# Microeconomic Theory III
# Preferences and Utility

Adam M. Lavecchia

Department of Economics
University of Ottawa

January 2018

# Outline for Lecture

- Axioms of Rational Choice
- Utility
- Trades and Substitution
- The Mathematics of Indifference Curves
- Utility Functions for Specific Preferences
  - Cobb-Douglas
  - Linear (perfect substitutes)
  - Leontief (perfect complements)
  - Constant elasticity of substitution (CES)
  - Quasi-linear
- The Many Good Case
- End of chapter review questions
  - All except 3.14 and 3.15

# Axioms of Rational Choice

- Classical Consumer Theory assumes that individuals in the economic (consumers) are **rational** – what do we mean by this?
- We must specify a basic set of postulates or axioms that characterize how individuals make choices
- **Preferences:** concept that individuals are able to evaluate among two or more options and decide which option(s) make them better off (all else equal)
- Preferences must satisfy the following properties
  - Completeness
  - Transitivity
  - Continuity
  - Local Non-satiation

# Definitions and Notation

- **Bundle:** a basket (set) of goods and services; can be a single good or a vector of goods
  - denoted by $x$
- **Choice set:** the set of bundles of goods and services from which an individual can (feasibly) choose
  - denoted by $\mathbf{X} = \{x_1, x_2, ...., x_n\}$
- Preference relations
  - strictly prefers: $\succ$
  - weakly prefers: $\succeq$
  - indifferent: $\sim$

# Completeness

- Consumers are able to rank all pairs of bundles in their choice set
- For any two options (bundles) in the choice set, A and B, the consumer will chose one and only one of the following
  - $A \succeq B$
  - $B \succeq A$
  - $A \sim B$
- Without this property, preferences are undefined
- Implies no indecision (e.g. answering “I’m not sure” is **ruled out**)

# Transitivity

- Consumers' choices are **internally consistent**
- If $A \succeq B$ and $B \succeq C$, then $A \succeq C$
- Rules out cycling between options

# Transitivity

- Consumers' choices are **internally consistent**
- If $A \succeq B$ and $B \succeq C$, then $A \succeq C$
- Rules out cycling between options
- **Exercise:** Suppose that a consumer's choice set is initially $\mathbf{X} = \{x_1, x_2, x_3\}$. Her preferences are complete and transitive and $x_1 \succeq x_2 \succeq x_3$.
  - Prove that $x_1 \succeq x_3$
  - Suppose a new good, $x_4$, is introduced and that $x_1 \succeq x_4$ and $\text{x}_2 \sim x_4$. Does the consumer: (i) weakly prefer $x_3$ to $x_4$, (ii) weakly prefer $x_4$ to $x_3$, or (iii) indifferent between $x_3$ and $x_4$?

# Continuity

- If $A \succ B$ and $C$ lies within an $\epsilon$ radius of $B$ (i.e. $C$ is **close to** $B$), then $A \succ C$
- Implies that preferences don't "jump" discontinuously at various different points
- Example where continuity is violated?

# Continuity

- If $A \succ B$ and $C$ lies within an $\epsilon$ radius of $B$ (i.e. $C$ is **close to** $B$), then $A \succ C$
- Implies that preferences don't "jump" discontinuously at various different points
- Example where continuity is violated?
- Given the completeness, transitivity and continuity axioms, we can always define a utility function
    - Any utility function that satisfies these 3 axioms **cannot have indifference curves that cross**

# Local Non-satiation (LNS)

- LNS is the **more is better** property
- For any two bundles, A and B, composed of two goods X and Y we have:
  - $A = \{x_A, y_A\}$
  - $B = \{x_B, y_B\}$
  - Both X and Y are desirable (i.e. “good goods”)
- LNS requires that if $x_A = x_B$ and $y_A > y_B$, then $A \succ B$ regardless of the levels of $\{x_a, x_B, y_A, y_B\}$
- Implies that:
  - consumers places value on more consumption
  - no upper limit to utility

# Utility

- From now on, we will assume that the completeness, transitivity, continuity and LNS axioms are satisfied
  - Implies that consumers are able to rank all pairs of bundles from the most to the least desirable
- This ranking of consumption bundles is called **utility**
  - is a mapping from consumption bundles to this ranking/utility
  - formally: $x \rightarrow U(x)$
  - utility rankings are **ordinal** in that they capture the **relative desirability** of commodity bundles
  - this ranking is unique only up to an order-preserving transformation (e.g. monotonic transformation)
- Utility functions can be defined over up to an infinite number of goods
  - n-good case $U(x_1, x_2, ..., x_n)$
- Later, we will introduce another axiom, **convexity of preferences**, which will guarantee **diminishing marginal utility**

Walter Nicholson | Christopher Snyder 12th edition

# MICROECONOMIC THEORY

Basic Principles & Extensions

CHAPTER

3

# Preferences and Utility

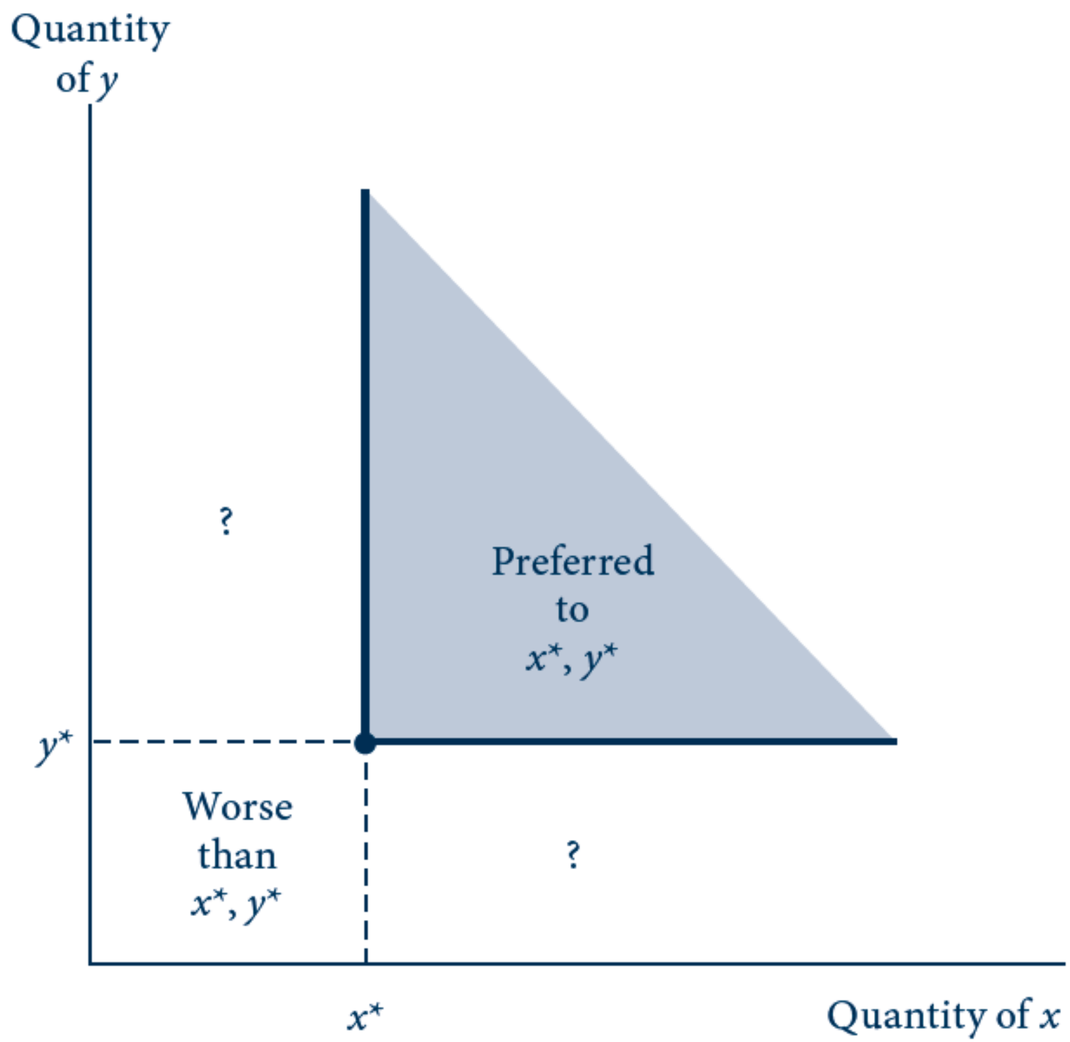
PowerPoint Slides prepared by:
V. Andreea CHIRITESCU
Eastern Illinois University

# Utility

- Utility is affected by
  - The consumption of physical commodities
  - Psychological attitudes
  - Peer group pressures
  - Personal experiences
  - The general cultural environment

# FIGURE 3.1 More of a Good Is Preferred to Less

The shaded area represents those combinations of x and y that are unambiguously preferred to the combination x*, y*. Ceteris paribus, individuals prefer more of any good rather than less.
Combinations identified by "?" involve ambiguous changes in welfare because they contain more of one good and less of the other.



# Trades and Substitution

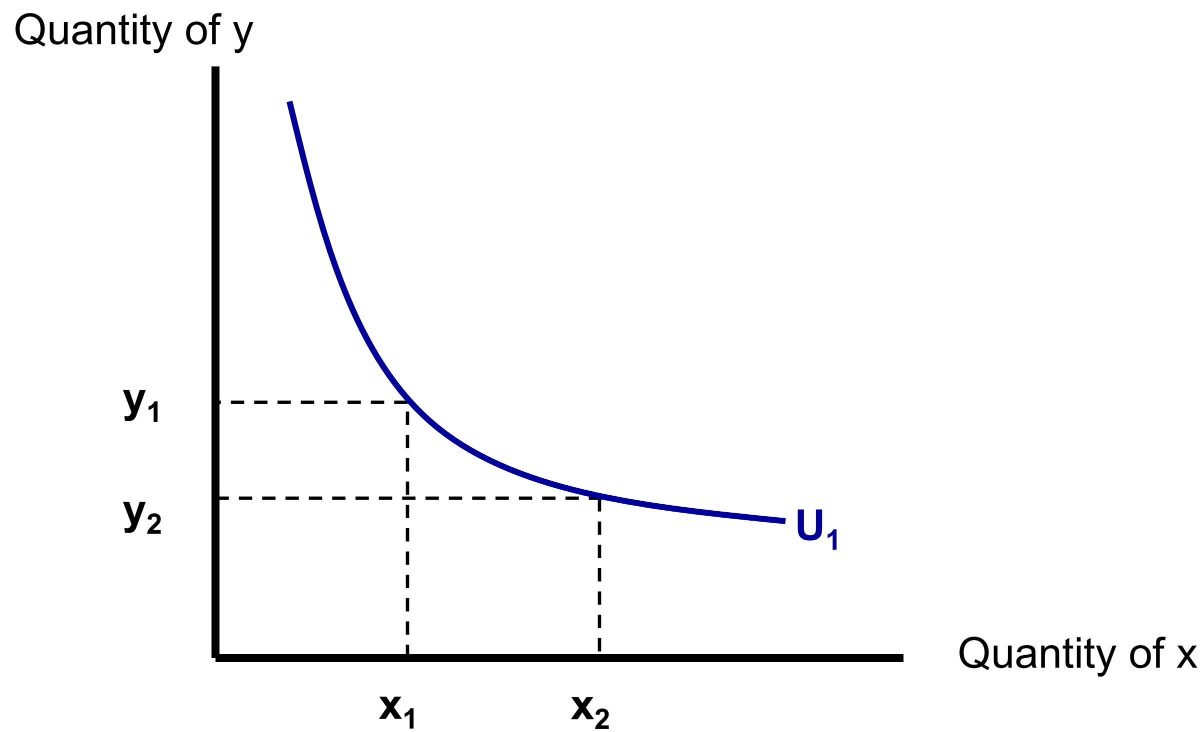
- Indifference curve
  - Shows a set of consumption bundles about which the individual is indifferent
  - All consumption bundles that the individual ranks equally
  - The bundles all provide the same level of utility

# Trades and Substitution

- Marginal rate of substitution, MRS
  - The negative of the slope of an indifference curve ($U_1$) at some point
    - Marginal rate of substitution at that point
  - MRS changes as *x* and *y* change
    - Reflects the individual's willingness to trade *y* for *x*

$$MRS = -\left.\frac{dy}{dx}\right|_{U=U_1}$$

# FIGURE 3.2 A Single Indifference Curve

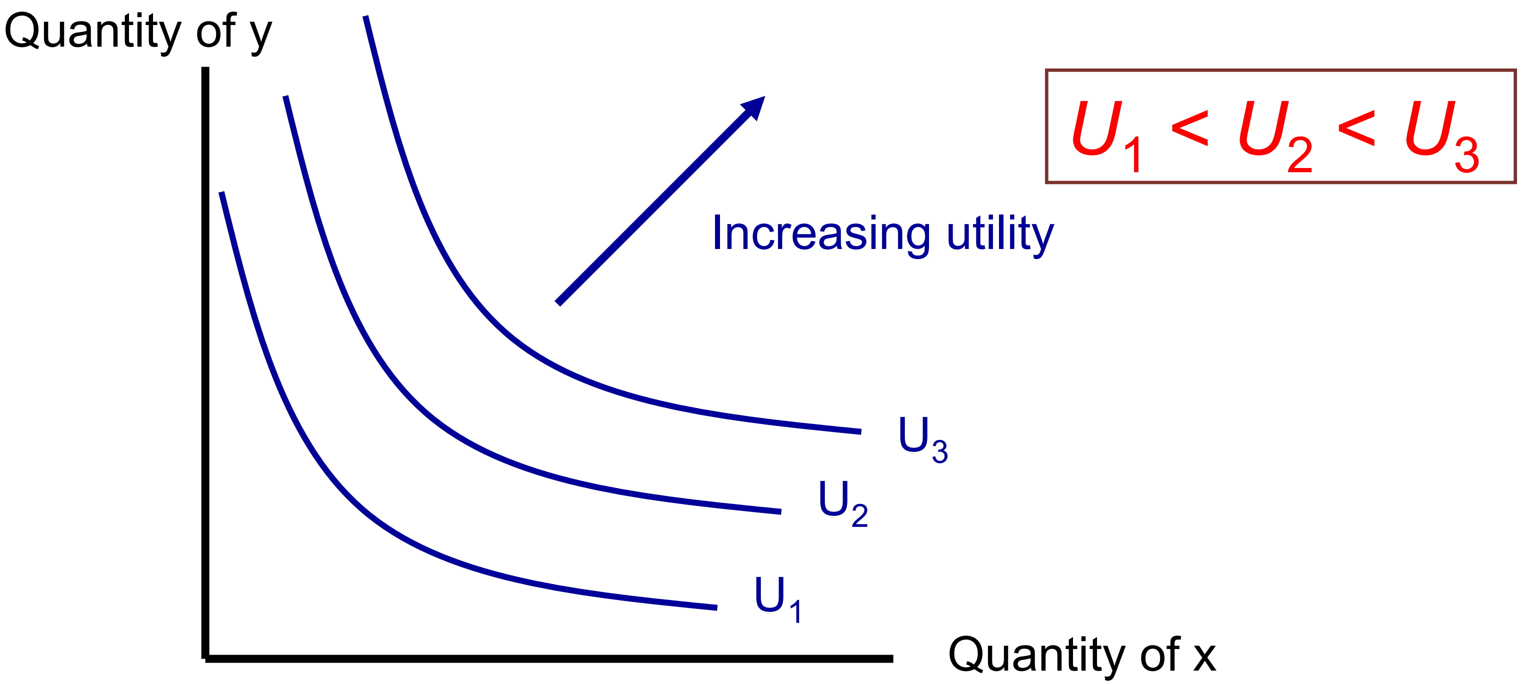



The curve $U_1$ represents those combinations of x and y from which the individual derives the same utility. The slope of this curve represents the rate at which the individual is willing to trade x for y while remaining equally well off. This slope (or, more properly, the negative of the slope) is termed the marginal rate of substitution. In the figure, the indifference curve is drawn on the assumption of a diminishing marginal rate of substitution.

# Trades and Substitution

- Indifference curve map
  - Several indifference curves
  - Level of utility represented by these curves increases as we move in a northeast direction
    - More of a good is preferred to less

# FIGURE 3.3 There Are Infinitely Many Indifference Curves in the x–y Plane



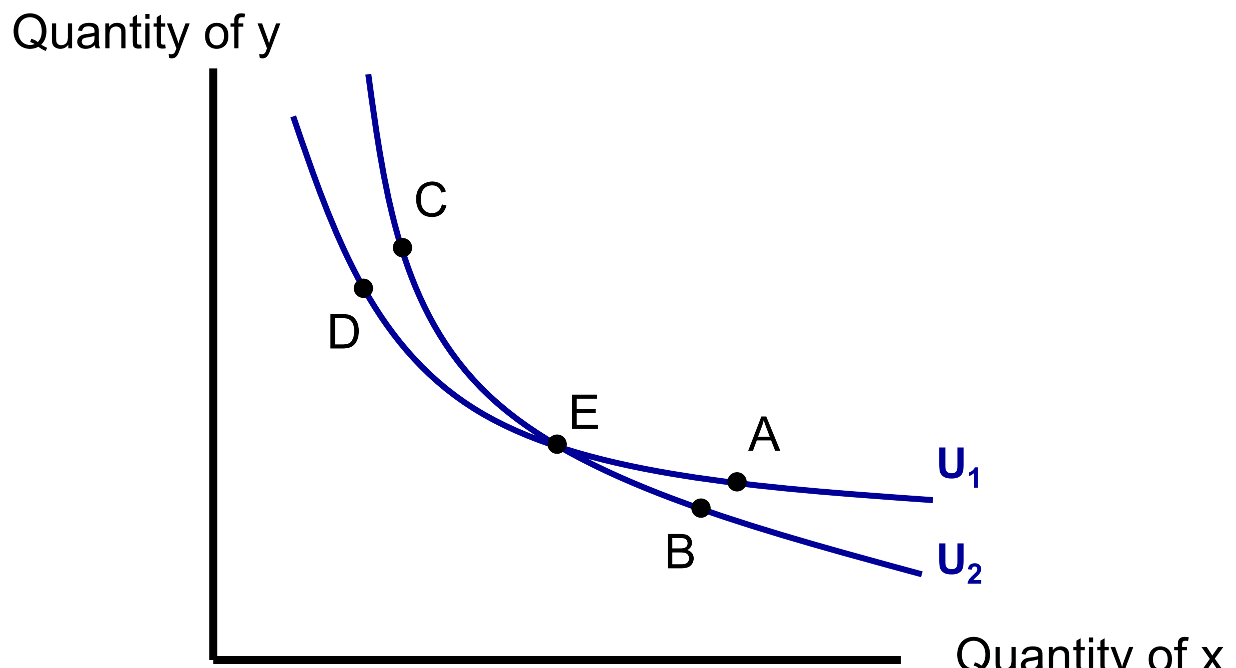

There is an indifference curve passing through each point in the x–y plane. Each of these curves records combinations of x and y from which the individual receives a certain level of satisfaction.

Movements in a northeast direction represent movements to higher levels of satisfaction.

# Trades and Substitution

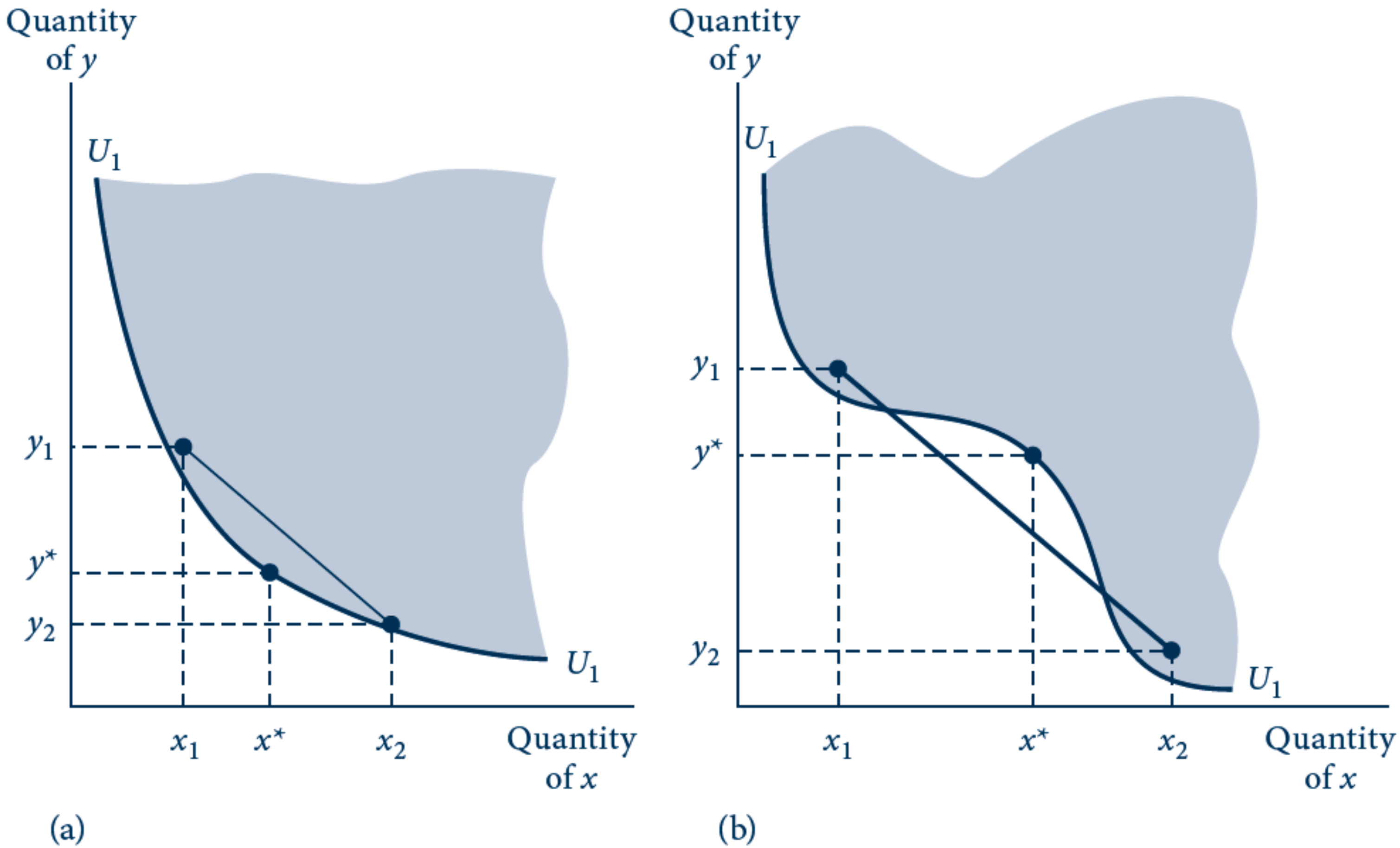
- Indifference curves and transitivity
  - Indifference curves cannot intersect
- A set of points is convex
  - If any two points can be joined by a straight line that is contained completely within the set
- Convexity of indifference curves
  - Indifference curves are convex
  - Diminishing MRS

# FIGURE 3.4 Intersecting Indifference Curves Imply Inconsistent Preferences




Combinations A and D lie on the same indifference curve and therefore are equally desirable. But the axiom of transitivity can be used to show that A is preferred to D. Hence intersecting indifference curves are not consistent with rational preferences. That is, point E cannot represent two different levels of utility.

# FIGURE 3.5 The Notion of Convexity as an Alternative Definition of a Diminishing MRS

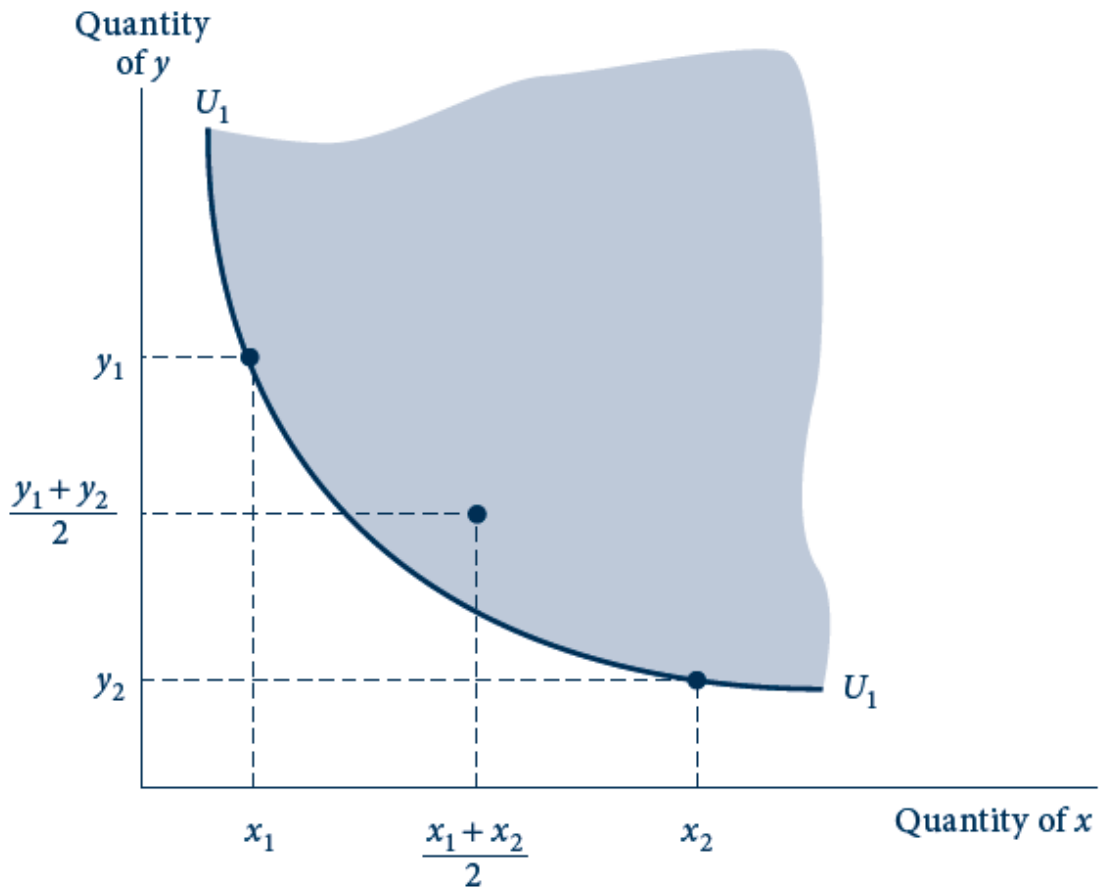



In (a) the indifference curve is convex (any line joining two points above $U_1$ is also above $U_1$). In (b) this is not the case, and the curve shown here does not everywhere have a diminishing MRS.

# Trades and Substitution

- Convexity and balance in consumption
  - Individuals prefer some balance in their consumption
  - "Well-balanced" bundles of commodities are preferred to bundles that are heavily weighted toward one commodity

# FIGURE 3.6 Balanced Bundles of Goods Are Preferred to Extreme Bundles

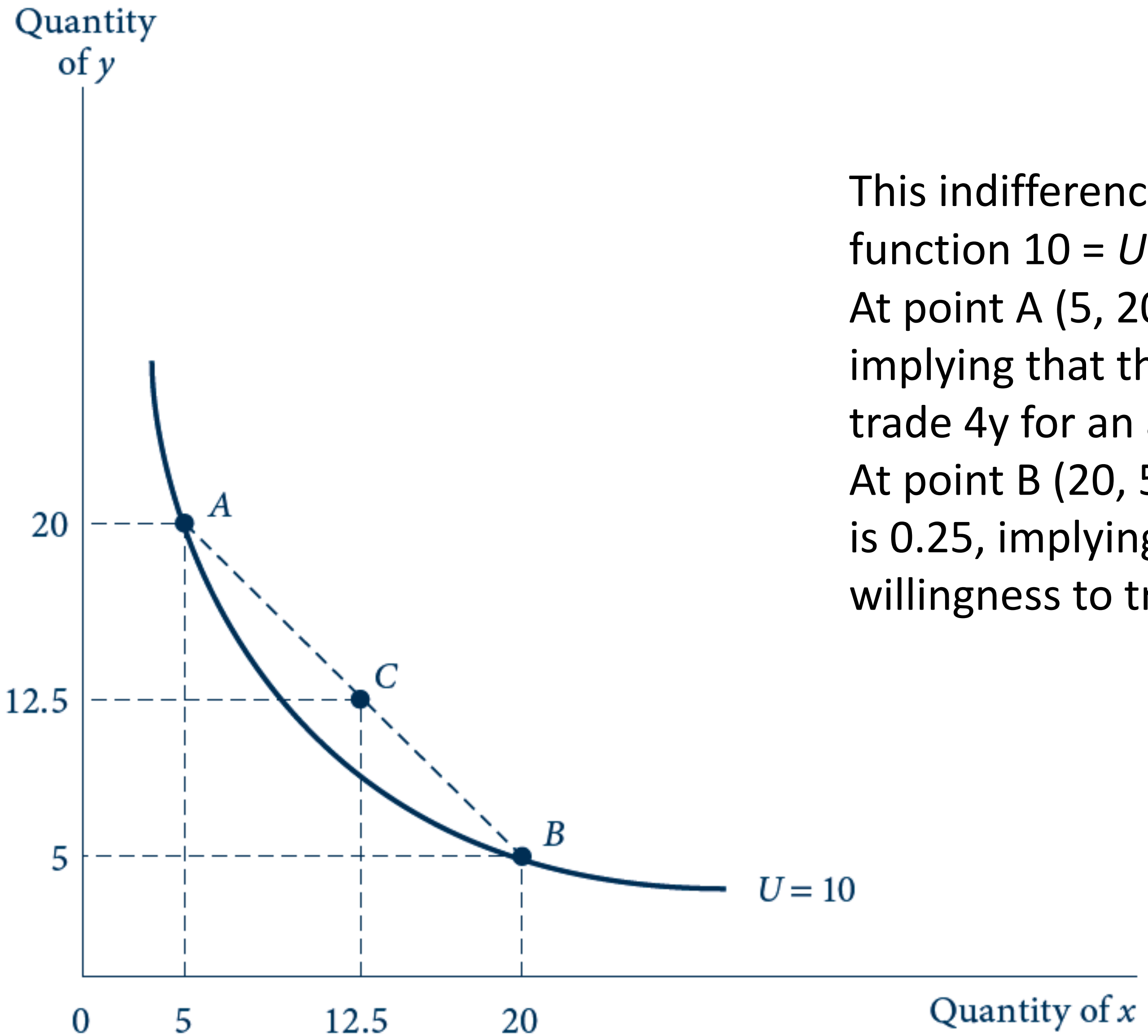



If indifference curves are convex (if they obey the assumption of a diminishing MRS), then the line joining any two points that are indifferent will contain points preferred to either of the initial combinations. Intuitively, balanced bundles are preferred to unbalanced ones.

# EXAMPLE 3.1 Utility and the MRS

- A person's ranking of hamburgers (y) and soft drinks (x), utility = $\sqrt{x \cdot y}$
- An indifference curve for this function
  - Identify that set of combinations of x and y for which utility has the same value
  - Utility = 10 = $\sqrt{x \cdot y}$, so 100=x·y, therefore y=100/x
- MRS = -dy/dx(along $U_1$)=100/$x^2$
  - As *x* rises, *MRS* falls
    - When $x = 5$, $MRS = 4$
    - When $x = 20$, $MRS = 0.25$

# FIGURE 3.7 Indifference Curve for Utility=$\sqrt{x \cdot y}$




This indifference curve illustrates the function 10 = $U = \sqrt{x \cdot y}$.
At point A (5, 20), the MRS is 4, implying that this person is willing to trade 4y for an additional x.
At point B (20, 5), however, the MRS is 0.25, implying a greatly reduced willingness to trade.

# The Mathematics of Indifference Curves

- An individual – consumes x and y
  - Utility = $U(x,y)$
  - Specific level of utility, k: $U(x,y)=k$
  - Trade-offs: the rate at which x can be traded for y
    - Is given by the negative of the ratio of the "marginal utility" of good x to that of good y

$$MRS = -\left.\frac{dy}{dx}\right|_{U(x,y)=k} = \frac{U_x}{U_y}$$

# EXAMPLE 3.2 Showing Convexity of Indifference Curves

$$1.\ U(x,y)=\sqrt{x\cdot y}$$

$$\text{Let}\quad U^*(x,y)=\ln[U(x,y)]=0.5\ln x+0.5\ln y$$

$$MRS=\frac{\partial U^*/\partial x}{\partial U^*/\partial y}=\frac{y}{x}$$

- MRS is diminishing as x increases and y decreases
- Therefore, the indifference curves are convex

# EXAMPLE 3.2 Showing Convexity of Indifference Curves

$$2.\ U(x, y) = x + xy + y$$

$$MRS = \frac{\partial U / \partial x}{\partial U / \partial y} = \frac{1 + y}{1 + x}$$

- MRS is diminishing as x increases and y decreases
- Therefore, the indifference curves are convex

# EXAMPLE 3.2 Showing Convexity of Indifference Curves

3. $U(x, y) = \sqrt{x^2 + y^2}$

Let $U^*(x, y) = [U(x, y)]^2 = x^2 + y^2$

$$MRS = \frac{\partial U^* / \partial x}{\partial U^* / \partial y} = \frac{x}{y}$$

- As x increases and y decreases, the MRS *increases*!
- The indifference curves are concave, not convex
- This is not a quasi-concave function

# Utility Functions for Specific Preferences

- Cobb-Douglas Utility, $U(x,y) = x^{\alpha}y^{\beta}$
  - $\alpha$ and $\beta$ are positive constants, each <1
  - The relative sizes of $\alpha$ and $\beta$ indicate the relative importance of the goods
  - Normalize so that $\alpha + \beta = 1$

    $U(x,y) = x^{\delta}y^{1-\delta}$

    - Where $\delta=\alpha/(\alpha+\beta)$ and $1-\delta=\beta/(\alpha+\beta)$

# Utility Functions for Specific Preferences

- Perfect substitutes
  - Linear indifference curves

    $$U(x,y) = \alpha x + \beta y$$

  - Where $\alpha$ and $\beta$ are positive constants
  - The MRS will be constant along the indifference curves
    - MRS = $\alpha/\beta$ along the entire indifference curve

# Utility Functions for Specific Preferences

- Perfect complements
  - L-shaped indifference curves

    $U(x,y) = \min(\alpha x, \beta y)$
  - Where $\alpha$ and $\beta$ are positive parameters

# Utility Functions for Specific Preferences

- CES Utility (constant elasticity of substitution)

  $U(x,y) = [x^\delta + y^\delta]^{1/\delta}$, where $\delta \leq 1$, $\delta \neq 0$

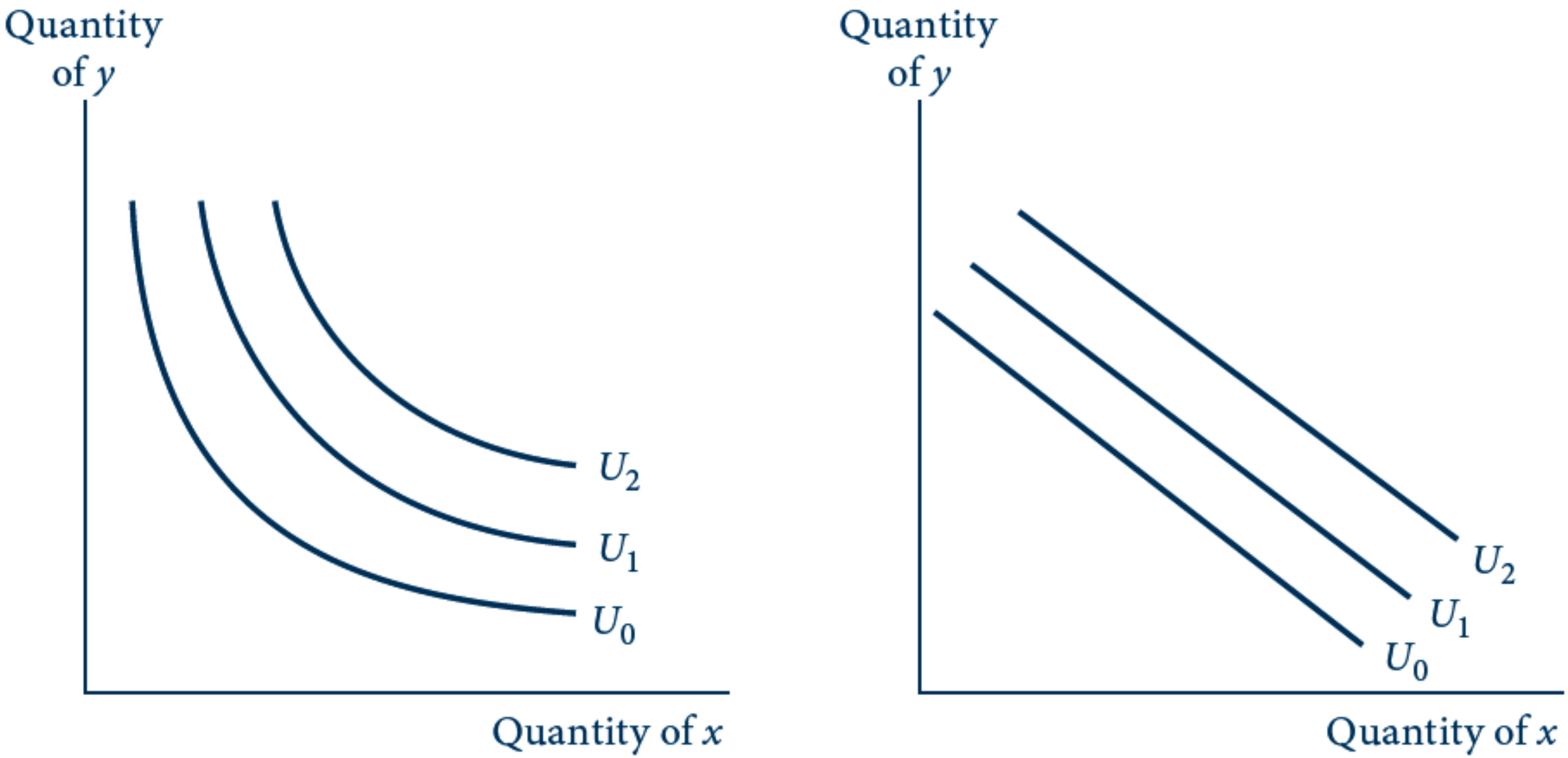
  - For $\delta = 1$: $U(x,y)$ corresponds to perfect substitutes case
  - For $\delta$ approaching 0: $U(x,y)$ approaches Cobb-Douglas
  - For $\delta$ approaching $-\infty$: $U(x,y)$ approaches the case of perfect complements
  - Monotonic transformation $U^* = U^\delta/\delta$, so

    $U(x,y) = x^\delta/\delta + y^\delta/\delta$

# Utility Functions for Specific Preferences

- The elasticity of substitution, $\sigma$
  - CES utility $\Rightarrow \sigma = 1/(1 - \delta)$
  - Perfect substitutes $\Rightarrow \sigma = \infty$
  - Perfect complements $\Rightarrow \sigma = 0$

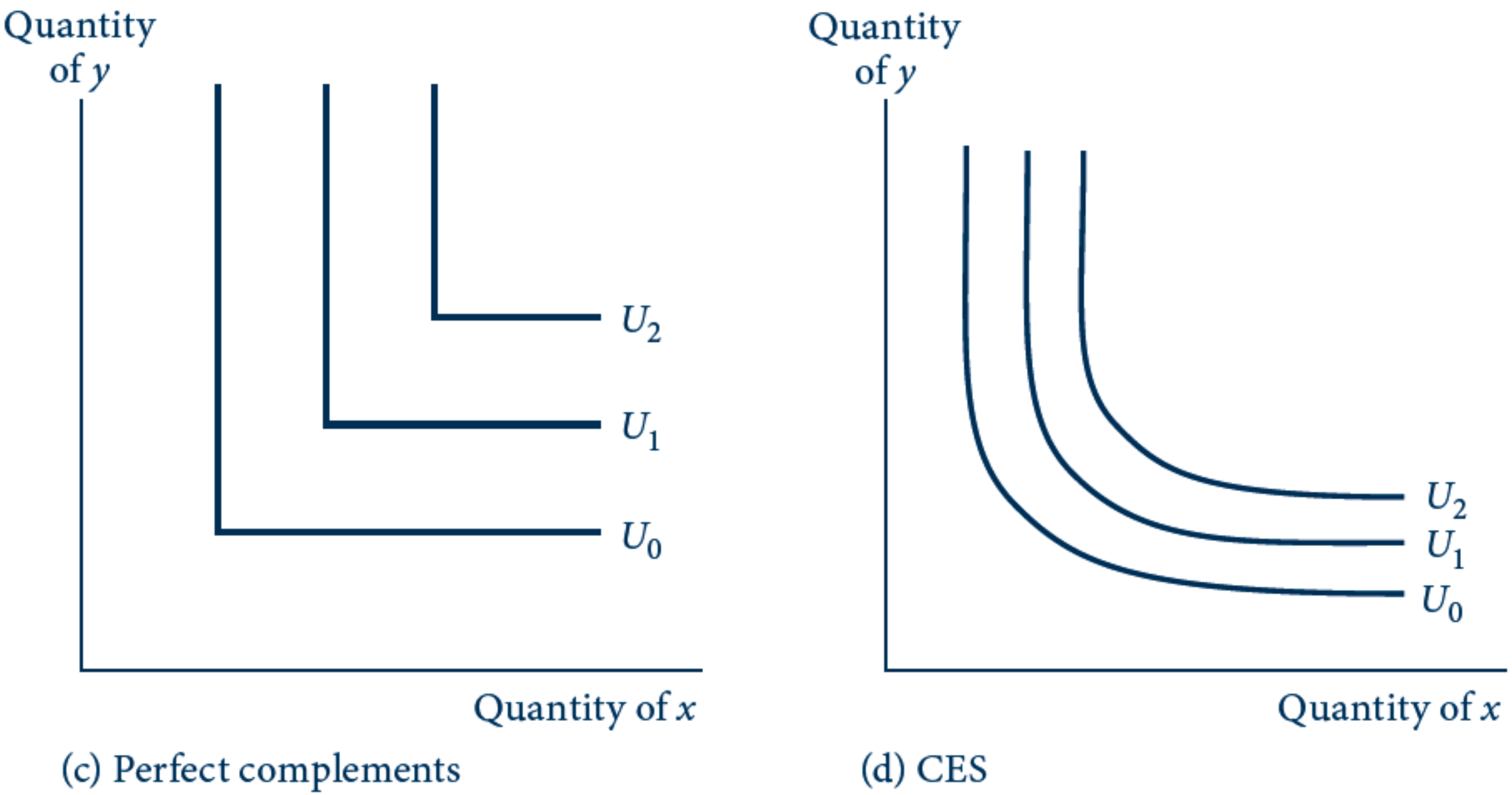
# FIGURE 3.8 Examples of Utility Functions (a, b)




(a) Cobb-Douglas (b) Perfect substitutes

The four indifference curve maps illustrate alternative degrees of substitutability of x for y. The Cobb–Douglas and constant elasticity of substitution (CES) functions (drawn here for relatively low substitutability) fall between the extremes of perfect substitution (b) and no substitution (c).

# FIGURE 3.8 Examples of Utility Functions (c, d)




The four indifference curve maps illustrate alternative degrees of substitutability of x for y. The Cobb–Douglas and constant elasticity of substitution (CES) functions (drawn here for relatively low substitutability) fall between the extremes of perfect substitution (b) and no substitution (c).

# EXAMPLE 3.3 Homothetic Preferences

- Utility function is homothetic
  - If the *MRS* depends only on the ratio of the amounts of the two goods
    - Not on the total quantities of the good
- Perfect substitutes
  - *MRS* is the same at every point
- Perfect complements
  - *MRS* is $\infty$ if $y/x > \alpha/\beta$
  - *MRS* is undefined if $y/x = \alpha/\beta$
  - *MRS* is 0 if $y/x < \alpha/\beta$

# EXAMPLE 3.3 Homothetic Preferences

- General Cobb-Douglas function
  - The *MRS* depends only on the ratio y/x

$$MRS = \frac{\partial U / \partial x}{\partial U / \partial y} = \frac{\alpha x^{\alpha-1} y^{\beta}}{\beta x^{\alpha} y^{\beta-1}} = \frac{\alpha}{\beta} \cdot \frac{y}{x}$$

# EXAMPLE 3.4 Nonhomothetic Preferences

- Some utility functions do not exhibit homothetic preferences

  utility = $U(x,y) = x + \ln y$

  - Good y exhibits diminishing marginal utility, but good x does not
  - The MRS diminishes as the chosen quantity of y decreases, but it is independent of the quantity of x consumed

$$MRS = \frac{\partial U / \partial x}{\partial U / \partial y} = \frac{1}{1/y} = y$$

# The Many-Good Case

- Suppose utility is a function of *n* goods given by $U(x_1, x_2,\dots, x_n)$
- Equation $U(x_1, x_2,\dots, x_n)=k$
  - Defines an indifference surface in n dimensions
    - All those combinations of the n goods that yield the same level of utility (Convex surface)
  - Quasi-concave

# The Many-Good Case

- MRS with many goods

$$MRS = -\left.\frac{dx_2}{dx_1}\right|_{U(x_1, x_2, \ldots, x_n) = k} = \frac{U_{x_1}(x_1, x_2, \ldots, x_n)}{U_{x_2}(x_1, x_2, \ldots, x_n)}$$