MATH 1007E **Term test #1 – solution** **25 September 2017**

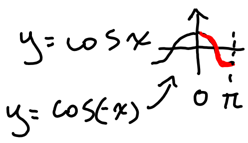
This test was written in room: ________________ Total marks: 30 marks

This test is closed book. **No calculators** or electronic aids are permitted. **Please supply your answers on this sheet.**

PLEASE PRINT ______________________________ ____________
First name Last name Student number

Please show your work where appropriate! TA's have extra paper if you need it. Test duration: 50 minutes.

1. Answer-only questions: $\cos(x-\pi/2) = \cos(-(\pi/2-x)) = \cos(\pi/2-x) = \sin x$



a. [1] TRUE or FALSE: $\cos(x-\frac{\pi}{2}) = \sin x$ ... TRUE FALSE

b. [1] TRUE or FALSE: $y=\cos(-x)$ is decreasing on $[0,\pi]$ ... TRUE FALSE

$g\circ f(x) = g(f(x))$, and $g\circ f(-x) = g(f(-x)) = g(f(x)) = g\circ f(x)$

c. [1] If f is even and g is odd, then what is the symmetry of $g\circ f$? even

d. [1] $\sin^2(\frac{\sqrt{2}}{2}) + \cos^2(\frac{\sqrt{2}}{2}) =$ 1

e. [1] TRUE or FALSE: $|x-a| = -(x-a)$ if $x<a$ ... TRUE FALSE

2. [2] Simplify as much as possible.

a. [2] $\dfrac{\frac{1}{x}-\frac{1}{a}}{x-a} = \dfrac{\frac{a-x}{ax}}{x-a} = \dfrac{-\cancel{(x-a)}}{ax\cancel{(x-a)}} = \dfrac{-1}{ax}$

3. [3] What is the equation of the line whose x–intercept is –3 and which goes through point (3, 2)?

$m = \dfrac{y_1-y_0}{x_1-x_0}$, with $(x_0,y_0) = (-3,0)$, $(x_1,y_1) = (3,2)$

$m = \dfrac{2-0}{3-(-3)} = \dfrac{1}{3}$

$\therefore\ y = \frac{1}{3}x + b \Rightarrow 2 = \frac{1}{3}(3) + b \Rightarrow b = 1$
or $0 = \frac{1}{3}(-3) + b \Rightarrow b = 1$

$\therefore\ y = \frac{1}{3}x + 1$ is the equation of the line

4. [3] Solve the inequality $|2x-5| \geq 7$

NOTE: $|A| \geq b$ means $A \geq b$ or $A \leq -b$ $(b>0)$
so: either $2x-5 \geq 7$ or $2x-5 \leq -7$
... this stems from the 2 cases: $2x-5 \geq 0$ or $2x-5<0$

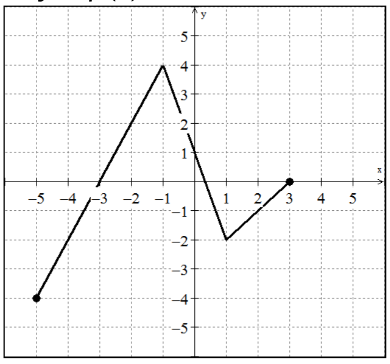
CASE 1: $2x-5 \geq 0 \Rightarrow |2x-5| = 2x-5$
$|2x-5| \geq 7 \Rightarrow 2x-5 \geq 7 \Rightarrow \boxed{x \geq 6}$

CASE 2: $2x-5<0 \Rightarrow |2x-5| = -(2x-5)$
$|2x-5| \geq 7 \Rightarrow -(2x-5) \geq 7$, $2x-5 \leq -7$ $\Rightarrow \boxed{x \leq -1}$

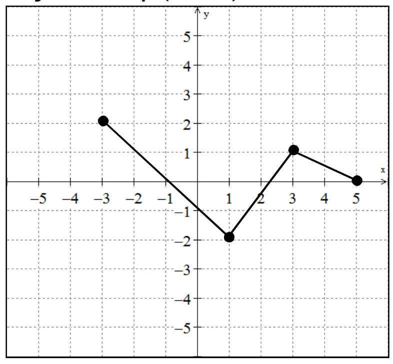
SOLUTION is $(-\infty,-1] \cup [6,\infty)$

5. [3] The graph of f is plotted. Plot the graphs of $y = -\frac{1}{2} f(x-2)$ and $y = f(-x-2) = f(-(x+2))$

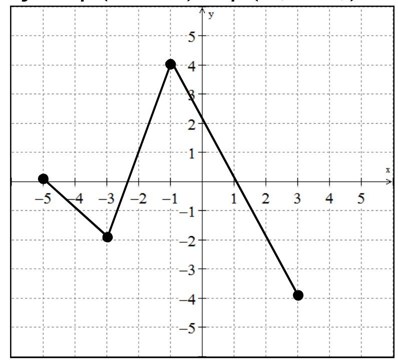
$y = f(x)$



$y = -\frac{1}{2} f(x-2)$



$y = f(-x-2) = f(-(x+2))$



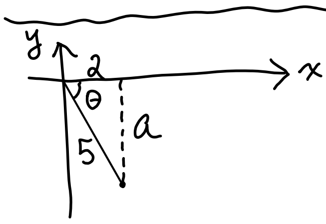
6. Let $\cos\theta = \frac{2}{5}$, with $\frac{3\pi}{2} \le \theta \le 2\pi$

a. [1] Complete the sentence: θ is situated in quadrant 4.

b. [3] Determine $\sin\theta$

$\sin\theta < 0$ in Q4, so: $\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\left(\frac{2}{5}\right)^2} = \boxed{-\frac{\sqrt{21}}{5}}$ $\quad \frac{25}{25} - \frac{4}{25}$

OR



$a = \sqrt{5^2 - 2^2} = \sqrt{21}$

$\therefore \sin\theta = -\frac{a}{5} = \boxed{-\frac{\sqrt{21}}{5}}$

c. [2] Determine $\tan\theta$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\sqrt{21}/5}{2/5} = \boxed{-\frac{\sqrt{21}}{2}}$$

(*For question #7 you may use a graphical technique or one involving identities*)

7. Let $f(x) = \sqrt[4]{5-x}$ and $g(x) = \sqrt{x+4}$

a. [1.5] What are the domains of f and of g? (*answer only ok*)

for g: $x+4 \ge 0 \Rightarrow x \ge -4 \Rightarrow \operatorname{dom}(g) = [-4, \infty) = \{x \in \mathbb{R} \mid x \ge -4\}$

for f: $5-x \ge 0 \Rightarrow x \le 5 \Rightarrow \operatorname{dom}(f) = (-\infty, 5] = \{x \in \mathbb{R} \mid x \le 5\}$

b. [2] What is the domain of $f+g$? (*answer only ok*)

$f+g(x) = \sqrt[4]{5-x} + \sqrt{x+4}$

$5-x \ge 0 \Rightarrow x \le 5$ and $x \ge -4$

$\operatorname{dom}(f+g) = [-4, 5]$

c. [1.5] What is the rule of $f \circ g(x) = f(g(x))$? Do not try to simplify.

$$f \circ g(x) = f(g(x)) = \sqrt[4]{5 - g(x)} = \sqrt[4]{5 - \sqrt{x+4}}$$

d. [3] What is the domain of $f \circ g(x)$?

• presence of $\sqrt{x+4} \Rightarrow x+4 \ge 0 \Rightarrow x \ge -4$

and

• " " $\sqrt[4]{5-\sqrt{x+4}} \Rightarrow 5 - \sqrt{x+4} \ge 0$

$\ldots \Rightarrow \sqrt{x+4} \le 5$

$x + 4 \le 25$

$x \le 21$

$$\boxed{\therefore \operatorname{dom}(f \circ g) = [-4, 21]}$$