TEST-III MATH 2107B, LINEAR ALGEBRA II, WINTER 2012

NAME: .... ID: ....

TIME: 50 MINUITES TOTAL POINTS: 25

Part A

Circle the correct answers.

Problem 1.(2 points) Let A be a 2×2 matrix with eigenvalues $\lambda_1 = 4$, $\lambda_2 = 1$ corresponding to eigenvectors $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$,$\mathbf{v}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$,respectively. Then $A^3\mathbf{v}_1 = \lambda^3 v_1$

a)$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ b)$\begin{pmatrix} 12 \\ 0 \end{pmatrix}$ c)$\begin{pmatrix} 64 \\ 0 \end{pmatrix}$ d)$\begin{pmatrix} 16 \\ 0 \end{pmatrix}$.

Problem 2.(2 points) A 5×5 matrix has two eigenvalues. The eigenspace corresponding to one of them is 3- dimensional. What must the dimension of the eigenspace of the second eigenvalue be if the matrix is diagonalizable?

a) 5 b)4 c) 3 d)2.

Problem 3.(2 points.) The coordinates of the vector $w = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ with respect to the basis $\left\{ \underset{v_1}{\begin{pmatrix} 3 \\ -1 \end{pmatrix}}, \underset{v_2}{\begin{pmatrix} 1 \\ 3 \end{pmatrix}} \right\}$ are $(c_1, c_2) = \left(\frac{w \cdot v_1}{v_1 \cdot v_1}, \frac{w \cdot v_2}{v_2 \cdot v_2}\right) = \left(\frac{2}{10}, \frac{4}{10}\right) = \left(\frac{1}{5}, \frac{2}{5}\right)$

a) $(\frac{1}{5}, \frac{2}{5})$ b)$(-\frac{1}{5}, \frac{2}{5})$ c) (3,1) d)(1,1).

Problem 4.(2 points) A square matrix Q is orthogonal if and only if $Q^{-1} = Q^T$.
a) True b) False

Problem 5.(2 points) If W is a subspace of $\mathbb{R}^n$, then $\dim W^{\perp} = n - \dim W$.
a) True b) False

Part B

Problem 6.(5 points) Let Q be an orthogonal matrix. Prove that $\det Q = \pm 1$.

Since Q is orthogonal,

$Q^TQ = I$

$\Rightarrow \det(Q^TQ) = \det I = 1$

$\Rightarrow (\det Q^T)(\det Q) = 1$ $[\because \det Q^T = \det Q]$

$\Rightarrow (\det Q)(\det Q) = 1$

$\Rightarrow (\det Q)^2 = 1 \Rightarrow \det Q = \pm 1.$ //