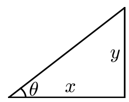
Formula Sheet

Trigonometry



$\sin\theta = \frac{y}{\sqrt{x^2+y^2}}$; $\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$;

Calculus

$\frac{d}{d\theta}\sin\theta = \cos\theta$; $\frac{d}{d\theta}\cos\theta = -\sin\theta$;

$\int x^n dx = \frac{x^{n+1}}{n+1} + constant$

$\frac{d}{dx}x^n = nx^{n-1}$

Vectors

$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$

$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$\mathbf{u}_A = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$

$\mathbf{u}_A = \cos(\alpha)i + \cos(\beta)j + \cos(\gamma)k$

$\mathbf{u}_A = \sin(\phi)\cos(\theta)i + \sin(\phi)\sin(\theta)j + \cos(\phi)k$

$\mathbf{F}_A = F\mathbf{u}_A$

$\mathbf{r}_{B/A} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$

$\mathbf{A}.\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$= (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$

Statics

Moment of a force

Scalar Formulation: $M_o = Fd$

Vector Formulation: $\mathbf{M}_o = \mathbf{r}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$

Moment of a force about a specified axis:

$$\mathbf{M}_{aa} = \mathbf{u}_{aa}.(\mathbf{r}\times\mathbf{F}) = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Simplification of a force and couple system:

$\mathbf{F}_R = \sum\mathbf{F}$

$(\mathbf{M}_R)_o = \sum\mathbf{M} + \sum\mathbf{M}_o$

Particle equilibrium:

$\sum F_x = 0$; $\sum F_y = 0$; $\sum F_z = 0$.

Rigid body equilibrium in 2D:

$\sum F_x = 0$; $\sum F_y = 0$; $\sum M_o = 0$.

Rigid body equilibrium in 3D:

$\sum F_x = 0$; $\sum F_y = 0$; $\sum F_z = 0$;

$\sum M_x = 0$; $\sum M_y = 0$; $\sum M_z = 0$.

Equilibrium equations: Vector formulation:

$\sum\mathbf{F} = 0$; $\sum\mathbf{M}_o = 0$.

Centroid:

$\bar{x} = \frac{\sum x_iA_i}{\sum A_i}$; $\bar{x} = \frac{\int x dA}{\int dA}$

Dynamics

Particle Rectilinear Motion:

Basic relationships	Constant acceleration
$a = \frac{dv}{dt}$	$v = v_0 + a_ct$
$v = \frac{ds}{dt}$	$s = s_0 + v_ot + \frac{1}{2}a_ct^2$
$ads = vdv$	$v^2 = v_0^2 + 2a_c(s - s_0)$

Particle Curvilinear Motion:

x, y, z Coordinates:

$v_x = \dot{x}$ $\quad a_x = \dot{v_x} = \ddot{x}$

$v_y = \dot{y}$ $\quad a_y = \dot{v_y} = \ddot{y}$

$v_z = \dot{z}$ $\quad a_z = \dot{v_z} = \ddot{z}$

n, t, b Coordinates:

$v = \frac{ds}{dt} = \dot{s}$ $\quad a_t = \dot{v} = v\frac{dv}{ds}$

$\rho = \frac{[1+(\frac{dy}{dx})^2]^{3/2}}{|\frac{d^2y}{dx^2}|}$ $\quad a_n = \frac{v^2}{\rho}$

r, θ, z Coordinates:

$v_r = \dot{r}$ $\quad a_r = \ddot{r} - r\dot{\theta}^2$

$v_\theta = r\dot{\theta}$ $\quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$v_z = \dot{z}$ $\quad a_z = \dot{v_z} = \ddot{z}$

$\tan\psi = \frac{r}{dr/d\theta}$

Relative Motion:

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$ $\quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Equations of Motion:

Particle	Rigid body as a particle
$\sum\mathbf{F} = m\mathbf{a}$	$\sum\mathbf{F} = m\mathbf{a}_G$
$\sum F_x = ma_x$	$\sum F_x = m(a_G)_x$
$\sum F_y = ma_y$	$\sum F_y = m(a_G)_y$
$\sum F_z = ma_z$	$\sum F_z = m(a_G)_z$

Work-Energy:

Work: $U_{1-2} = \int\mathbf{F}.d\mathbf{r} = \int F\cos(\theta)ds$

Work-energy principle: $\frac{1}{2}mv_1^2 + U_{1-2} = \frac{1}{2}mv_2^2$

Conservation of mechanical energy:

$\frac{1}{2}mv^2 + mgh + \frac{1}{2}ks^2 = constant$