

Question #1: (10 points)

Consider an LTI filter described by the transfer function

$$H(z) = \frac{z^{-1}}{(1 - 0.5z^{-1})(1 - 2z^{-1})}$$

(a) Give the region of convergence (ROC) for this filter if it is stable.

poles: $(1 - 0.5z^{-1})(1 - 2z^{-1}) = 0$

$$z_1 = 0.5, \quad z_2 = 2.$$

to make this filter stable,

ROC should contain unit circle.

\therefore ROC: $0.5 < |z| < 2$ ✓

(b) Determine the impulse response $h[n]$ of the stable filter for $H(z)$.

ROC: $0.5 < |z| < 2$. $A_1 = (1 - 2z^{-1})H(z)|_{z=2} = +\frac{2}{3}$ $A_2 = (1 - 0.5z^{-1})H(z)|_{z=0.5} = -\frac{2}{3}$.

$$\therefore H(z) = \frac{z^{-1}}{1 - 2.5z^{-1} + z^{-2}} = -\frac{2}{3} \cdot \frac{1}{1 - 0.5z^{-1}} + \frac{2}{3} \cdot \frac{1}{1 - 2z^{-1}}$$

$$\therefore h[n] = -\frac{2}{3} \cdot (0.5)^n u[n] - \frac{2}{3} \cdot (2)^n u[-n-1] \quad \checkmark$$