

Section A

Answer all questions. Each question is worth 1 mark. Please circle your answer in the box.

A1

A 12.0 N force with a fixed orientation does work on a particle, as the particle moved through a displacement $\mathbf{d} = (2.00 \text{ m})\mathbf{i} - (4.00 \text{ m})\mathbf{j} + (3.00 \text{ m})\mathbf{k}$. The change in the particle's kinetic energy is +30.0 J. What is the angle between the force vector and the displacement vector?

(A) 12.1°	(B) 16.8°	(C) 21.9°	(D) 46.1°	(E) 62.3°
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A2

A 3.0 kg particle is moving in simple harmonic motion in one dimension and moves according to the equation:

$$x = (5.0 \text{ m})\cos\left[\left(\frac{\pi}{3} \text{ rad/s}\right)t - \left(\frac{\pi}{4} \text{ rad}\right)\right]$$

where t is measured in seconds. At what value of x is the potential energy of the particle exactly half of the mechanical energy?

(A) 0.1 m	(B) 2.8 m	(C) 3.5 m	(D) 4.6 m	(E) 4.9 m
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A3

The linear density of a string is 16 g/cm. A transverse wave on the string is described by the equation

$$y = (0.021 \text{ m})\sin\left[(2.0 \text{ m}^{-1})x + (32 \text{ s}^{-1})t\right]$$

Calculate the tension in the string

(A) 0.35 N	(B) 2.6 N	(C) 24 N	(D) 410 N	(E) 830 N
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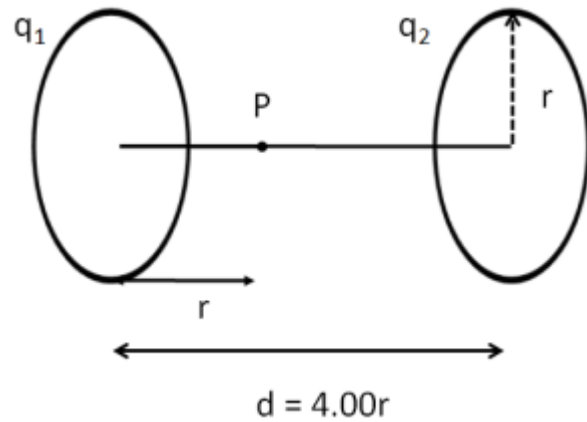
A4

If a metal conductor has a charge $-1.45 \times 10^{-7} \text{ C}$, how many excess electrons are there on it?

(A) 1.44×10^{-3}	(B) 2.40×10^8	(C) 9.05×10^{11}	(D) 8.31×10^{13}	(E) 1.05×10^{18}
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A5

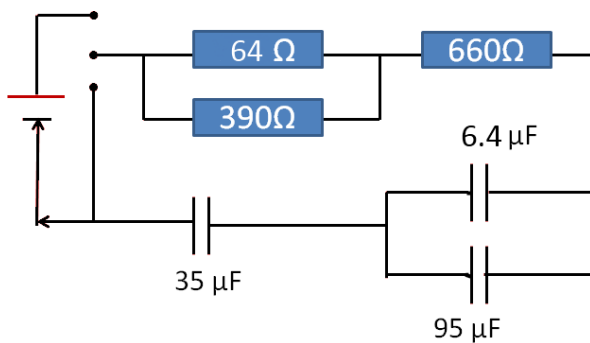
The figure shows two parallel non-conducting rings with their central axes along a common line. Ring 1 has a uniform charge q_1 and ring 2 has a uniform charge q_2 . Both disks have a radius R , and the separation between the disks is $4R$. The net electric field is zero at point P , which is R away from disk 1 and $3R$ from disk 2. What is the charge ratio q_1/q_2 ?



(A) 0.268	(B) 0.333	(C) 0.500	(D) 0.750	(E) 1.00
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A6

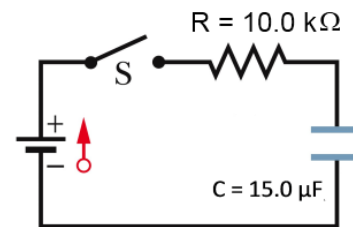
What is the time constant of this RC circuit?



(A) $26 \mu\text{s}$	(B) 19 ms	(C) 1.2 s	(D) 11 s	(E) 230 s
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A7

The switch S in the circuit pictured on the right is closed at $t = 0$ and the uncharged capacitor ($C = 15.0 \mu\text{F}$) starts to charge. At what point does the potential across the resistor ($R = 10.0 \text{ k}\Omega$) equal that of the capacitor?



(A) $525 \mu\text{s}$	(B) 950 μs	(C) 0.100 ms	(D) 0.208 ms	(E) 0.350 ms
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A8

A solenoid of length 15 cm, and with 45 turns, has a current of 1.25 amps flowing in it. What is the magnitude of the magnetic field inside the solenoid?

(A) 1.0×10^{-5} T	(B) 1.2×10^{-4} T	(C) 3.8×10^{-3} T	(D) 7.0×10^{-3} T	(E) 9.5×10^{-2} T
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A9

A coil is connected in series with a 10.0 k Ω resistor. An ideal 50.0V battery is connected across the two devices, and the current reaches 2.00 mA after 5.00 milliseconds. Find the inductance of the coil.

(A) 97.9 H	(B) 108 H	(C) 979 H	(D) 2.61×10^3 H	(E) 2.61×10^6 H
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A10

In an oscillating LC circuit with L = 65 mH and C = 4.0 μ F. The current is initially a maximum. How long will it take before the capacitor is fully charged for the first time?

(A) 7.1×10^{-7} s	(B) 2.6×10^{-6} s	(C) 4.0×10^{-5} s	(D) 6.4×10^{-4} s	(E) 8.0×10^{-4} s
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Here are the worked solutions (note that I may have changed the letter for the answer for the final exam, just look at the method and final value!)

Section A

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Question 1

A 12.0 N force with a fixed orientation does work on a particle, as the particle moved through a displacement $\mathbf{d} = (2.00 \text{ m})\mathbf{i} - (4.00 \text{ m})\mathbf{j} + (3.00 \text{ m})\mathbf{k}$. The change in the particle's kinetic energy is +30.0 J. What is the angle between the force vector and the displacement vector?

(A) 12.1°	(B) 16.8°	(C) 21.9°	(D) 46.1°	(E) 62.3°
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Solution:

Using the work-kinetic energy theorem, we have

$$\Delta K = W = \vec{F} \cdot \vec{d} = Fd \cos \phi.$$

In addition, $F = 12 \text{ N}$ and $d = \sqrt{(2.00 \text{ m})^2 + (-4.00 \text{ m})^2 + (3.00 \text{ m})^2} = 5.39 \text{ m}$.

If $\Delta K = +30.0 \text{ J}$, then

$$\phi = \cos^{-1} \left(\frac{\Delta K}{Fd} \right) = \cos^{-1} \left(\frac{30.0 \text{ J}}{(12.0 \text{ N})(5.39 \text{ m})} \right) = 62.3^\circ.$$

Answer (E)

Question 2

A 3.0 kg particle is moving in simple harmonic motion in one dimension and moves according to the equation:

$$x = (5.0 \text{ m})\cos[(\pi/3 \text{ rad/s})t - \pi/4 \text{ rad}]$$

where t is measured in seconds. At what value of x is the potential energy of the particle exactly half of the mechanical energy?

(A) 0.1 m	(B) 2.8 m	(C) 3.5 m	(D) 4.6 m	(E) 4.9 m
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$$U = \frac{1}{2} E$$

$$\frac{1}{2} kx^2 = \frac{1}{2} \left(\frac{1}{2} kx_m^2 \right)$$

$$x^2 = \frac{1}{2} x_m^2 = \frac{1}{2} (5.0\text{m})^2$$

$$x = 3.5\text{m}$$

Answer (C)

Question 3

The linear density of a string is 16 g/cm. A transverse wave on the string is described by the equation

$$y = (0.021 \text{ m}) \left[\sin(2.0 \text{ m}^{-1})x + (32 \text{ s}^{-1})t \right]$$

Calculate the tension in the string

(A) 0.35 N	(B) 2.6 N	(C) 24 N	(D) 410 N	(E) 830 N
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Solution

The wave speed is given by $v = \lambda/T = \omega/k$, where λ is the wavelength, T is the period, ω is the angular frequency ($2\pi/T$), and k is the angular wave number ($2\pi/\lambda$). The displacement has the form $y = y_m \sin(kx + \omega t)$, so $k = 2.0 \text{ m}^{-1}$ and $\omega = 32 \text{ rad/s}$. Thus

$$v = (32 \text{ rad/s}) / (2.0 \text{ m}^{-1}) = 16 \text{ m/s}.$$

Since the wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, the tension is

$$\tau = \mu v^2 = (16 \text{ g/cm}) \frac{1 \text{ kg}}{1000 \text{ g}} \frac{100 \text{ cm}}{1 \text{ m}} (16 \text{ m/s})^2 = 410 \text{ N}$$

Question 4

If a metal conductor has a charge $-1.45 \times 10^{-7} \text{ C}$, how many excess electrons are there on it?

(A) 1.44×10^{-3}	(B) 2.40×10^8	(C) 9.05×10^{11}	(D) 8.31×10^{13}	(E) 1.05×10^{18}
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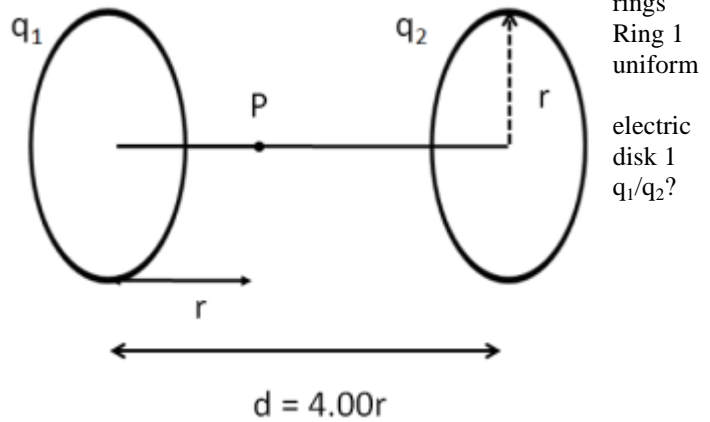
Solution

The charge on the electron is $1.602 \times 10^{-19} \text{ C}$, so to make up a charge of $-1.45 \times 10^{-7} \text{ C}$

$$n = \frac{-1.45 \times 10^{-7} \text{ C}}{-1.602 \times 10^{-19} \text{ C}} = 9.05 \times 10^{11}$$

Question 5

The figure shows two parallel non-conducting rings with their central axes along a common line. Ring 1 has a uniform charge q_1 and ring 2 has a charge q_2 . Both disks have a radius R , and the separation between the disks is $4R$. The net field is zero at point P , which is R away from and $3R$ from disk 2. What is the charge ratio



(A)	(B)	(C)	(D)	(E)
0.268	0.333	0.500	0.750	1.00

Solution

We use Eq. 22-16, assuming both charges are positive. At P , we have for the magnitudes of E

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1 R}{(R^2 + R^2)^{\frac{3}{2}}}$$

And these magnitudes are equal at the point P

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2 3R}{((3R)^2 + R^2)^{\frac{3}{2}}}$$

Answer (A)

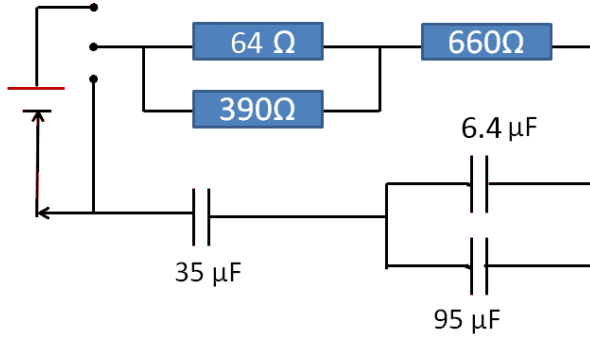
$$\frac{1}{4\pi\epsilon_0} \frac{q_2 3R}{(10R)^{\frac{3}{2}}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 R}{(2R)^{\frac{3}{2}}}$$

$$\frac{q_2 3}{(10R)^{\frac{3}{2}}} = \frac{q_1}{(2R)^{\frac{3}{2}}}$$

$$3 \left(\frac{2}{10} \right)^{\frac{3}{2}} = \frac{q_1}{q_2} = 0.268$$

Question 6

What is the time constant of this RC circuit?



(A) 26 μs	(B) 19 ms	(C) 1.2 s	(D) 11 s	(E) 230 s
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Solution:

Equivalent capacitance of the parallel capacitors:

Equivalent capacitance of the series

Equivalent resistance of the parallel resistors

Equivalent resistance of the series.

Time constant:

$$RC = 715\Omega \times 26.0 \mu\text{F} = 19 \text{ ms}$$

Answer (b)

$$C_{\parallel} = 6.4 \mu\text{F} + 95 \mu\text{F} = 101.4 \mu\text{F}$$

$$\frac{1}{C_{eq}} = \frac{1}{101.4 \mu\text{F}} + \frac{1}{35 \mu\text{F}}$$

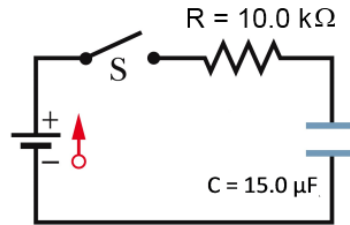
$$\frac{1}{C_{eq}} = \frac{1}{26.0 \mu\text{F}}$$

$$R_p = \frac{64\Omega \cdot 390\Omega}{64\Omega + 390\Omega} = 55.0\Omega$$

$$R_{eq} = 55.0\Omega + 660\Omega = 715\Omega$$

Question 7

The switch S in the circuit pictured on the right is closed at $t = 0$ and the uncharged capacitor ($C = 15.0 \mu\text{F}$) starts to charge. At what time does the potential across the resistor ($R = 10.0 \text{ k}\Omega$) equal that of the capacitor?



0 and what equal

(A) 525 μs	(B) 950 μs	(C) 0.100 ms	(D) 0.208 ms	(E) 0.350 ms
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Solution:

The battery emf is V . Then the requirement stated in the problem that the resistor voltage be equal to the capacitor voltage becomes $iR = V_{\text{cap}}$, or

$$Ve^{-t/RC} = V(1 - e^{-t/RC})$$

$$e^{-t/RC} = 0.5$$

$$t = -RC \ln(0.5) = RC \ln(2) = 0.208 \text{ ms}$$

where Eqs. 27-34 and 27-35 have been used. **Answer (D)**

Question 8

A solenoid of length 15 cm, and with 45 turns, has a current of 1.25 amps flowing in it. What is the magnitude of the magnetic field inside the solenoid?

(A) $1.0 \times 10^{-5} \text{ T}$	(B) $1.2 \times 10^{-4} \text{ T}$	(C) $3.8 \times 10^{-3} \text{ T}$	(D) $7.0 \times 10^{-3} \text{ T}$	(E) $9.5 \times 10^{-2} \text{ T}$
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Solution

$$B = \mu_0 i n = (4\pi \times 10^{-7} \text{ N}\cdot\text{m}^2)(1.25 \text{ A}) \frac{45 \text{ turns}}{0.15 \text{ m}} = 1.2 \times 10^{-4} \text{ T}$$

Answer (B)

Question 9

A coil is connected in series with a 10.0 kΩ resistor. An ideal 50.0V battery is connected across the two devices, and the current reaches 2.00 mA after 5.00 milliseconds. Find the inductance of the coil.

(A) 97.9 H	(B) 108 H	(C) 979 H	(D) 2.61×10^3 H	(E) 2.61×10^6 H
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Solution:

If the battery is applied at time $t = 0$ the current is given by

$$i = \frac{\varepsilon}{R} \left(1 - e^{-\frac{t}{\tau_L}} \right)$$

$$1 - \frac{iR}{\varepsilon} = e^{-\frac{t}{\tau_L}}$$

$$\ln \left[1 - \frac{iR}{\varepsilon} \right] = -\frac{t}{\tau_L}$$

$$\tau_L = -\frac{t}{\ln \left[1 - \frac{iR}{\varepsilon} \right]} = -\frac{5.00 \times 10^{-3} \text{ s}}{\ln \left[1 - \frac{(2.00 \times 10^{-3} \text{ A})(10.0 \times 10^3 \Omega)}{50.0 \text{ V}} \right]} = 9.79 \times 10^{-3} \text{ s}$$

$$L = \tau_L R = 9.79 \times 10^{-3} \text{ s} \times 10.0 \times 10^3 \Omega = 97.9 \text{ H}$$

Answer (A)

Question 10

In an oscillating LC circuit with $L = 65 \text{ mH}$ and $C = 4.0 \mu\text{F}$. The current is initially a maximum. How long will it take before the capacitor is fully charged for the first time?

(A) $7.1 \times 10^{-7} \text{ s}$	(B) $2.6 \times 10^{-6} \text{ s}$	(C) $4.0 \times 10^{-5} \text{ s}$	(D) $6.4 \times 10^{-4} \text{ s}$	(E) $8.0 \times 10^{-4} \text{ s}$
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Solution: The time required is $t = T/4$, where the period is given by $T = 2\pi / \omega = 2\pi\sqrt{LC}$.

$$t = \frac{T}{4} = \frac{2\pi\sqrt{LC}}{4} = \frac{2\pi\sqrt{(0.065 \text{ H})(4.0 \times 10^{-6} \text{ F})}}{4} = 8.0 \times 10^{-4} \text{ s}$$

Answer (D)