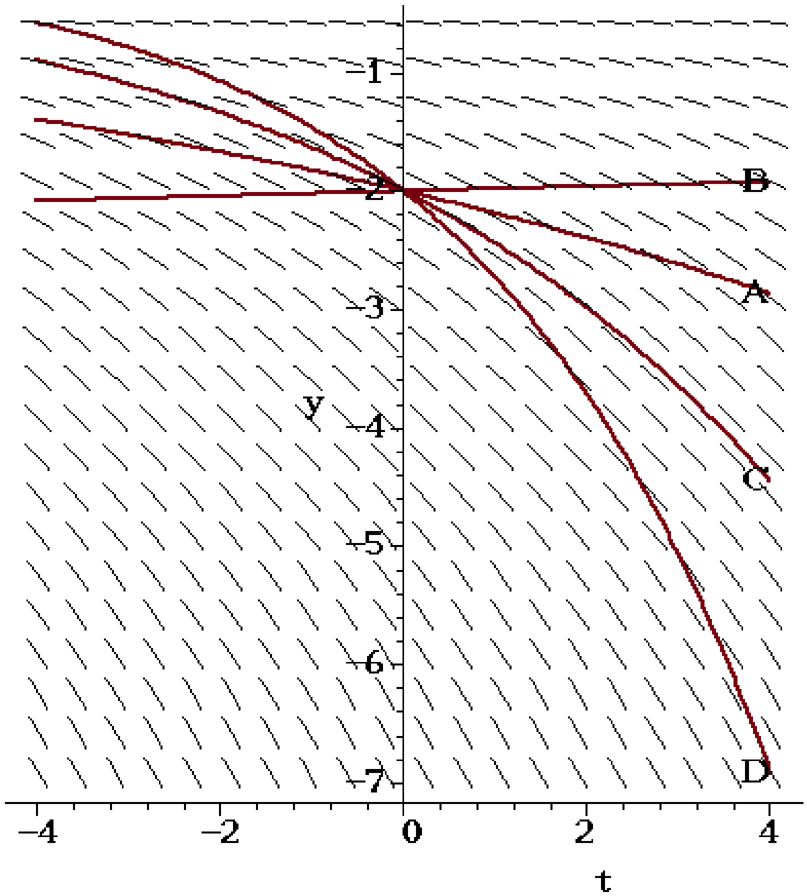
If we substitute $y = e^{rx}$ into the differential equation, we get $re^{rx} = 5e^{rx}$. Thus $r = 5$ is the only value that gives a solution.

Question 3: *Score 0/1*

We have sketched the slope field for the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = F(t, y)$$

in the graphic below. Which of the four curves (labeled A, B, C, D) that we have drawn over the slope field could be the solution to this differential equation with the initial condition $y(0) = -2$?



Answer:

Your response	Correct response
	C

Grade: 0/1.0

Total grade: 0.0×1/1 = 0%

Comment:

The little straight lines indicate the slopes of the solutions as they pass that point. A solution curve which is close to a little straight line must have a tangent line which is almost parallel to the little straight line. If a solution curve touches a little straight line, it must be tangent to it.

Question 4: *Score 0/1*

Consider the initial-value problem

$$y\prime = 0.1xy\,, \quad y(\text{-}2) = 0.2\,.$$

(a) Use Euler's method to estimate $y(\text{-}1)$ with step size $h = 0.5$.

Give your approximation for $y(\text{-}1)$ with a precision of ± 0.01 .

$y(\text{-}1) =$

Your response	Correct response
	0.1665±0.01

Grade: 0/1.0

(b) Use Euler's method to estimate $y(\text{-}1)$ with step size $h = 0.25$.

Give your approximation for $y(\text{-}1)$ with a precision of ± 0.01 .

$y(\text{-}1) =$

Your response	Correct response
	0.169409±0.01

Grade: 0/2.0

Total grade: 0.0×1/3 + 0.0×2/3 = 0% + 0%

Comment:

The differential equation we must solve is of the form $y\prime = F(x, y)$ where $F(x, y) = 0.1xy$.

(a) For the step size $h = 0.5$, Euler's method consists of letting $x_0 = \text{-}2$ and $y_0 = 0.2$ (since $y(\text{-}2) = 0.2$), and defining recursively

$$x_{n+1} = x_n + h = x_n + 0.5$$

and

$$\begin{aligned} y_{n+1} &= y_n + F(x_n, y_n) h \\ &= y_n + (0.1 x_n y_n)(0.5) \\ &= y_n + 0.05 x_n y_n \\ &= y_n (1 + 0.05 x_n) , \end{aligned}$$

for each $n = 0, 1, 2, \ldots$ so that we have $y(x_n) \cong y_n$ for every n .

Since we want to approximate $y(\text{-}1)$, we stop once we arrive at $x_n = \text{-}1$.

We find:

$x_1 = \text{-}2 + 0.5 = \text{-}1.5$, $y_1 = 0.2(1 + (0.05)(\text{-}2)) = 0.18$

$x_2 = x_1 + 0.5 = \text{-}1$, $y_2 = 0.18(1 + (0.05)(\text{-}1.5)) = 0.1665$

Hence the answer is $y(\text{-}1) \cong 0.1665$.

(b) For a step size of $h = 0.25$, once again, we let $x_0 = \text{-}2$ and $y_0 = 0.2$ (since $y(\text{-}2) = 0.2$), and we define recursively

$$x_{n+1} = x_n + h = x_n + 0.25$$

and

$$\begin{aligned} y_{n+1} &= y_n + F(x_n, y_n) h \\ &= y_n + (0.1 x_n y_n)(0.25) \\ &= y_n + 0.025 x_n y_n \\ &= y_n (1 + 0.025 x_n) , \end{aligned}$$

for each $n = 0, 1, 2, \ldots$ Such that we have $y(x_n) \cong y_n$ for every n .

Since we want to approximate $y(\text{-}1)$, we stop once we arrive at $x_n = \text{-}1$.

We find:

$x_1 = \text{-}2 + 0.25 = \text{-}1.75$, $y_1 = 0.2(1 + (0.025)(\text{-}2)) = 0.19$

$x_2 = x_1 + 0.25 = \text{-}1.5$, $y_2 = 0.19(1 + (0.025)(\text{-}1.75)) = 0.181688$

$x_3 = x_2 + 0.25 = \text{-}1.25$, $y_3 = 0.181688(1 + (0.025)(\text{-}1.5)) = 0.174874$

$x_4 = x_3 + 0.25 = \text{-}1$, $y_4 = 0.174874(1 + (0.025)(\text{-}1.25)) = 0.169409$

Hence the answer is $y(\text{-}1) \cong 0.169409$.

Question 5: *Score 0/1*

Solve the initial-value problem shown below:

$$\frac{dy}{dx} = 12x^3\ y\ , \quad y\ (0) = 2$$

Give an exact formula for y .

$y =$

Your response	Correct response
No answer	**Correct Answer not defined**

Grade: 0/1.0

Total grade: 0.0×1/1 = 0%

Comment:

After separation of variables, the differential equation becomes

$$\int \frac{1}{y}\ dy = \int 12x^3\ dx$$

We deduce $\ln|y| = \frac{12}{3+1}\ x^{3+1} + C = 3x^4 + C$ where C is a constant. Next, we isolate y :

$|y| = e^{3x^4 + C}$

$\Rightarrow \quad y = \pm e^C\ e^{3x^4} = Ae^{3x^4}$

where $A = \pm e^C$.

To determine A , we use the initial condition $y(0) = 2$. This gives

$$2 = Ae^0 = A\ ,$$

hence

$$y = 2e^{3x^4}$$
