

Question #3: (10 points)

Consider an impulse response $h[n]$ for a causal linear-phase FIR filter where the first three coefficients are

$$h[0] = 1, h[1] = 3, h[2] = -2.$$

- (a) For each of the four linear-phase FIR filter types (Type I, II, III, and IV), find the coefficients of the smallest length linear-phase FIR filter that satisfies the above start values of $h[n]$.

Type I: $h[n] = h[M-n], M=4, 0 \leq n \leq 4$

$$h[0]=1, h[1]=3, h[2]=-2, h[3]=3, h[4]=1 \quad \checkmark$$

Type II: $h[n] = h[M-n], M=5, 0 \leq n \leq 5$

$$h[0]=1, h[1]=3, h[2]=-2, h[3]=-2, h[4]=3, h[5]=1 \quad \checkmark$$

Type III: $h[n] = -h[M-n], M=4, 0 \leq n \leq 4$

$$h[0]=1, h[1]=3, h[2]=-2, h[3]=-3, h[4]=-1$$

Type IV: $h[n] = -h[M-n], M=5, 0 \leq n \leq 5$

$$h[0]=1, h[1]=3, h[2]=-2, h[3]=2, h[4]=-3, h[5]=-1 \quad \checkmark$$

- (b) For each of the four filters found in (a), give the slope of the phase for the frequency response of the filter as well as the corresponding group delay.

Type I: $\angle H(e^{j\omega}) = -\omega M/2 = -2\omega$ slope = -2 \checkmark
 $\tau_g(\omega) = -\frac{d\angle H(e^{j\omega})}{d\omega} = 2$

Type II: $\angle H(e^{j\omega}) = -\omega M/2 = -2.5\omega$ slope = -2.5 \checkmark
 $\tau_g(\omega) = -\frac{d\angle H(e^{j\omega})}{d\omega} = 2.5$

Type III: $\angle H(e^{j\omega}) = -\omega M/2 = -2\omega$ slope = -2 \times
 $\tau_g(\omega) = -\frac{d\angle H(e^{j\omega})}{d\omega} = 2$

Type IV: $\tau_g(\omega) = -\frac{d\angle H(e^{j\omega})}{d\omega} = 2.5$ slope = 2.5 \checkmark