MAT1374 Probability and Games of Chance: Poker101
Midterm Examination Solutions

Part 1: Multiple Choice Questions

Indicate your answers in the box on the first page of this booklet. Each question is worth one point.

1. In a game tree, what do the edges represent?

   A. The players

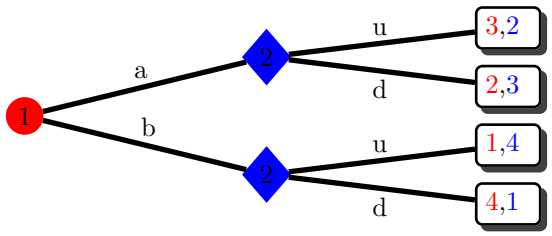
   B. The information available to the players

   C. The moves available to the players

   D. The payoffs

   Answer: C

2. Consider the following game tree:



   Which of the following is correct?

   A. Player 1 should choose b, because that is the only way (s)he can get a payoff of 4.

   B. Player 1 should choose b, because that gives the highest chance of a payoff of 4.

   C. Player 1 should choose b, because (b,d) is the only rollback equilibrium in the game.

   D. None of the above.

   Answer: D

3. What was John von Neumann's role in the development of game theory?

   A. His 1928 article on parlor games laid out the mathematical foundations of game theory.

   B. He proved that all games have Nash Equilibria.

   C. He added a new dimension to the theory of games by introducing strategic moves.

   D. His 1944 book with Oskar Morgenstern *Theory of Games and Economic Behavior* was the first text to consider the competitive nature of the free market.

   Answer: A

4. Thomas Schelling performed an experiment in which he had pairs of strangers try to meet each other somewhere in New York City. What was the purpose of this experiment and what can be concluded from the results?

   A. The purpose was to find out how adept people are at navigating maps. The conclusion was that most people are good at it.

   B. The purpose was to find out whether extrasensory perception plays a role in complex search tasks. The conclusion was that it didn't.

   C. The purpose was to find out whether people can do better than guessing randomly by using focal points. The conclusion was that people rely heavily on such focal points.

   D. The purpose was to find out whether people use game-theoretically optimal strategies in complex search tasks. The conclusion was that most people fall short of complete rationality.

   Answer: C

5. Which of the following statements about dominant strategies is correct? (Assume that all you care about is maximizing your payoffs.)

   A. You should always play a dominant strategy, even if your opponent is irrational.

   B. You should always play a dominant strategy, but only when your opponent is rational.

   C. You should always play a dominant strategy when your opponent does the same thing, but if (s)he doesn't you must change your strategy in order to take maximal advantage.

   D. You should only play a dominant strategy if you are the only player who has one.

   Answer: A

6. What does the "Winner's Curse" in auctions refer to?

   A. In many auctions there are hidden costs and fees for the winner.

   B. The winner of an auction frequently does worse in subsequent auctions.

   C. Many auctions are set up in a way as to encourage excessively high bids.

   D. Sometimes the fact that you win the auction means that you overvalue the item.

   Answer: D

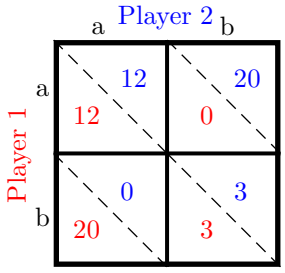
7. What game is this?

   A. Chicken

   B. Battle of the Sexes

   C. Prisoner's Dilemma

   D. Coordination Game



   Answer: C

8. Which of the following behaviours is not rational, in the game-theoretic sense?

   (A) Playing a game of pure chance.

   (B) Accepting a bet in which the chances of losing are higher than the chances of winning.

   (C) Playing a game against an opponent whose behaviour is erratic and unpredictable.

   (D) Playing a dominated strategy in a simultaneous game.

   Answer: D

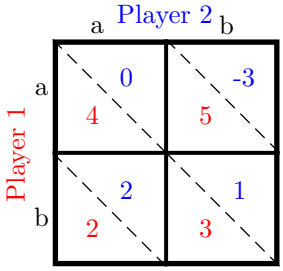
9. In this 2x2 game, which of the following is true?

   A. Neither player has a dominant strategy.

   B. Player 1 has a dominant strategy, player 2 doesn't.

   C. Player 2 has a dominant strategy, player 1 doesn't.

   D. Both players have a dominant strategy.



   Answer: D

10. Consider the following variation on the 21 flags game: each turn, players take 1,2,3, or 4 flags. The player to take the last flag loses. If there are 16 flags left, how many should you take in order to win?

    A. 1

    B. 2

C. 3

D. 4

E. Doesn't matter, this is a lost position.

Answer: E

11. Which of the following statements about Nash Equilibria in a 2-player game is correct?

A. In a Nash Equilibrium, both players play a dominant strategy.

B. In a Nash Equilibrium, no player can improve his/her payoff by unilaterally changing strategies.

C. In a Nash Equilibrium, both players receive the highest payoff in the game.

D. If there is more than one Nash Equilibrium in a game, then all such equilibria must result in the same payoffs for both players.

Answer: B

12. Which of the following most accurately describes the concept of expected value of a bet?

(A) It is the likelihood that you win the bet.

(B) It is the amount you will win when you win the bet.

(C) It is the amount you win on average when you repeat the bet a large number of times.

(D) It is the difference between the payoff when you win and the payoff when you lose.

Answer: C

13. Roulette (European). We bet \$100 on red and \$100 on even (at the same time). Which is most accurate?

(A) The chances of winning are below 50%, and the EV is negative.

(B) The chances of winning are above 50%, and the EV is negative.

(C) The chances of winning are below 50%, and the EV is positive.

(D) The chances of winning are above 50%, and the EV is positive.

Answer: B

14. Suppose we roll two fair six-sided dice. What is the probability that the total is 4 or less?

A. 3/36

B. 4/36

C. 6/36

D. 8/36

E. None of the above.

Answer: B

15. The conclusion drawn from the St Petersburg Paradox is that we need to consider expected utility instead of expected monetary payoffs. Why?

A. Because the St Petersburg bet demonstrates that most people are not willing to take any risks with their money.

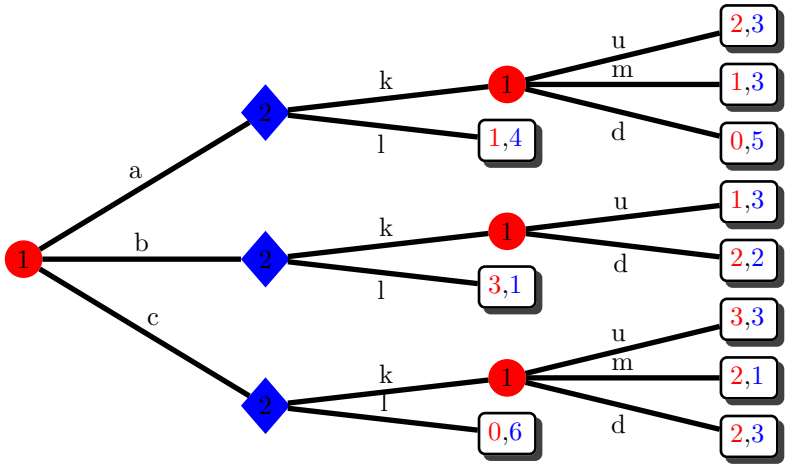
B. Because the St Petersburg bet demonstrates that the risk/reward properties of the bet make it not as appealing to people as the expected monetary payoff suggests.

C. Because the St Petersburg bet demonstrates that some people do not understand how to calculate expected payoffs.

D. Because the St Petersburg bet demonstrates that some bets are inherently unfair and cannot be made appealing regardless of the payoffs.

Answer: B

16. Consider the following game tree.



What is the rollback equilibrium in this game?

A. (a,k,u).

B. (b,k,d).

C. (c,k,u).

D. (a,k).

E. (b,l).

Answer: B

17. In the game of the previous question, how many strategies does Player 1 have?

A. 3

B. 6

C. 11

D. 54

E. None of the above.

Answer: D

18. In class, we discussed the Effort Game, where each player has to choose an effort level, but where the payout depended on the smaller of the two effort levels chosen. What 2x2 game is the Effort Game most similar to?

A. Prisoner's Dilemma

B. Chicken

C. Coordination Game

D. Battle of the Sexes

Answer: C

19. Roulette (European). Suppose I bet \$50 on even and \$50 on number 23. What is the expectation of this bet?

A. $-\$100$

B. $-\$\frac{50}{37}$

C. $-\$\frac{17}{37}$

D. $-\$\frac{100}{37}$.

Answer: D

20. Which of the following statements about the chicken game is correct?

A. Because neither player has a dominant strategy there are no Nash Equilibria.

B. No player has a dominant strategy, but there is exactly one Nash Equilibrium.

C. Both players have a dominant strategy, and there is exactly one Nash Equilibrium.

D. No player has a dominant strategy, and there are two Nash Equilibria.

Answer: D

21. What are the paradoxes in rationality meant to demonstrate?

A. That Expected Utility Theory is too complicated for most people to understand and apply.

B. That most people do not believe mathematics can be used to make correct decisions, even though Expected Utility Theory can be mathematically proven to be correct.

C. That most people can make rational decisions without using Expected Utility Theory.

D. That most people deviate from the principles of Expected Utility Theory because they make mistakes or take other considerations than utility into account.

Answer: D

22. Texas Hold'em. Player 1 holds 6♣6♥, Player 2 holds A♥Q♥, and Player 3 holds Q♦J♦. The board on the river is Q♠4♦A♦6♦4♥. Which of the following is true?

A. Player 1 wins with a full house.

B. Player 2 wins with a full house.

C. Player 3 wins with a flush.

D. None of the above.

Answer: A

23. Texas Hold'em. Player 1 holds 6♥7♥, Player 2 holds Q♥Q♦. On the turn, the board is 4♥5♥Q♠T♠. How many river cards give Player 1 the winning hand?

A. 9

B. 12

C. 13

D. 14

E. None of the above.

Answer: C

24. Texas Hold'em. You have a weak hand, and on the river your opponent bets $800 into the $1000 pot. (You have just enough left to call.) You know that this either either signifies a very strong hand or a complete bluff, and your best estimate is that this is a bluff about one in three times. What is the best reasoning?

A. Call, because the size of the pot is larger than the size of the bet.

B. Call, because the expected value of the call is higher than that of folding.

C. Fold, because the chance of winning the hand is too small.

D. Fold, because the expected value of the call is negative.

Answer: B

25. Which of the following statements about the history of probability theory is most accurate?

A. Probability theory has its origins in Ancient Greece, where mathematicians and philosophers were eager to develop a theory of randomness and probability.

B. Galileo was the first scientist who tried to develop a mathematical approach to probability.

C. The Roman statesman and philosopher Cicero applied mathematics to better understand uncertainty and randomness.

D. Cardano's Law was the first systematic approach to reasoning about events with uncertain outcomes.

Answer: D

26. Why did the popular opinion on game theory become less favourable during the cold war?

(A) Game theory hadn't been able to solve the economic challenges that people were facing.

(B) Game theory was perceived to embody a cold and inhumane perspective on people and their interactions.

(C) Game theory was considered unsuccessful in adequately analysing the nuclear arms race.

(D) Game theory, unlike other hard sciences, did not have a solid mathematical foundation.

Answer: B

27. In class, we played the "Public Investment Game", where you had to decide how much to invest (0-10 chips) and where the payouts depended only on the overall investment. Which of the following best describes the nature of that game?

A. This is a multi-person prisoner's dilemma

B. This is a multi-person coordination game

C. This is a multi-person battle of the sexes game

D. This is a multi-person chicken game

Answer: A

28. What conclusion can be correctly drawn from the Coordination game?

A. That games may have more than one Nash Equilibrium, but that it can be difficult to decide which one to aim for.

B. That in some games rationality doesn't help you in finding the equilibrium strategy.

C. That some games don't have Nash Equilibria.

D. That dominant strategies don't always help in achieving the best outcome.

Answer: A

29. What is the main lesson from the "Truel" as studied in class?

A. That in multi-person games it is sometimes better for the weakest player to have the strong competitors eliminate each other.

B. That in games with uncertainty you can never hope to find a strategy which is always successful.

C. That some games are inherently unfair and are stacked in favour of the strongest player(s).

D. That in multi-player games, the person to move first has a big advantage.

Answer: A

30. Which of the following examples is a good argument that just looking at money gained or lost is not enough for determining someone's payoffs in a game?

A. In last night's poker game, Mr. F made several bad decisions and lost $100.

B. In last night's poker game, Mr. F was outclassed by his opponents but got lucky several times and ended up winning $100.

C. The first time Mr. F went to the casino he was thrilled to win $100, but on subsequent visits he noticed that he needed to win larger and larger amounts in order to get the same thrill.

D. The first time Mr. F went to the casino he played roulette, not knowing that the odds in this game are unfavourable.

Answer: C

BONUS QUESTION [1 point] Which actor plays the role of The Good in "The Good, The Bad and The Ugly?"

Answer: Clint Eastwood

Part 2: Long Answer Questions: choose 2 out of 3 questions.

Write your answers in the space below the questions. Each question is worth 5 points. You get points for clear, structured and relevant motivation of your answers.

Question 1. Consider the following game matrix:

(a) Is (b,q) a Nash Equilibrium? Why (not)?

(b) Find all Nash Equilibria.

Player 1 \ Player 2	p	q	r
a	2, 1	-2, 3	-1, -1
b	1, 2	3, 2	2, 2
c	1, 3	-1, 0	0, 1

Answer: For both parts, use the methods of best responses.

Move by p1	best response by p2
a	p
b	q
c	p
Move by p2	**best response by p1**
p	c
q	a
r	b

Thus (b,q) is not a NE because b is not the best response by p1 to q; in other words, after learning p2 played q, p1 would like to switch to a instead.

The only mutual best response is (c,p), and this is the only NE in the game.

Note: for full marks you need to (i) clearly indicate which method you use and how and (ii) draw the correct conclusion from the method.

Question 2. Explain in no more than 200 words what the difference between normative and behavioural game theory is. Include an example in your explanation.

Answer: Normative game theory describes and analyses how people *should* play. It uses mathematical methods to find optimal strategies and equilibria in games. It operates under the assumption that players are rational, in the sense that they will act in a way that maximizes their expected utility. By contrast, behavioural game theory describes how people actually play in practice, and investigates the psychological, social and cultural reasons why people deviate from optimal play. A good example is the ultimatum game, where the normative theory dictates that player 1 offers the minimal amount to player 2 and player 2 accepts. In practice, cultural standards about fairness, pride and other factors make people play differently, and behavioural game theory performs experiments to study what factors influence people's decision making in such situations.

Question 3. Hold'em. Player 1 has 4♥6♥, Player 2 has A♣A♥, and the board on the turn is 3♥7♥Q♣A♦.

(a) Determine the probability that the river card gives Player 1 a winning hand.

(b) Suppose that the pot on the turn is \$500, and that Player 2 bets his last \$300. Should Player 1 call? (Assume Player 1 also has \$300 left, and that there are no considerations except for making the mathematically correct play.)

Answer: Player 1 can win with a straight or a flush. However, the Q♥is a tainted out, giving player 1 a full house. We thus have the following clean outs:

$$2♥,\ 5♥,\ 5♦,\ 5♠,\ 5♣,\ 8♥,\ 9♥,\ T♥,\ J♥,\ K♥$$

This gives 10 outs. There are $52 - 8 = 44$ unseen cards, hence the probability is $10/44$.

For the second part, we need to find the EV of a call. If player 1 wins, then (s)he gains \$500+\$300= \$800. If (s)he loses, (s)he gains \$300. Thus

$$\text{EV(call)} = \frac{10}{44}(\$800) - \frac{34}{44}(\$300) \approx -\$50$$

This is negative, so folding is better than calling.