MATH 3705C
Test 1
January 31, 2014

NAME: **ID#:**

Questions 1-5 are multiple choice. Circle the correct answer. Only the answer will be marked. [20 marks]

1. $\mathcal{L}\{e^{4t}\cos(5t)\} =$

   $(a)\ \dfrac{s+4}{(s+4)^2+25}$ $(b)\ \dfrac{s-4}{(s-4)^2+25}$ $(c)\ \dfrac{s+4}{s^2+25}$ (d) None of the above

2. $\mathcal{L}\{t\sin(2t)\} =$

   $(a)\ \dfrac{4-s^2}{(s^2+4)^2}$ $(b)\ \dfrac{s^2-4}{(s^2+4)^2}$ $(c)\ \dfrac{4s}{(s^2+4)^2}$ (d) None of the above

3. $\mathcal{L}^{-1}\left\{\dfrac{6}{s^2-9}\right\} =$

   $(a)\ e^{3t} - e^{-3t}$ $(b)\ e^{3t} + e^{-3t}$ $(c)\ -\sin(3t)$ (d) None of the above

4. $\mathcal{L}^{-1}\left\{\dfrac{3s+6}{s^2-4s+13}\right\} =$

   $(a)\ 3e^{2t}\cos(3t) + 4e^{2t}\sin(3t)$ $(b)\ 3e^{-2t}\cos(3t) + \dfrac{9}{4}e^{-2t}\sin(3t)$

   $(c)\ 3e^{2t}\cos(3t) + \dfrac{9}{4}e^{2t}\sin(3t)$ (d) None of the above

5. $\mathcal{L}^{-1}\left\{\dfrac{e^{-2s}}{s^2-6s+25}\right\} =$

   $(a)\ u(t-2)e^{3(t-2)}\sin[4(t-2)]$ $(b)\ \dfrac{1}{4}u(t-2)e^{3t}\sin[4(t-2)]$

   $(c)\ \dfrac{1}{4}u(t-2)e^{3t-6}\sin(4t-8)$ (d) None of the above

bcaac

6. [13 marks] Employ the Laplace transform to solve the initial-value problem

$$y'' + 2y' + 10y = 0, \quad y(0) = 1, \; y'(0) = 2.$$

LT 2 $\quad s^2 Y(s) - sy(0) - y'(0) + 2[sY(s) - y(0)] + 10Y(s) = 0$

Y(s) separatⁿ 2 $\quad (s^2+2s+10)\,Y(s) - s - 2 - 2 = 0$

1 $\quad (s^2+2s+10)\,Y(s) = s+4$ or

$(s+1)^2+9$ 2 $\quad Y(s) = \dfrac{s+4}{s^2+2s+1+9} = \dfrac{(s+1)+3}{(s+1)^2+9}$

0 $\quad \mathcal{L}^{-1}\{Y(s)\} = y(t) = \mathcal{L}^{-1}\left\{\dfrac{(s+1)+3}{(s+1)^2+9}\right\}$

Separatⁿ 3 $\quad = \mathcal{L}^{-1}\left\{\dfrac{s+1}{(s+1)^2+9}\right\} + \mathcal{L}^{-1}\left\{\dfrac{3}{(s+1)^2+9}\right\}$

inverse LT 3 $\quad y(t) = e^{-t}\cos 3t + e^{-t}\sin 3t$

7. [12 marks] Employ the Laplace transform to solve the initial-value problem

$$y'' - 2y' + 5y = 2\delta(t-3), \quad y(0) = 1, \ y'(0) = 1.$$

L.T. 2 $s^2Y(s) - sy(0) - y'(0) - 2[sY(s) - y(0)] + 5Y(s) = 2e^{-3s}$

1 $(s^2 - 2s + 5)Y(s) - s - 1 + 2 = 2e^{-3s}$

Y(s) & move over

1 $(s^2 - 2s + 5)Y(s) = s - 1 + 2e^{-3s}$

separatn 3 $Y(s) = \dfrac{s-1}{s^2-2s+5} + \dfrac{2e^{-3s}}{s^2-2s+5}$

0 $\mathcal{L}^{-1}\{Y(s)\} = y(t) = \mathcal{L}^{-1}\left\{\dfrac{s-1}{s^2-2s+1+4}\right\} + \mathcal{L}^{-1}\left\{\dfrac{2e^{-3s}}{s^2-2s+1+4}\right\}$

$(s-1)^2+4$ 2 $y(t) = \mathcal{L}^{-1}\left\{\dfrac{s-1}{(s-1)^2+4}\right\} + \mathcal{L}^{-1}\left\{\dfrac{2e^{-3s}}{(s-1)^2+4}\right\}$

inverse LT 3 $= e^{t}\cos 2t + u(t-3)e^{t-3}\sin 2(t-3)$