Question 1 (10 points):

Water flows steadily downward in the pipe shown in Figure 1 with negligible losses. Determine the flowrate.

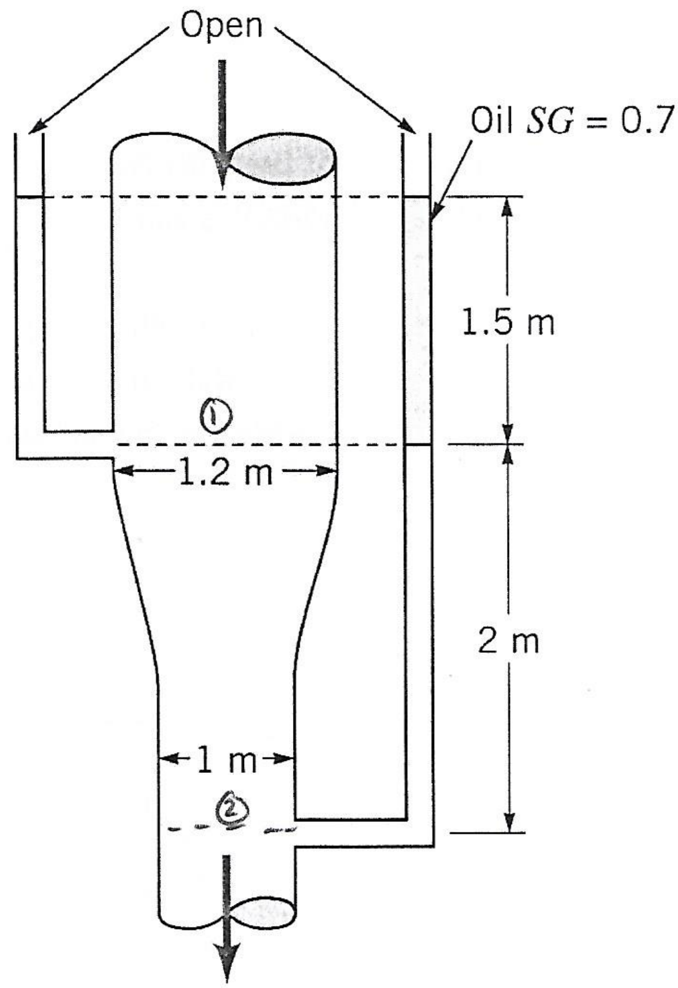


Figure 1.

i) $\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_1$ ✓

$\underline{P_1}$

$P_1 = \gamma_{H2O}(1.5)$

$= 9800(1.5)$

$= 14700$ Pa ✓

$\underline{P_2}$

$P_2 = 0.7\gamma_{H2O}(1.5) + \gamma_{H2O}(2)$

$= 10290 + 19600$

$= 29890$ Pa ✓

ii)

$Q_{in} = Q_{out}$

$V_1 = \frac{V_2 A_2}{A_1}$

$A_1 = \left(\frac{\pi(1.2)^2}{4}\right) = 1.1309\, m^2$

$A_2 = \left(\frac{\pi}{4}\right) = 0.7854\, m^2$

$V_1 = 0.6945 V_2$ ✓

iii)

$$\frac{14700}{9800} + \frac{0.6945^2 V_2^2}{19.62} + 2 = \frac{29890}{9800} + \frac{V_2^2}{19.62}$$

$$1.5 + 0.02458 V_2^2 + 2 = 3.05 + 0.05096 V_2^2$$

$$-0.02638 V_2^2 = -0.45$$

$$V_2 = 4.13 \text{ m/s}$$ ✓

iv)

$Q = V_2 A_2$

$Q = (4.13)(0.7854)$

$\boxed{Q = 3.2438 \frac{m^3}{s}}$ ✓

10

Question 2 (10 points):

CV since Volume is maintained constant

Water is added to the tank shown in Figure 2 through a vertical pipe to maintain a constant (water) level. The tank is placed on a horizontal plane which has a frictionless surface. Neglect all losses.

a) Determine V_1 the speed of the water leaving the tank on the right.
b) Determine V_2 the speed of the water leaving the tank on the left.
c) Determine the horizontal force, F, required to hold the tank stationary.

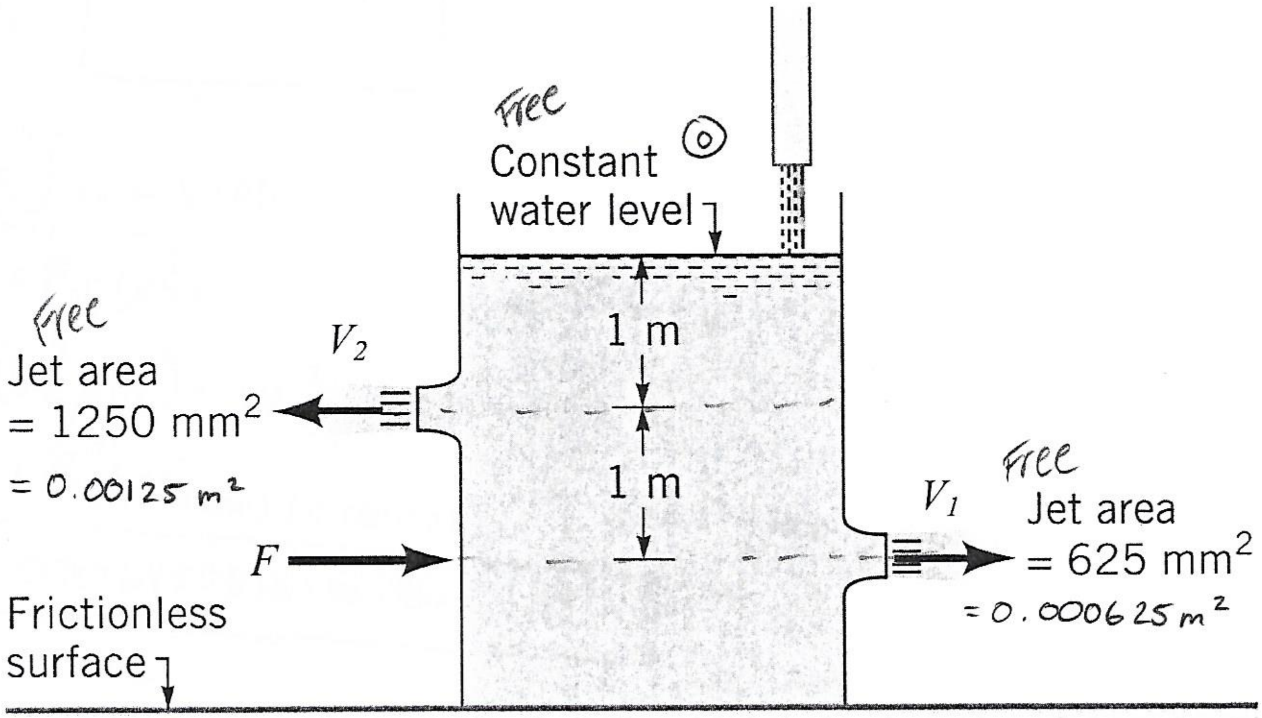


Figure 2.

a) Free jet, large tank, Free jet

$$\cancel{\frac{P_0}{\gamma}} + \cancel{\frac{V_0^2}{2g}} + z_0 = \cancel{\frac{P_1}{\gamma}} + \frac{V_1^2}{2g} + \cancel{z_1}$$

$$2 = \frac{V_1^2}{2(9.81)}$$

$$\boxed{V_1 = 6.2642 \text{ m/s}}$$

✓

b) Free jet, large tank, Free jet

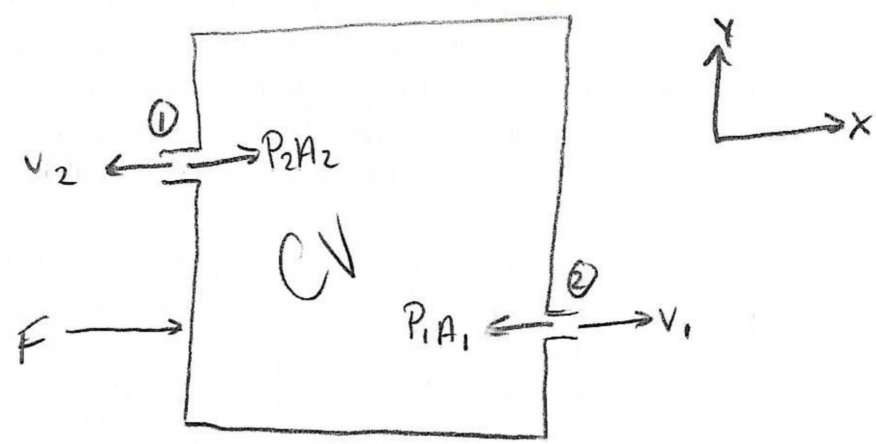
$$\cancel{\frac{P_0}{\gamma}} + \cancel{\frac{V_0^2}{2g}} + z_0 = \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$$1 = \frac{V_2^2}{2(9.81)}$$

$$\boxed{V_2 = 4.4294 \text{ m/s}}$$

✓

C)



$$\Sigma F_x = \int u_x \rho V \cdot dA$$

$$\cancel{P_1 A_1}^{0} + F - \cancel{P_2 A_2}^{0} = -(V_2^2 \rho A_1) + (V_1^2 \rho A_2)$$

$\leftarrow$ Free jet $\rightarrow$

$$F = -(V_2^2 \rho A_2) + (V_1^2 \rho A_1)$$

$$F = -(4.4294^2 (1000)(0.001250)) + (6.2642^2 (1000)(0.000625))$$

$$\boxed{F = 0.00064558\text{N} \text{ to the right}}$$

10

Question 3 (10 points):

Water exits the diffuser at 3 m/s as shown in figure 3.

a) Calculate the power (kW) generated by the turbine if the head losses are 15 m.

b) Consider adding a second turbine on the same pipe line. The second turbine has a shaft power of 60 kW. Calculate the power of the first turbine if the exit velocity and head losses remain the same.

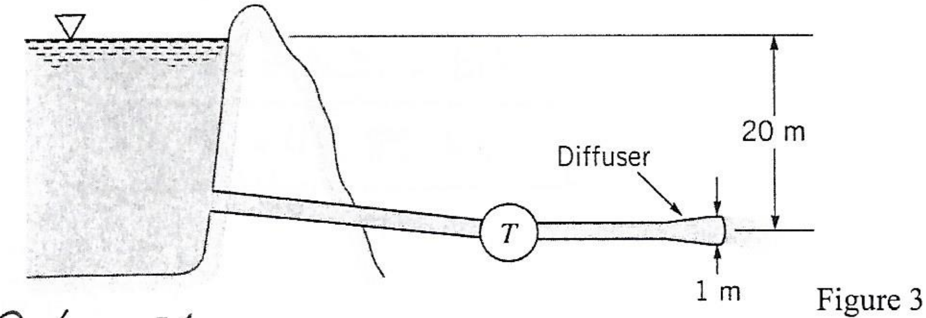


Figure 3

$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + h_s - h_L + z_1 = \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + \cancel{z_2}$$ ✓

10

a)

$$h_s - 15 + 20 = \frac{3^2}{19.62}$$

$h_T = -4.5412$ m (Negative makes sense since turbine uses energy).

$\dot{W}_{Turbine} = \gamma Q h_T$ ✓

$$Q = 3\left(\frac{\pi (1)^2}{4}\right)$$

$= 2.356 \ \frac{m^3}{s}$ ✓

$\dot{W}_{Turbine} - (9800)(2.356)(-4.5412)$

$\dot{W}_{Turbine} = -104850.8586$

$\boxed{\dot{W}_{Turbine} = -104.850 \text{ kW}}$ ✓

b) If the exit velocity and head losses remain the same then the work produced by the turbine calculate in Part a, becomes the Total work produced by both Turbines

$$W_{Total} = W_1 + W_2$$

$$-104.85 = W_1 - 60$$

$$\boxed{W_1 = -44.85 \text{ kW}}$$ ✓