

1. Calculate the area of a solid whose base lies within the region bounded between the  $y$ -axis and the parabola  $x = 1 - y^2$ . The cross-sections perpendicular to the  $x$ -axis are semi-circles.
2. Find the volume of a solid of revolution obtained by rotating about the line  $y = -1$  the region bounded by the curves  $y = x$  and  $y = 2\sqrt{x}$ .
3. Let  $\mathcal{R}$  be the area enclosed by  $y = \cos(x)$  and  $y = 2 - \cos(x)$  with  $0 \leq x \leq 2\pi$ 
  - (1) Sketch the region  $\mathcal{R}$  and calculate its area.
  - (2) Let  $\mathcal{S}$  be the solid obtained by a rotating about the line  $x = 2\pi$  the region  $\mathcal{R}$ . Sketch the cross-section of the solid  $\mathcal{S}$  in the  $xy$ -plane, and the thin cylinder obtained by this rotation of  $\mathcal{R}$  between  $x$  and  $x + \Delta x$  for a general  $x$  and  $\Delta x$  very small (i.e. a typical shell at a sample point  $x_i^* \in [x_{i-1}, x_i]$ ). Label its dimensions. What is a good estimate for the radius  $r$ , the height  $h$  and the volume  $V_i$  of this one cylindrical shell?
  - (3) Write down the integral that computes the volume of  $\mathcal{S}$  and evaluate it.
4. Assume you have a chain of uniform density 80 g/m to lift a bag of coal of 400 N from the bottom to the top of a mine of height 170 m. Compute the work done.
5. Consider a reservoir filled with water, created by inverting a 10-m tall pyramid with rectangular base, where the tip of the pyramid is cut off 5 m from the base. The (un-inverted) pyramid's base rectangle is 4 by 5 metres. Compute the work required to pump the water to 1 m over the top of the reservoir.
6. A leaky bucket with mass 5 kg filled with water is lifted from 0 m to 13 m with a rope of negligible weight. At the beginning, the bucket contains 40 L of water and when it arrives at 13 m it only contains 1 L of water. Assume it loses water at a constant rate. What work is required to lift it?