Multiple-Choice Questions
Please choose only one answer and insert in PENCIL in your Scantron sheet.

1. **[3 marks]** Evaluate $\lim_{x\to 1} \frac{x^3-1}{x^2-1}$.

   (a) -1
   (b) $1/2$
   (c) $3/2$
   (d) 0

2. **[4 marks]** Let $f(x) = (\ln x)^x$. Evaluate $f'(e)$, where $e = 2.718\ldots$ is Euler's number. In other words, find the derivative of f at $x = e$.

   (a) $f'(e) = 0$
   (b) $f'(e) = 1$
   (c) $f'(e) = -1$
   (d) $f'(e) = 2$

3. **[3 marks]** Let $f(x) = 3|x-1|$. Calculate

$$L = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}.$$

   (a) $L = 0$
   (b) $L = 1$
   (c) $L = -1$
   (d) This limit does not exist

4. **[4 marks]** Find the derivative of the function f defined by $f(x) = \sqrt{1 + (\ln x)^2}$.

   (a) $\dfrac{\ln x}{x\sqrt{1+(\ln x)^2}}$
   (b) $\dfrac{1}{2\sqrt{1+(\ln x)^2}}$
   (c) $\dfrac{1}{2x\sqrt{1+(\ln x)^2}}$
   (d) $\dfrac{\ln x}{2x^2\sqrt{1+(\ln x)^2}}$

5. **[3 marks]** Let $f(x) = 3^x\, x^3$. Evaluate $f'(1)$. In other words, find the derivative of f at $x = 1$.

(a) $9 + \ln 27$
(b) $2 \ln 3$
(c) $1 + \ln 3$
(d) 0

6. **[3 marks]** A differentiable function f with a differentiable inverse, F, has the property that $f'(1) = -2$ and $F(0) = 1$. What is the value of the derivative of the inverse of f at $x = 0$? That is, calculate $F'(0)$.

(a) 2
(b) $-1/2$
(c) 0
(d) $1/2$

7. **[3 marks]** Find the derivative of the function f defined by $f(x) = \text{Arcsin}\,(\frac{2}{x})$ for $x > 0$.

(a) $f'(x) = -(\frac{2}{x^2})\text{Arccos}(\frac{2}{x})$
(b) $f'(x) = \sqrt{4 - x^2}$
(c) $f'(x) = -\dfrac{2}{x\sqrt{x^2 - 4}}$
(d) $f'(x) = \dfrac{1}{x\sqrt{x^2 - 4}}$

8. **[4 marks]** Evaluate the limit: $L = \lim_{x \to \infty} x \sin\left(\frac{2}{x}\right)$.

(a) $L = 1/2$
(b) $L = 0$
(c) $L = 2$
(d) $L = -1$

9. **[4 marks]** Let y be given implicitly as a differentiable function of x by $y = \cos(x + y)$. Calculate the value of the derivative $\dfrac{dy}{dx}$ at the point (x, y) where $x = \pi/2$, $y = 0$:

(a) $1/3$,
(b) -1,
(c) 0,
(d) $-1/2$

10. **[4 marks]** Evaluate $L = \lim_{x\to\infty} \frac{2^x - 1}{3^x - 1}$ using any method.

    (a) $L = 0$
    (b) $L = 1$
    (c) $L = 2$
    (d) $L = -1$

11. **[4 marks]** Evaluate

$$I = \lim_{t\to 0^+} \frac{d}{dt} \int_1^{\sqrt{t}} \frac{\sin x^2}{x}\, dx$$

    (a) $I = 0$
    (b) $I = \frac{1}{2}$
    (c) This limit does not exist
    (d) $I = \frac{5}{2}$

12. **[4 marks]** The function f defined by $f(x) = 2x^3 + 3x^2 - 12x + 1$ is decreasing at each point of which of the following intervals?

    (a) $0 < x < 7$
    (b) $-6 < x < -4$
    (c) $3 < x < 5$
    (d) $-2 < x < 1$

13. **[4 marks]** For what values of a and b does the function f defined by

$$f(x) = a \ln x + bx^2 + x$$

have the two critical points, $x = 1$ and $x = 2$?

    (a) $a = 0$, $b = 1$
    (b) $a = -2/3$, $b = -1/6$
    (c) $a = -1/2$, $b = 1/3$
    (d) $a = 1$, $b = -1/2$

14. **[4 marks]** Find all the points of inflection of the function f defined by $f(x) = (x+1)^4 + e^{-x}$.

(a) $x = 0$ only.
(b) $x = e$ and $x = -1$ only.
(c) $x = 0$ and $x = 1$ only.
(d) There are no points of inflection.

15. **[4 marks]** Count the **number of points of inflection** of the graph of the function defined by

$$f(x) = \frac{x+1}{x^2+1}.$$

(a) 3
(b) 1
(c) 4
(d) 2

16. **[4 marks]** Determine the interval in which the graph of the function f defined by $f(x) = e^{-x^2}$ is concave up.

(a) $x > \sqrt{2}/2$
(b) $x < 0$
(c) $x < \sqrt{2}/2$
(d) This graph is never concave up.

17. **[4 marks]** Find ALL the horizontal asymptotes of the graph of the function f defined by

$$f(x) = \frac{4x + \sin x}{2x + \cos x}.$$

(a) $y = -1,\ y = 1$ only
(b) $y = 0,\ y = 1$ only
(c) $y = 2$ only
(d) $y = 3$ only

18. **[4 marks]** An antiderivative of $f(x) = \cot(2x+1)$ is given by

(a) $\ln(\sec(2x+1)) - 1$

(b) $\frac{1}{2}\ln|\cos(2x+1)| + 2$

(c) $\frac{1}{2}\ln|\sin(2x+1)|$

(d) $\frac{1}{2}\ln|\csc^2(2x+1)|$

19. **[4 marks]** Evaluate $\int_0^1 x\, e^{-x}\, dx$

(a) 1

(b) $1 - \frac{2}{e}$

(c) $\frac{e^2-1}{2}$

(d) $\frac{e-1}{2}$

20. **[4 marks]** Evaluate $\int t^2 \ln 3t\, dt$.

(a) $\frac{1}{3}t^3\ln(3t) - \frac{t^3}{9} + C$

(b) $\frac{1}{3}t^3 \ln t - \frac{t}{9} + C$

(c) $3(\ln 3t)^2 + \frac{1}{9} + C$

(d) $2t^2(\ln 3t) + \frac{t}{9} + C$

21. **[4 marks]** The improper integral $\int_0^\infty x^2\, 2^{-x^3}\, dx$ has the value

(a) $-\frac{1}{2\ln 3}$

(b) $\frac{1}{2}$

(c) $\frac{1}{9}$

(d) $\frac{1}{3\ln 2}$

22. **[4 marks]** Evaluate and simplify the indefinite integral: $\int x^2 \, e^{3x} \; dx$.

(a) $e^{3x} \; \left(\frac{2}{27} - \frac{2x}{9} + \frac{x^2}{3}\right) + \; C$

(b) $e^{3x} \; \left(\frac{1}{27} - \frac{2x}{9} + \frac{x^2}{27}\right) + \; C$

(c) $e^{-3x} \; \left(\frac{2}{9} - \frac{2x}{27} + \frac{x^2}{6}\right) + \; C$

(d) $e^{3x} \; \left(\frac{1}{27} - \frac{2x}{27} + \frac{x^2}{9}\right) + \; C$

23. **[4 marks]** Evaluate

$$\int \frac{x\,dx}{(x+1)(x-2)}$$

using the method of partial fractions.

(a) $\frac{2}{3}\ln|x+1| + \frac{4}{3}\ln|x-2| + C$

(b) $\frac{1}{3}\ln|x+1| - \frac{2}{3}\ln|x-2| + C$

(c) $\frac{1}{3}\ln|x+1| + \frac{2}{3}\ln|x-2| + C$

(d) $\frac{1}{2}\ln|x+1| + \frac{1}{3}\ln|x-2| + C$

24. **[4 marks]** Evaluate the indefinite trigonometric integral

$$\int \sin^2 x \;\; \cos^3 x \; dx.$$

(a) $\frac{\sin^3 x \;\; \cos^3 x}{3} + \; C$

(b) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + \; C$

(c) $\frac{\sin^3 x}{3} + \frac{\sin^5 x}{5} + \; C$

(d) $\frac{\sin^3 x}{5} - \frac{\sin^5 x}{3} + \; C$

25. **[4 marks]** Find an expression for the volume of the solid of revolution obtained by rotating the region in the first quadrant and between the curves defined by $y = \sqrt{x}$ and $y = x^2$ about the $y-$axis.

(a) $\displaystyle\int_0^1 \pi(x^2 - \sqrt{x})\ dx$

(b) $\displaystyle\int_0^1 \sqrt{x}\ dx$

(c) $\displaystyle\int_0^1 2\pi x\left(\sqrt{x} - x^2\right)\ dx$

(d) $\displaystyle\int_0^1 \pi\left(\sqrt{x} - x^2\right)\ dx$

26. **[5 marks]** Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} = \frac{x^2}{y^3}$$

subject to the initial condition $y(0) = 1$. Then its value at the point $x = 1$, that is, $y(1)$ is given approximately by

(a) $y(1) = \sqrt[4]{\dfrac{7}{3}} \approx 1.236$

(b) $y(1) = 0$

(c) $y(1) = \sqrt[4]{\dfrac{1}{3}} \approx 0.760$

(d) $y(1) = \sqrt[4]{\dfrac{5}{12}} \approx 0.803$

END