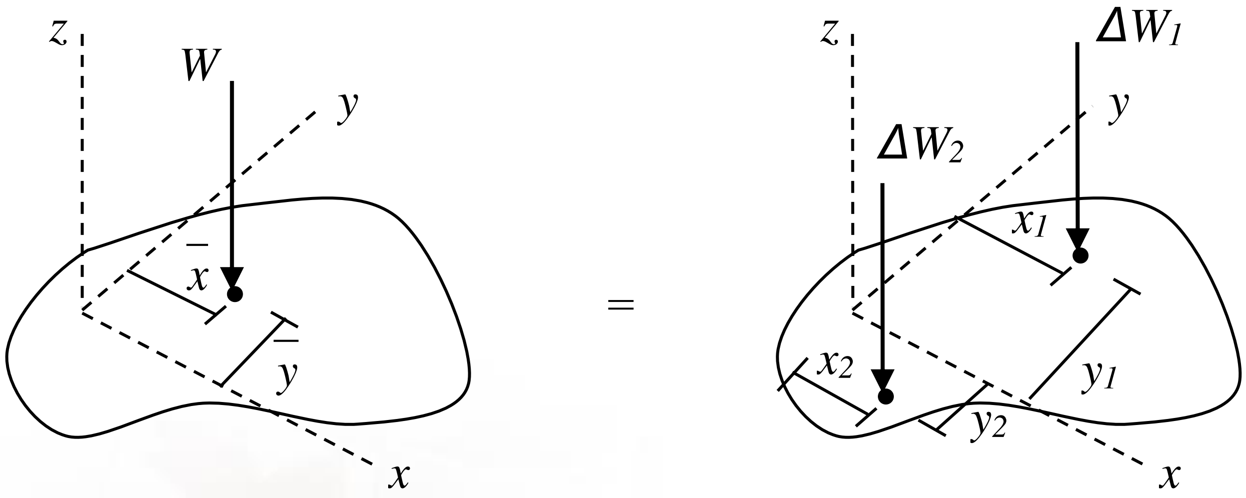
GNG 1105: Section C

Engineering Mechanics

Fall 2016

Chapter 5: Centre of Gravity



The total weight of the plate acting at *G* is the sum of all the weights of the particles forming the rigid body.

$$\sum F_Z = W = \Delta W_1 + \Delta W_2 + \ldots + \Delta W_n$$

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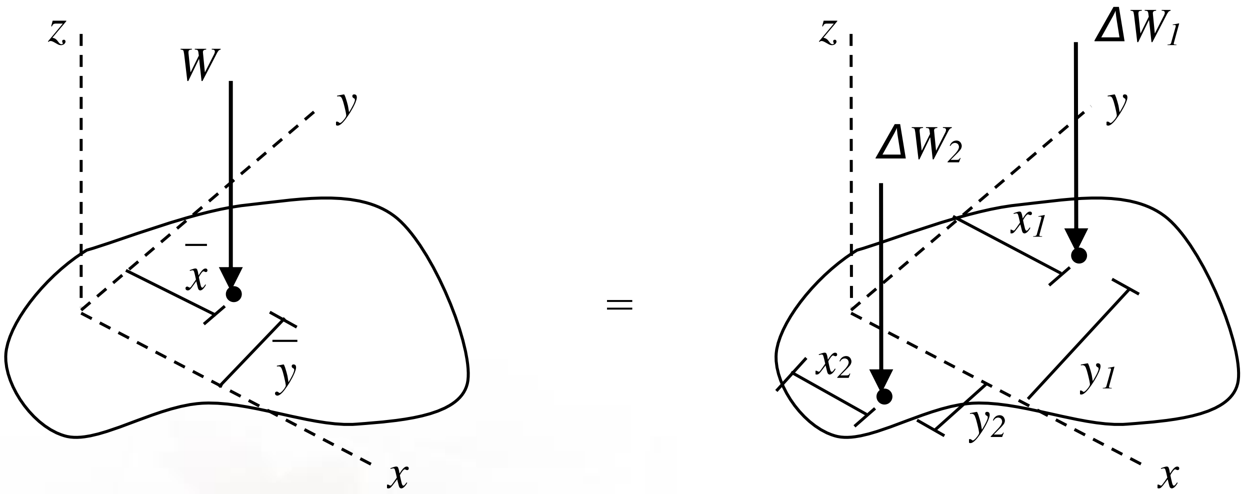

To find the position of *G*, we take the moment about the x and y axes.

$$\sum My = \bar{x}W = x_1\Delta W_1 + x_2\Delta W_2 + \ldots + x_n\Delta W_n$$

$$\sum Mx = \bar{y}W = y_1\Delta W_1 + y_2\Delta W_2 + \ldots + y_n\Delta W_n$$

If we decrease the size of each particle, at the limit we get:

$$W = \int dW, \quad \bar{x}W = \int xdW, \quad \bar{y}W = \int ydW$$

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Centroids

The centroid is what we consider the centre of an area or a line.

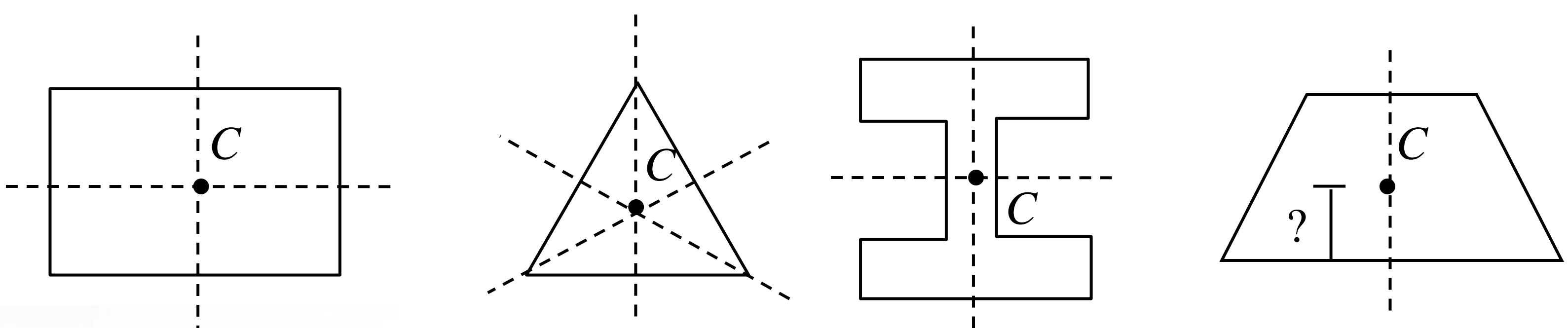
If $W = \rho g t A$ and $\Delta W = \rho g t \Delta A$ where ρ = density, g = gravitational acceleration, t = thickness (constant), A = area

Then $\bar{x}\,\cancel{\rho g t}\,A = \cancel{\rho g t}\int x\,dA$ and $\bar{y}\,\cancel{\rho g t}\,A = \cancel{\rho g t}\int y\,dA$

$\bar{x}A = \int x\,dA$ and $\bar{y}A = \int y\,dA$

Note: the centroid always lies on lines of symmetry.

Ex:



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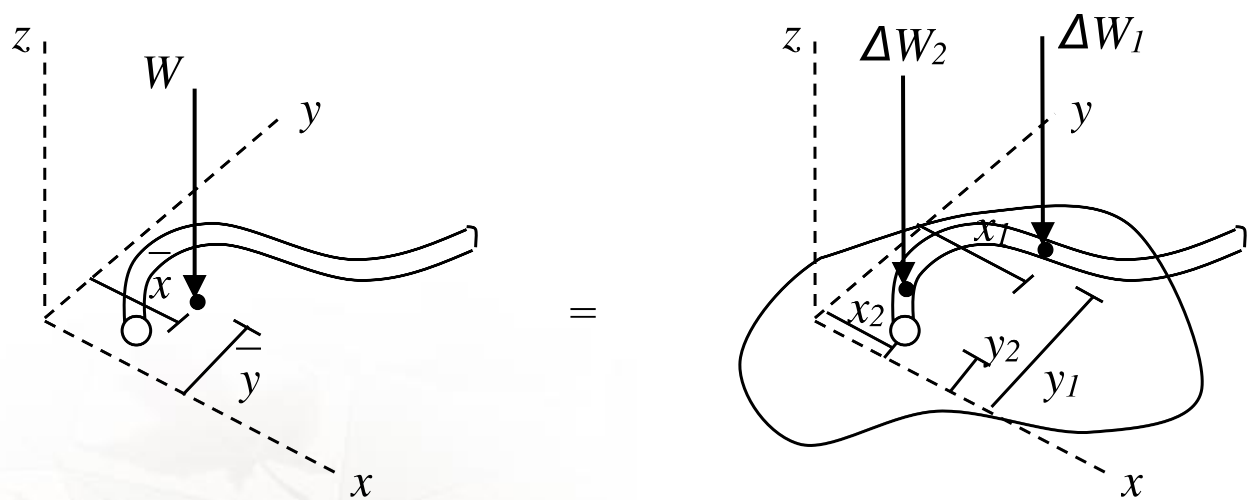
Similarly for Lines



If $W = \rho g a L$ and $\Delta W = \rho g a \Delta L$ where a = cross-sectional area (constant)

L = length

Then $\bar{x}L = \int x\,dL$ and $\bar{y}L = \int y\,dL$

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First Moment of Area

(A measure of the distribution of area w.r.t. an axis)

The first moment of area with respect to the y axis is Qy and with respect to the x axis is Qx.

Defined as: $Qy = \int x dA$ and $Qx = \int y dA$

Thus, from above: $Qy = \bar{x}A$ and $Qx = \bar{y}A$

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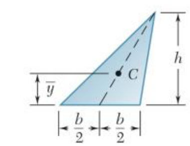
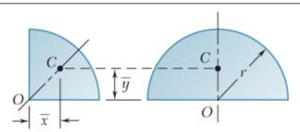
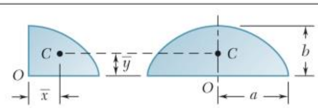
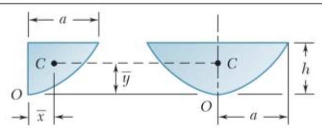
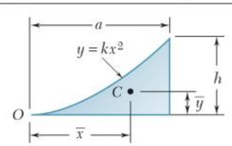
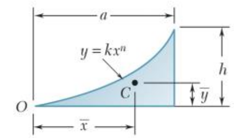
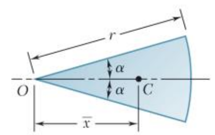
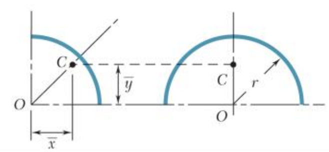
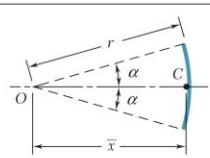
First Moment of Area for Simple Shapes

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2
Shape		$\bar{x}$	$\bar{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r\sin\alpha}{\alpha}$	0	$2\alpha r$

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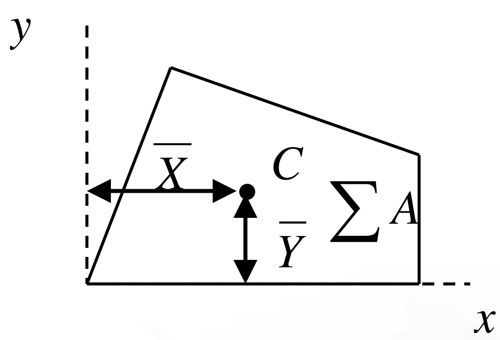
Composite Plates



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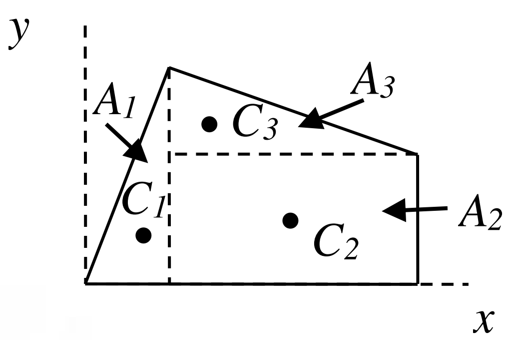

$$\bar{X}\sum A=\sum \bar{x}A \quad \text{and} \quad \bar{Y}\sum A=\sum \bar{y}A$$

or

$$\boxed{Qy=\bar{X}\sum A=\sum \bar{x}A \quad \text{and} \quad Qx=\bar{Y}\sum A=\sum \bar{y}A}$$

Note: area of holes are negative!!

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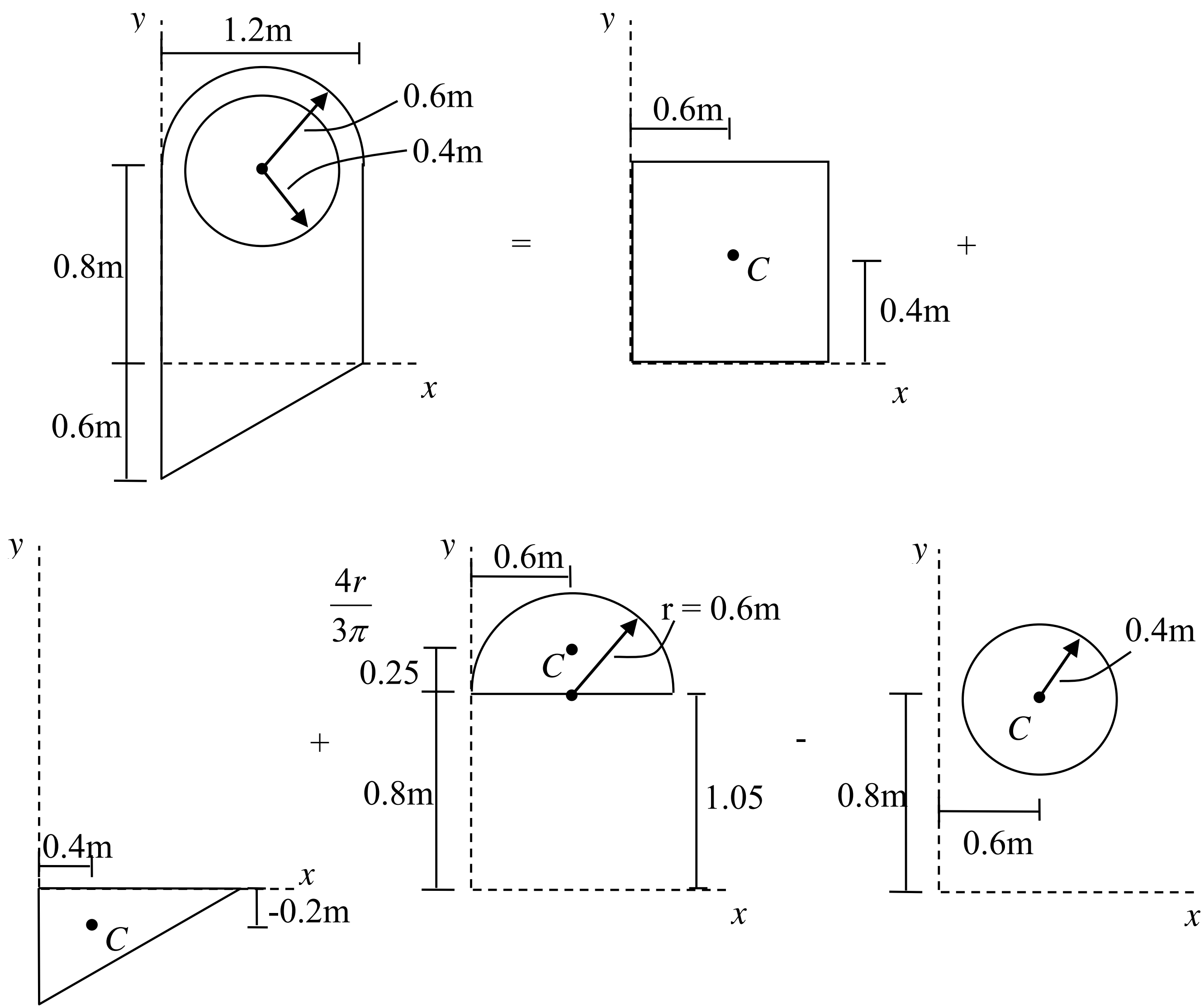
Example:

Determine the centroid



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Make a table:

Components	A **(m²)**	$\overline{x}$ **(m)**	$\overline{y}$ **(m)**	$\overline{x}A$ **(m³)**	$\overline{y}A$ **(m³)**
Rectangle	$1.2\times 0.8=0.96$	0.6	0.4	0.58	0.38
Triangle	$\dfrac{1.2\times 0.6}{2}=0.36$	0.4	-0.2	0.14	-0.07
Semicircle	$\dfrac{\pi(0.6)^2}{2}=0.57$	0.6	1.05	0.34	0.6
Circle	$-\pi(0.4)^2=-0.5$	0.6	0.8	-0.3	-0.4
	$\sum A=1.39$			$\sum\overline{x}A=0.76$	$\sum\overline{y}A=0.51$

$$\overline{X}\sum A=\sum\overline{x}A \quad\Rightarrow\quad \overline{X}=\frac{\sum\overline{x}A}{\sum A}=\frac{0.76\text{m}^3}{1.39\text{m}^2} \quad\Rightarrow\quad \boxed{\overline{X}=0.55\text{m}}$$

$$\overline{Y}\sum A=\sum\overline{y}A\Rightarrow \quad \overline{Y}=\frac{\sum\overline{y}A}{\sum A}=\frac{0.51\text{m}^3}{1.39\text{m}^2} \quad\Rightarrow\quad \boxed{\overline{X}=0.37\text{m}}$$

Therefore: