

Tutorial #2 Solutions**Problem 1.**Given:

Pipe	L (m)	D (m)	λ
a	3000	1.00	0.014
b	600	0.45	0.024
c	1000	0.60	0.020

$$y_A = 30 \text{ m}$$

$$y_B = 18 \text{ m}$$

$$y_C = 9 \text{ m}$$

Hints:

- net discharge at junction = 0
- set up energy question for each pipe to/from the junction
- assume values of H_j
- calculate discharge at junction
- iterate initial assumption until inflow = outflow

Solution:*Assume flow directions:*

$$A \rightarrow J \quad B \rightarrow J \quad J \rightarrow C$$

Energy equation for pipe a:

$$\frac{P_A}{\gamma} + y_A + \frac{V_A^2}{2g} = \frac{P_J}{\gamma} + y_J + \frac{V_J^2}{2g} + h_{fa} + h_{La}$$

$$0 + y_A + 0 = H_J + 0 + h_{fa} + 0$$

$$y_A - H_J = h_{fa}$$

$$30 \text{ m} - H_J = \frac{V_a^2}{2g} \frac{\lambda_a L_a}{D_a}$$

Where:

$$V_a^2 = \left(\frac{Q_a}{A_a}\right)^2 \quad \text{and} \quad A_a = \frac{\pi D_a^2}{4}$$

Thus:

$$30 \text{ m} - H_f = \frac{\left(\frac{Q_a}{A_a}\right)^2 \lambda_a L_a}{2g D_a}$$

$$Q_a = \sqrt{\frac{(30 \text{ m} - H_f) * 2g D_a * \left(\frac{\pi D_a^2}{4}\right)^2}{\lambda_a L_a}}$$

$$Q_a = \sqrt{\frac{(30 \text{ m} - H_f) * 2 * 9.81 \frac{\text{m}}{\text{s}^2} * 1 \text{ m} * \left(\frac{\pi * 1 \text{ m}^2}{4}\right)^2}{0.014 * 3000 \text{ m}}}$$

$$Q_a = \sqrt{(30 \text{ m} - H_f) * 0.287} \rightarrow \text{Equation \#1}$$

Following same steps, energy equation for pipe b:

$$Q_b = \sqrt{\frac{(18 \text{ m} - H_f) * 2g D_b * \left(\frac{\pi D_b^2}{4}\right)^2}{\lambda_b L_b}}$$

$$Q_b = \sqrt{\frac{(18 \text{ m} - H_f) * 2 * 9.81 \frac{\text{m}}{\text{s}^2} * 0.45 \text{ m} * \left(\frac{\pi * 0.45 \text{ m}^2}{4}\right)^2}{0.024 * 600 \text{ m}}}$$

$$Q_b = \sqrt{(18 \text{ m} - H_f) * 0.0155} \rightarrow \text{Equation \#2}$$

Energy equation for pipe c:

$$\frac{P_J}{\gamma} + y_J + \frac{V_J^2}{2g} = \frac{P_C}{\gamma} + y_C + \frac{V_C^2}{2g} + h_{fc} + h_{Lc}$$

$$\frac{P_J}{\gamma} + y_J + 0 = 0 + y_C + 0 + h_{fc} + 0$$

$$H_J - y_C = h_{fc}$$

$$H_J - 9m = \frac{V_C^2}{2g} \frac{\lambda_c L_c}{D_c}$$

$$H_J - 9m = \frac{\left(\frac{Q_c}{A_c}\right)^2}{2g} \frac{\lambda_c L_c}{D_c}$$

$$Q_c = \sqrt{\frac{(H_J - 9m) * 2g D_c * \left(\frac{\pi D_c^2}{4}\right)^2}{\lambda_c L_c}}$$

$$Q_c = \sqrt{\frac{(H_J - 9m) * 2 * 9.81 \frac{m}{s^2} * 0.6m * \left(\frac{\pi * 0.6m^2}{4}\right)^2}{0.020 * 1000m}}$$

$$Q_c = \sqrt{(H_J - 9m) * 0.047} \rightarrow \text{Equation \#3}$$

Continuity Equation:

$$Q_a + Q_b + Q_c = 0$$

Convention is (+) into junction, (-) out of junction

Thus, continuity equation becomes:

$$Q_a + Q_b - Q_c = 0$$

Assume values of H_J and calculate discharges:

Initial Assumption: $H_J = 18$ m

$$\begin{aligned} \text{Equation \#1} \rightarrow Q_a &= 1.856 \text{ m}^3/\text{s} \\ \text{Equation \#2} \rightarrow Q_b &= 0 \text{ m}^3/\text{s} \\ \text{Equation \#3} \rightarrow Q_c &= 0.650 \text{ m}^3/\text{s} \end{aligned}$$

$$Q_a + Q_b - Q_c = 0$$

$$1.856 \text{ m}^3/\text{s} + 0 \text{ m}^3/\text{s} - 0.650 \text{ m}^3/\text{s} = 1.206 \text{ m}^3/\text{s} \neq 0$$

In order to decrease Q_a , and increase Q_c , H_J must be increased above 18 m. This will cause H_J to be higher than z_B , thus the flow direction will be reversed to $J \rightarrow B$ in pipe b.

Energy equation for pipe b with reversed flow:

$$\frac{P_J}{\gamma} + y_J + \frac{V_J^2}{2g} = \frac{P_B}{\gamma} + y_B + \frac{V_B^2}{2g} + h_{fb} + h_{Lb}$$

$$\frac{P_J}{\gamma} + y_J + 0 = 0 + y_B + 0 + h_{fb} + 0$$

$$H_J - y_B = h_{fb}$$

$$H_J - 18 = \frac{\left(\frac{Q_b}{A_b}\right)^2 \lambda_b L_b}{2g D_b}$$

$$Q_b = \sqrt{\frac{(H_J - 18m) * 2gD_b * \left(\frac{\pi D_b^2}{4}\right)^2}{\lambda_b L_b}}$$

$$Q_b = \sqrt{\frac{(H_J - 18m) * 2 * 9.81 \text{ m}^2/\text{s} * 0.45\text{m} * \left(\frac{\pi * 0.45\text{m}^2}{4}\right)^2}{0.024 * 600\text{m}}}$$

$$Q_b = \sqrt{(H_J - 18m) * 0.0155} \rightarrow \text{New Equation \#2}$$

Iterate assumptions for H_J :

Iteration #2: $H_J = 25 \text{ m}$

$$\text{Equation \#1} \rightarrow Q_a = 1.198 \text{ m}^3/\text{s}$$

$$\text{Equation \#2} \rightarrow Q_b = 0.329 \text{ m}^3/\text{s}$$

$$\text{Equation \#3} \rightarrow Q_c = 0.867 \text{ m}^3/\text{s}$$

$$Q_a + Q_b - Q_c = 0$$

$$1.198 \text{ m}^3/\text{s} - 0.329 \text{ m}^3/\text{s} - 0.867 \text{ m}^3/\text{s} = 0.001 \text{ m}^3/\text{s} \approx 0$$

Reasonably close to 0, thus answer is acceptable.

Calculate h_f in each pipe, sample calculations for Pipe AB:

$$k_s/D_{AB} = \frac{6 \times 10^{-5} \text{ m}}{0.25 \text{ m}} = 2.4 \times 10^{-4}$$

$$V_{AB} = \frac{q_{AB}}{A_{AB}} = \frac{q_{AB}}{\pi \frac{(D_{AB})^2}{4}} = \frac{0.12 \frac{\text{m}^3}{\text{s}}}{\pi \frac{(0.25 \text{ m})^2}{4}} = 2.445 \text{ m/s}$$

$$Re_{AB} = \frac{V_{AB} D_{AB}}{\nu} = \frac{2.445 \text{ m/s} \times 0.25 \text{ m}}{1.13 \times 10^{-6} \text{ m}^2/\text{s}} = 5.41 \times 10^5$$

From Moody Diagram $\rightarrow \lambda_{AB} = 0.0158$

$$h_{f-AB} = \frac{\lambda_{AB} L_{AB} V_{AB}^2}{2gD_{AB}} = \frac{0.0158 \times 600 \text{ m} \times (2.445 \text{ m/s})^2}{2 \times \frac{9.81 \text{ m}}{\text{s}^2} \times 0.25 \text{ m}} = 11.58 \text{ m}$$

Sample for calculating δq increment for next iteration:

$$\delta q = - \frac{\sum h_{fi}}{2 \sum \frac{h_{fi}}{q}} = - \frac{-34.452 \text{ m}}{2 \times 1205.287 \frac{\text{s}}{\text{m}^2}} = 0.014 \text{ m}^3/\text{s}$$

$$q = q_{initial} + \delta q = 0.12 \text{ m}^3/\text{s} + 0.014 \text{ m}^3/\text{s} = 0.134 \text{ m}^3/\text{s}$$

Repeat process for all pipes in Loop #1, determine δq and alter discharge values:

Loop1

Pipe	q (m ³ /s)	L (m)	D (m)	ks/D (-)	V (m/s)	Re (-)	λ^* (-)	h_f (m)	h_f/q (-)	new q (m ³ /s)
AB	0.1200	600.0000	0.2500	2.4E-04	2.445	5.41E+05	0.0158	11.581	96.509442	0.1343
BE	0.0100	200.0000	0.1000	6.0E-04	1.273	1.13E+05	0.0206	3.411	341.14837	0.0243
EF	-0.0600	600.0000	0.1500	4.0E-04	3.395	4.51E+05	0.0174	-40.977	682.95742	-0.0457
FA	-0.1000	200.0000	0.2000	3.0E-04	3.183	5.63E+05	0.0164	-8.467	84.671318	-0.0857
Σ								-34.452	1205.287	
$\delta q =$								0.014		

After 1st iteration of Loop #1, repeat process for Loop #2 using values calculated for BE in Loop #1:

Loop2

Pipe	q (m ³ /s)	L (m)	D (m)	ks/D (-)	V (m/s)	Re (-)	λ^* (-)	h_f (m)	h_f/q (-)	new q (m ³ /s)	
BC	0.0500	600.0000	0.1500	4.0E-04	2.829	3.76E+05	0.0176	28.735	574.69088	0.0478	
CD	0.0100	200.0000	0.1000	6.0E-04	1.273	1.13E+05	0.0206	3.411	341.14837	0.0078	
DE	-0.0200	600.0000	0.1500	4.0E-04	1.132	1.50E+05	0.0190	-4.951	247.55903	-0.0222	
BE	-0.0243	200.0000	0.1000	6.0E-04	3.093	2.74E+05	0.0193	-18.780	773.09668	-0.0265	
								Σ	8.415	1936.495	
										$\delta q = -0.002173$	

Perform 2nd iteration on Loop #1 using value for BE calculated in Loop #2:

Loop1

Pipe	q (m ³ /s)	L (m)	D (m)	ks/D (-)	V (m/s)	Re (-)	λ^* (-)	h_f (m)	h_f/q (-)	new q (m ³ /s)	
AB	0.1343	600.0000	0.2500	2.4E-04	2.736	6.05E+05	0.0157	14.410	107.3012	0.1323	
BE	0.0265	200.0000	0.1000	6.0E-04	3.370	2.98E+05	0.0192	22.187	838.37454	0.0245	
EF	-0.0457	600.0000	0.1500	4.0E-04	2.587	3.43E+05	0.0177	-24.141	528.16078	-0.0477	
FA	-0.0857	200.0000	0.2000	3.0E-04	2.728	4.83E+05	0.0165	-6.272	73.174446	-0.0877	
								Σ	6.184	1547.011	
										$\delta q = -0.002$	

Repeat procedure until acceptable tolerance is achieved (i.e. $\sum h_{fi} < 0.01\text{m}$)

Final Iteration:

Loop1

Pipe	q (m ³ /s)	L (m)	D (m)	ks/D (-)	V (m/s)	Re (-)	λ^* (-)	h_f (m)	h_f/q (-)	new q (m ³ /s)	
AB	0.1316	600.0000	0.2500	2.4E-04	2.680	5.93E+05	0.0158	13.844	105.234	0.1316	
BE	0.0250	200.0000	0.1000	6.0E-04	3.183	2.82E+05	0.0192	19.859	794.36403	0.0250	
EF	-0.0484	600.0000	0.1500	4.0E-04	2.742	3.64E+05	0.0176	-27.027	557.85971	-0.0484	
FA	-0.0884	200.0000	0.2000	3.0E-04	2.815	4.98E+05	0.0165	-6.667	75.379084	-0.0884	
								Σ	0.009	1532.837	
										$\delta q = 0.000$	

Loop2

Pipe	q (m ³ /s)	L (m)	D (m)	ks/D (-)	V (m/s)	Re (-)	λ^* (-)	h_f (m)	h_f/q (-)	new q (m ³ /s)	
BC	0.0466	600.0000	0.1500	4.0E-04	2.634	3.50E+05	0.0177	25.014	537.32882	0.0465	
CD	0.0066	200.0000	0.1000	6.0E-04	0.834	7.38E+04	0.0217	1.540	234.97687	0.0065	
DE	-0.0234	600.0000	0.1500	4.0E-04	1.327	1.76E+05	0.0187	-6.695	285.5309	-0.0235	
BE	-0.0250	200.0000	0.1000	6.0E-04	3.183	2.82E+05	0.0192	-19.854	794.27546	-0.0250	
								Σ	0.005	1852.112	
										$\delta q = -1.42\text{E-}06$	

Thus:

Pipe	AB	BC	CD	DE	EF	AF	BE
Length (m)	600	600	200	600	600	200	200
Diameter (m)	250	150	100	150	150	200	100
Discharge (m^3/s)	0.131	0.047	0.007	0.024	0.048	0.088	0.025
Frictional Head Loss (m)	13.84	25.01	1.54	6.70	27.03	6.67	19.85