Question #2: (10 points)
Consider an input sequence to a filter given as

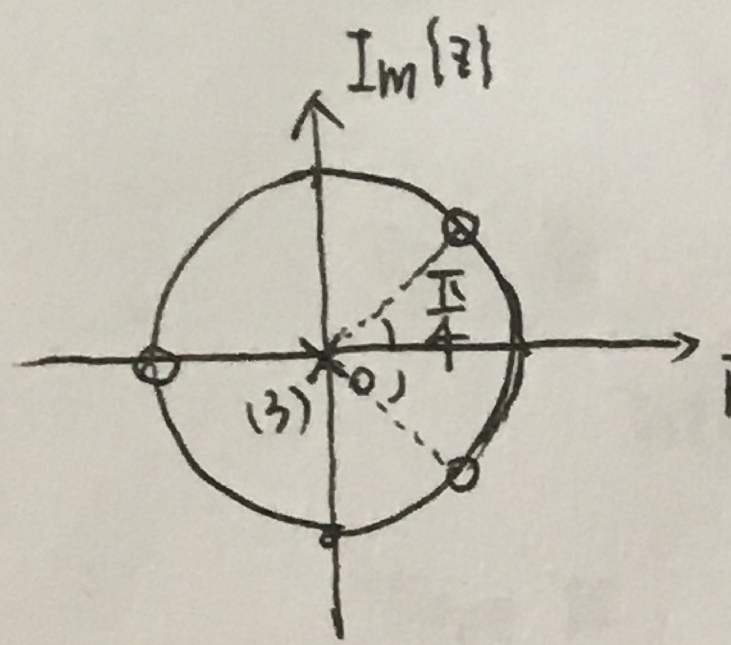
$$x[n] = 4\cos(0.25\pi n + \pi/3) - 5\sin(\pi n - \pi/2) \qquad n \in \mathbb{Z}.$$

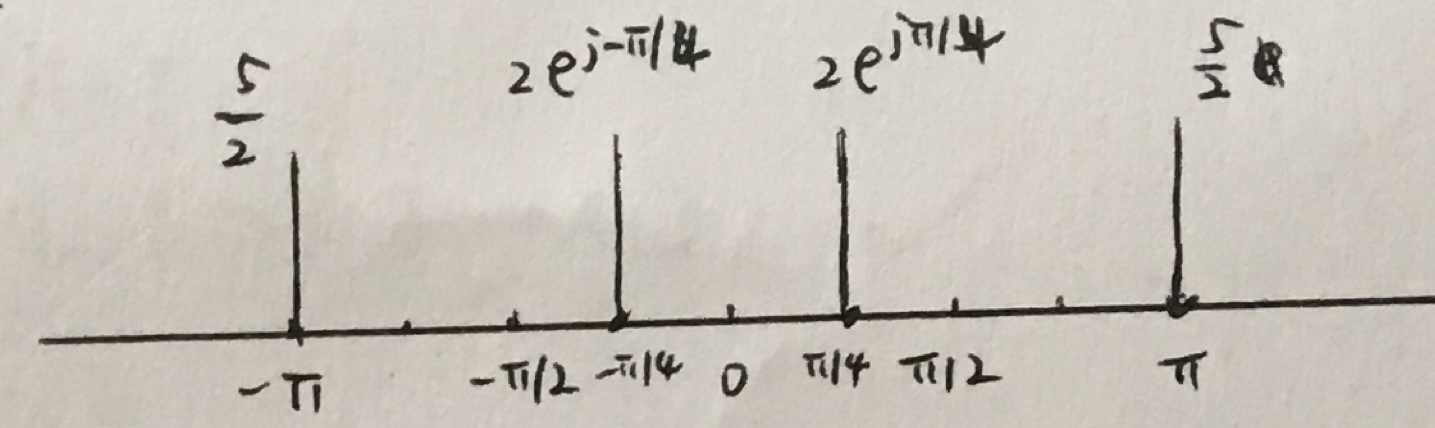
(a) Draw a pole-zero diagram for a filter $H(z)$ that will $\boxed{\text{null}}$ this input $x[n]$.

$x[n] = 4\cos(0.25\pi n + \pi/3) + 5\cos(\pi n)$

$\omega_1 = \pm 0.25\pi$

$\omega_2 = \pm\pi$





(b) Determine the impulse response $h[n]$ of an FIR filter with minimum number of coefficients (other than the trivial case $h[n] = 0$) such that $x[n] * h[n] = 0$.

Zeros: $z_1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j \qquad z_2 = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j \qquad z_3 = -1$

$\therefore H(z) = [1 - (\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j)z^{-1}] \cdot [1 - (\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j)z^{-1}] \cdot [1 + z^{-1}]$

$= (1 - \sqrt{2}z^{-1} + z^{-2})(1 + z^{-1}) = 1 + (1-\sqrt{2})z^{-1} + (1-\sqrt{2})z^{-2} + z^{-3}$

$\cancel{= 1 + (1-\sqrt{2})z^{-1} - (1+\sqrt{2})z^{-2} + z^{-3}}$

$\therefore \cancel{h[n] = \delta[n] + (1-\sqrt{2})\delta[n-1] - (1+\sqrt{2})\delta[n+1]} + \delta[n-3]]$

$\therefore h[n] = \delta[n] + (1-\sqrt{2})\delta[n-1] + (1-\sqrt{2})\delta[n-2] + \delta[n-3]$