Tutorial #3 Solutions

Problem 1.

Given:

Q = 400 L/s = 0.400 m^3/s
h = 12 m
r_1 = 0.25 m
r_2 = 0.50 m
n = 1200 RPM
V_{water} = 4 m/s
η = 90 %
ζ = 85 %
V_{F2} = 4 m/s

Solution:

a) Calculate the ideal pump head (H_{Ideal}):

Pump must meet design head requirement and overcome efficiency losses:

$$H_{ideal} = \frac{h}{\eta} = \frac{12\ m}{0.90} = 13.33\ m$$

b) Calculate the tangential velocity of the fluid (V_w)

V_w is the tangential component of the fluid velocity V_A:

Since

$$Power\ = \rho Q(V_{w2}V_{I2} - V_{w1}V_{I1})$$

And

$$H_{ideal} = \frac{Power}{\rho g Q}$$

Then

$$H_{ideal} = \frac{Power}{\rho g Q} = \frac{V_{w2}V_{I2} - V_{w1}V_{I1}}{g}$$

In a radial pump → $V_{w1} = 0$, and $V_{I2} = 2\pi r_2 n$

$$V_{I2} = 2\pi r_2 n = 2\pi\left(\frac{0.50\ m}{2}\right)\frac{1200\ RPM}{60\ sec/min} = 31.42\ m/s$$

$$H_{ideal} = \frac{V_{w2}V_{I2}}{g} \quad \rightarrow \quad V_{w2} = \frac{H_{ideal} * g}{V_{I2}}$$

$$V_{w2} = \frac{13.33\ m * 9.81\ m/s^2}{31.42\ m/s}$$

$$V_{w2} = 4.16\ m/s$$

c) Calculate the required impeller outer vane angle (α_2):

The outlet vane angle is a function of the difference between the impeller and whirl velocities, and the radial component of the water velocity:

$$\cot(\alpha_2) = \frac{V_{I2} - V_{w2}}{V_{F2}}$$

$$\cot(\alpha_2) = \frac{31.42\frac{m}{s} - 4.16\frac{m}{s}}{4\frac{m}{s}}$$

$$\cot(\alpha_2) = 6.815 = \frac{1}{\tan(\alpha_2)}$$

$$\alpha_2 = \tan^{-1}\left(\frac{1}{6.815}\right)$$

$$\alpha_2 = 8.3°$$

d) Calculate the total input power that must be provided to the pump (P_{in}^{total}):

$$Power \ = \rho g Q H_{ideal}$$

$$Power \ = 1000 \frac{kg}{m^3} * 9.81 \frac{m}{s^2} * 0.40 \frac{m^3}{s} * 13.33\, m$$

$$Power \ = 52{,}306.90\, W = 52.3\, kW$$

Correct for mechanical efficiency losses:

$$Power \ = 52.3 \frac{kW}{\zeta} = 52.3 \frac{kW}{0.85} = 61.5\, kW$$

Problem 2.

Given:

$Q_{required}$ = 60 L/s
D = 200 mm
L = 150 m
H_s = 2 m
λ = 0.028
N = 1385 RPM

Pump No. 1 Performance Data

Head (m)	8.6	8.35	7.56	6.35	4.95	3.7	2.3
Discharge (L/s)	0	18	39	60	75	88	100
Efficiency (%)	0	52	72	79	75	63	48

Pump No. 2 Performance Data

Head (m)	9.0	8.8	8.1	7.0	6.0	4.5	3.3
Discharge (L/s)	0	18	39	60	75	88	100
Efficiency (%)	0	52	75	76	67	58	46

<u>Solution:</u>

Develop system curve equation:

$$H_{SH} = H_S + \frac{\lambda L V^2}{2gD}$$

Assume values of Q and calculate the system head H_{SH}:

Assumption #1 → Q = 0 L/s = 0 m^3/s

$$V = 0\ m/s$$

$$h_f = \frac{\lambda L V^2}{2gD} = 0\ m$$

$$H_{SH} = H_S + h_f = 2.0\ m + 0\ m = 2.0\ m$$

Assumption #2 → Q = 20 L/s = 0.02 m^3/s

$$V = \frac{Q}{A} = \frac{0.02\,\frac{m^3}{s}}{\frac{\pi}{4}(0.2\ m)^2} = 0.637\ m/s$$

$$h_f = \frac{\lambda L V^2}{2gD} = \frac{0.028 * 150\ \mathrm{m} * (0.637\ m/s)^2}{2 * 9.81\,\frac{m}{s^2} * 0.2m} = 0.43\ m$$

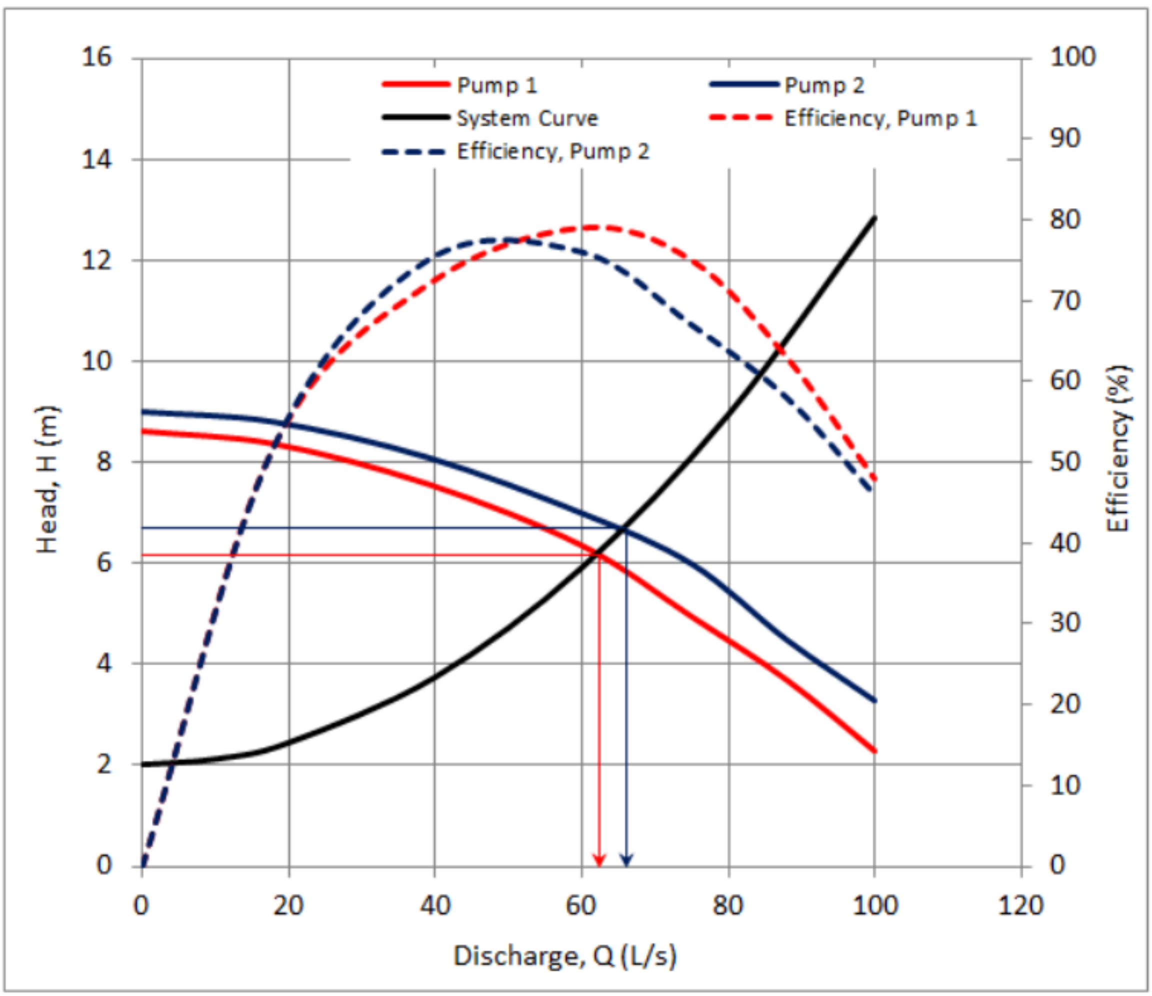
$$H_{SH} = H_S + h_f = 2.0\ m + 0.43\ m = 2.43\ m$$

Repeat these steps for Q = 40, 60, 80, & 100 L/s to make system curve:

System Curve

Q (L/s)	Q (m^3/s)	v (m/s)	λ (-)	h_f (m)	H_s (m)	H_{SH} (m)
0.00	0.00	0.000	0.0280	0.00	2.00	2.00
10.00	0.01	0.318	0.0280	0.11	2.00	2.11
20.00	0.02	0.637	0.0280	0.43	2.00	2.43
40.00	0.04	1.273	0.0280	1.74	2.00	3.74
60.00	0.06	1.910	0.0280	3.90	2.00	5.90
80.00	0.08	2.546	0.0280	6.94	2.00	8.94
100.00	0.10	3.183	0.0280	10.84	2.00	12.84

Plot the system curve with the given pump and efficiency curves:



Obtain the coordinates for the duty point of each pump as the point of intersection between the pump curve and system curve:

Pump #1 → *Q = 63 L/s, H_P = 6.15 m,* η = 79%
Pump #2 → *Q = 66 L/s, H_P = 6.60 m,* η = 73%

Choose Pump #1 because it will deliver the required discharge at a higher efficiency:

$$P_{in} = \frac{P_{out}}{\eta}$$

$$\frac{P_{out}}{\eta} = \frac{\rho g Q H_P}{\eta} = \frac{1000\frac{kg}{m^3} * 9.81\frac{m}{s^2} * 0.063\frac{m^3}{s} * 6.15\ m}{0.79}$$

$$P_{in} = 4.81\ kW$$

$$N_s = \frac{NQ^{1/2}}{H_P^{3/4}} = \frac{1385\ RPM * \left(0.063\ \frac{m^3}{s}\right)^{1/2}}{6.15^{3/4}}$$

$$N_s = 89$$

Choose Mixed Flow Pump ($70 < N_s = 89 < 165$)

Problem 3.

Given:

N = 1000 RPM
H_P = 14 m
Q = 150 L/s = 0.15 m^3/s
P_{atm} = 101.3 kPa
ρ = 1000 kg/m^3
T = 20 ° C
D = 0.20 m
$h_L + h_f$ = 0.3 m

Solution:

Calculate specific speed of pump:

$$N_s = \frac{NQ^{1/2}}{H_P^{3/4}} = \frac{1000\ RPM * \left(0.15\,\frac{m^3}{s}\right)^{\frac{1}{2}}}{(14\ m)^{3/4}} = 54$$

Since $10 < N_s < 70$, a centrifugal pump would be the best choice.

Estimate the critical cavitation parameter (σ_{crit}):

$$\sigma_{crit} = \left(\frac{N_s}{191}\right)^{4/3} = \frac{54}{191}^{4/3} = 0.19$$

Estimate the net positive suction head (NPSH):

$$NPSH_{crit} = H_p \sigma_{crit} = 14\ m * 0.19 = 2.66\ m$$

Set up equation for maximum pump height (z):

$$z \leq \frac{P_{atm} - P_{vap}}{\rho g} - \frac{V_{inlet}{}^2}{2g} - (h_L + h_f) - NPSH_{crit}$$

Where:

$$P_{vap} = 2300\ Pa\ (from\ given\ table)$$

$$V_{inlet} = \frac{0.15 \frac{m^3}{s}}{\frac{\pi}{4}(0.2\ m)^2} = 4.77 \frac{m}{s}\ (from\ continuity)$$

$$(h_L + h_f) = 0.3\ m\ (from\ given\ data)$$

Thus:

$$z \leq \frac{101300\ Pa - 2300\ Pa}{1000 \frac{kg}{m^3} * 9.81 \frac{m}{s^2}} - \frac{\left(4.77 \frac{m}{s}\right)^2}{2 * 9.81 \frac{m}{s^2}} - 0.3\ m - 2.66\ m$$

$$z \leq 5.97\ m \approx 6.0\ m$$

Therefore:

The pump can be located up to 6.0 m above the reservoir, or 106.0 meters above sea level (masl).