

ENGG 3116 Tutorial 4 - Solutions

Q1.

$$Q = 30 \text{ L/s} = 0.03 \text{ m}^3/\text{s}$$

$$H_s = 20 \text{ m}$$

$$L = 100 \text{ m}, \quad D = 15 \text{ cm} = 0.15 \text{ m}$$

$$k_s = 0.12 \text{ mm} = 0.12 \times 10^{-3} \text{ m}$$

Local losses: Ball check valve ($k_L = 70$), entrance ($k_L = 0.5$) and exit ($k_L = 1.0$)

$$\nu = 1.0 \times 10^{-6} \text{ m}^2/\text{s}$$

Q, Hp, $\eta = ?$

Solution:

To deliver the required discharge, each pump must provide a total energy head of:

$$H_p = H_s + \left(\frac{fL}{D} + \sum k_L \right) \frac{v^2}{2g}$$

$$\frac{k_s}{D} = \frac{0.12 \times 10^{-3}}{0.15} = 0.0008$$

$$v = \frac{Q}{A} = \frac{0.03}{\left(\frac{\pi \times 0.15^2}{4} \right)} = 1.698 \text{ m/s}$$

$$Re = \frac{Dv}{\nu} = \frac{0.15 \times 1.698}{1.0 \times 10^{-6}} = 2.55 \times 10^5$$

From Moody diagram: $f = 0.0204$

$$\sum k_L = 70 + 0.5 + 1.0 = 71.5$$

$$H_p = 20 + \left(\frac{0.0204 \times 100}{0.15} + 71.5 \right) \times \frac{1.698^2}{2 \times 9.81} = 32.50 \text{ m}$$

From Figure 5.23, with $Q = 30 \text{ L/s}$ and $H_p = 32.50 \text{ m}$ try Pump I or Pump II.

Obtain more points on the system curve (Q vs H_{SH}) by assuming different values of Q and calculating H_{SH} . Plot Q vs H_{SH} (system curve) on Figure 5.24, Pumps I and II.

The two selected pumps will have the following operating conditions:

Pump I: $n = 3850$ rpm, $Q = 30.5$ L/s, $H_p = 33$ m, $\eta = 41\%$
 $P_{in} = 37$ Hp

Pump II: $n = 3550$ rpm, $Q = 30$ L/s, $H_p = 34$ m, $\eta = 50\%$
 $P_{in} = 28$ Hp.

Pump II would be the better choice as it can supply the required head & discharge at a greater efficiency and lower power input requirements.

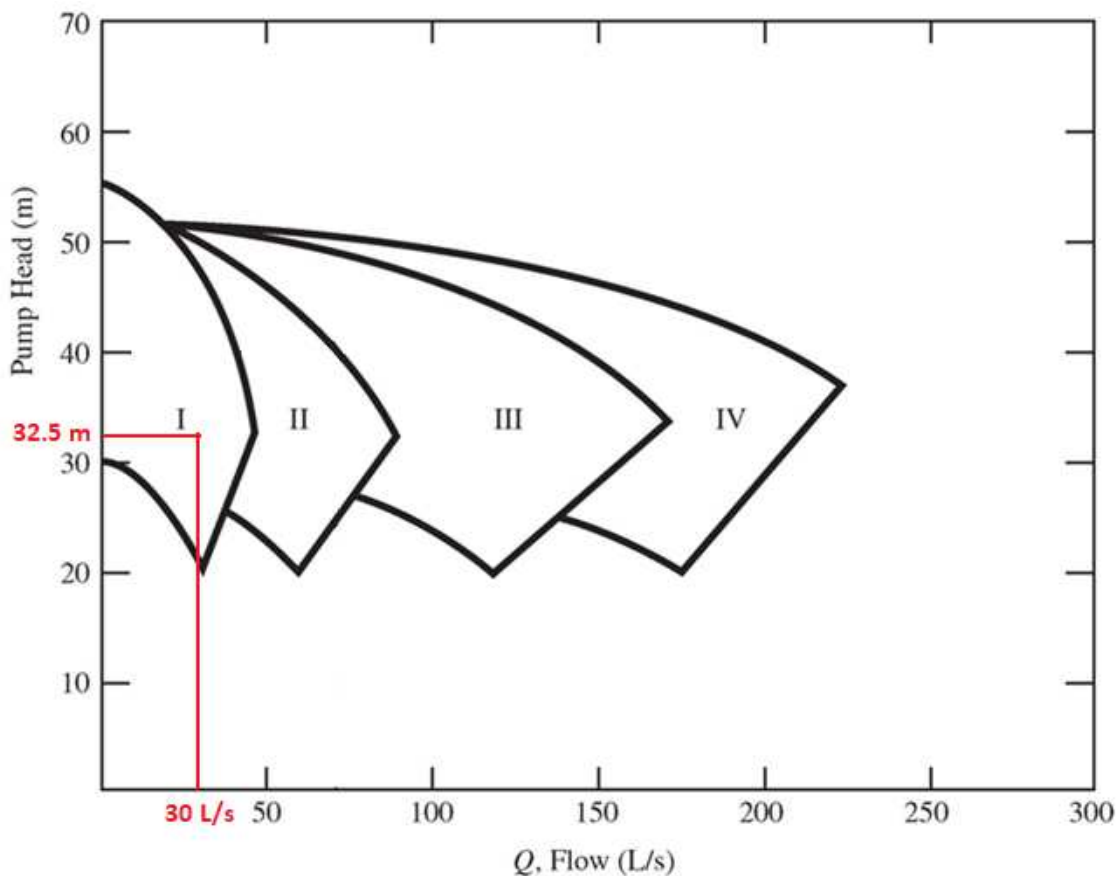
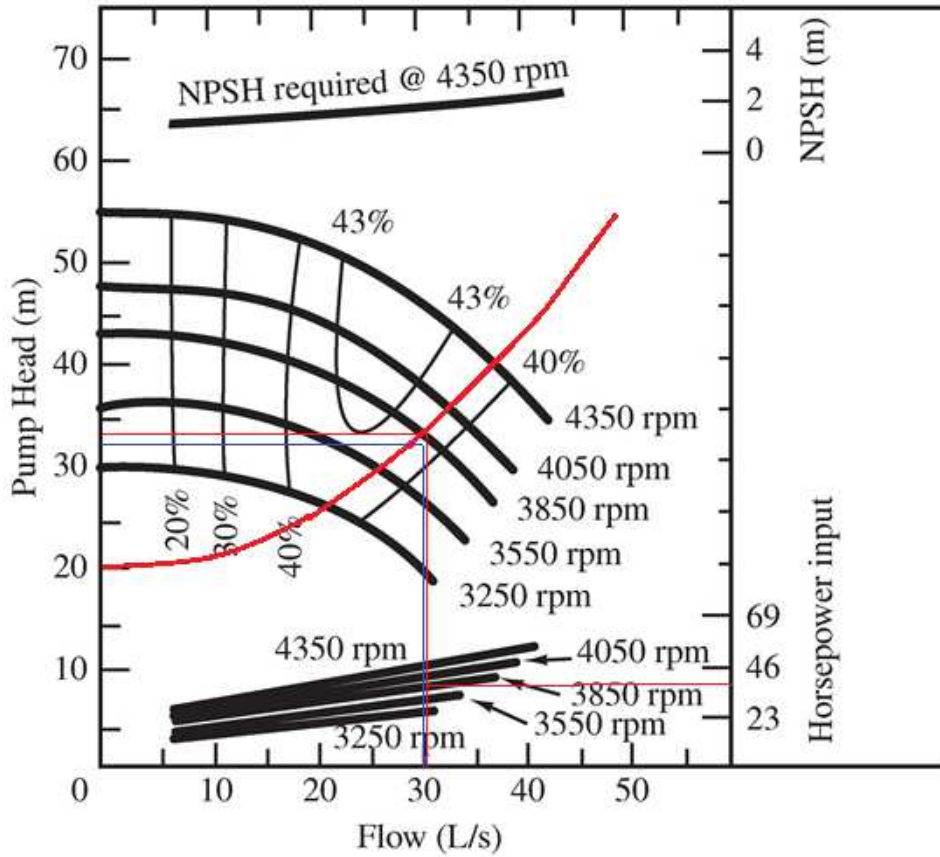
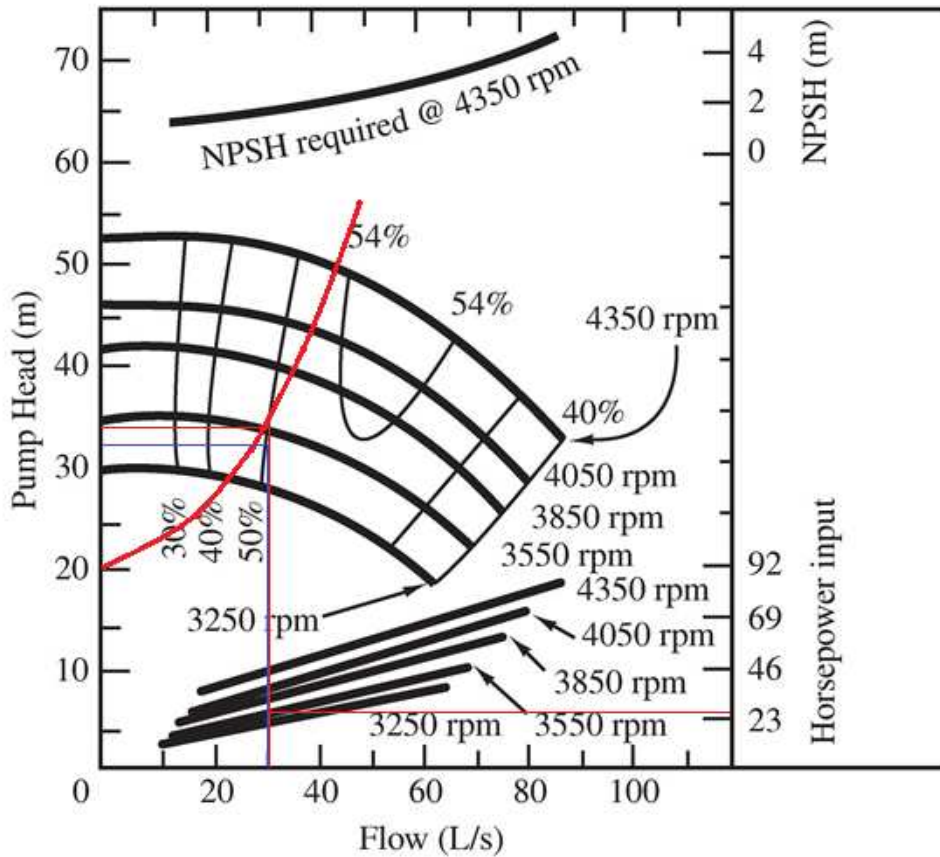


Figure 5.23 Pump model selection chart (Houghtalen et al., 2010)

Pump, I



Pump, II



Question 2

Given:

$$D = 0.4 \text{ m}$$

$$L = 1000 \text{ m}$$

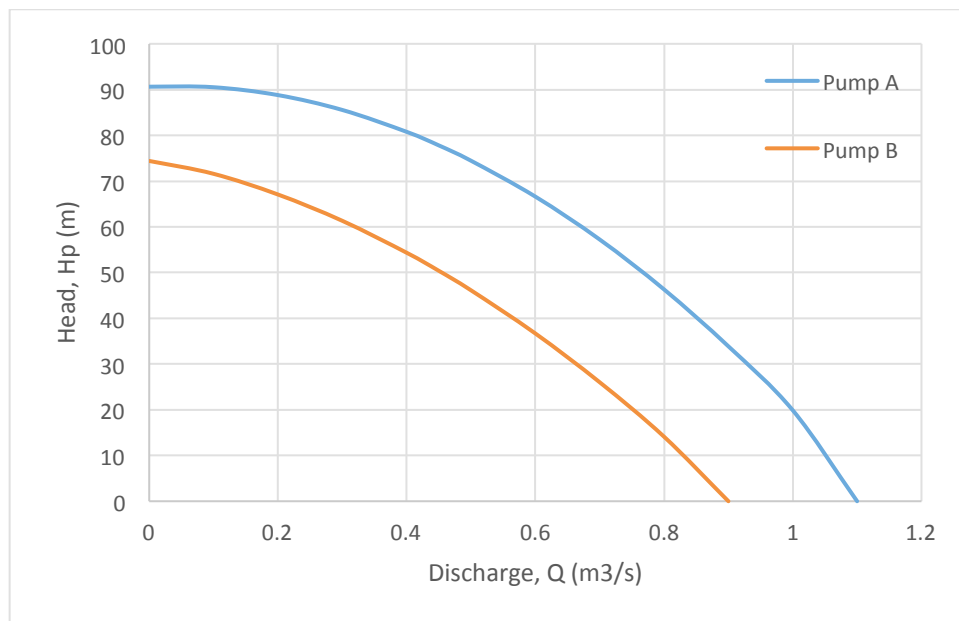
$$h_L = 10 \frac{v^2}{2g}$$

$$\lambda = 0.02$$

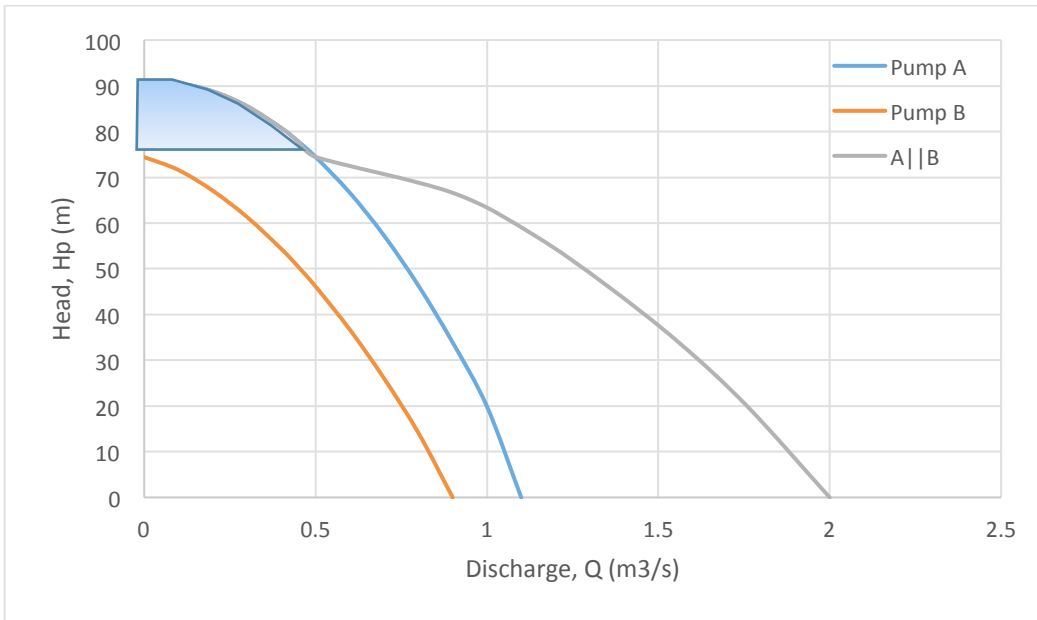
$$\Delta z = 60 \text{ m} \equiv z_{R2} - z_{R1}$$

a) Plot the combined pump curve (H_p vs Q)

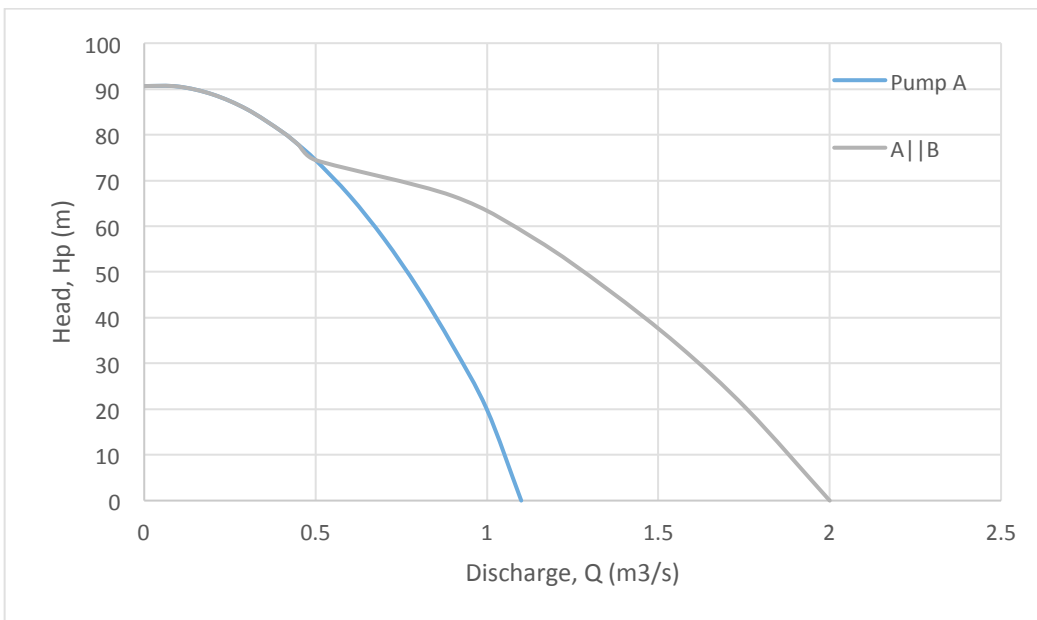
The individual pump performance curves can be generated using the pumps' data given in the tables.



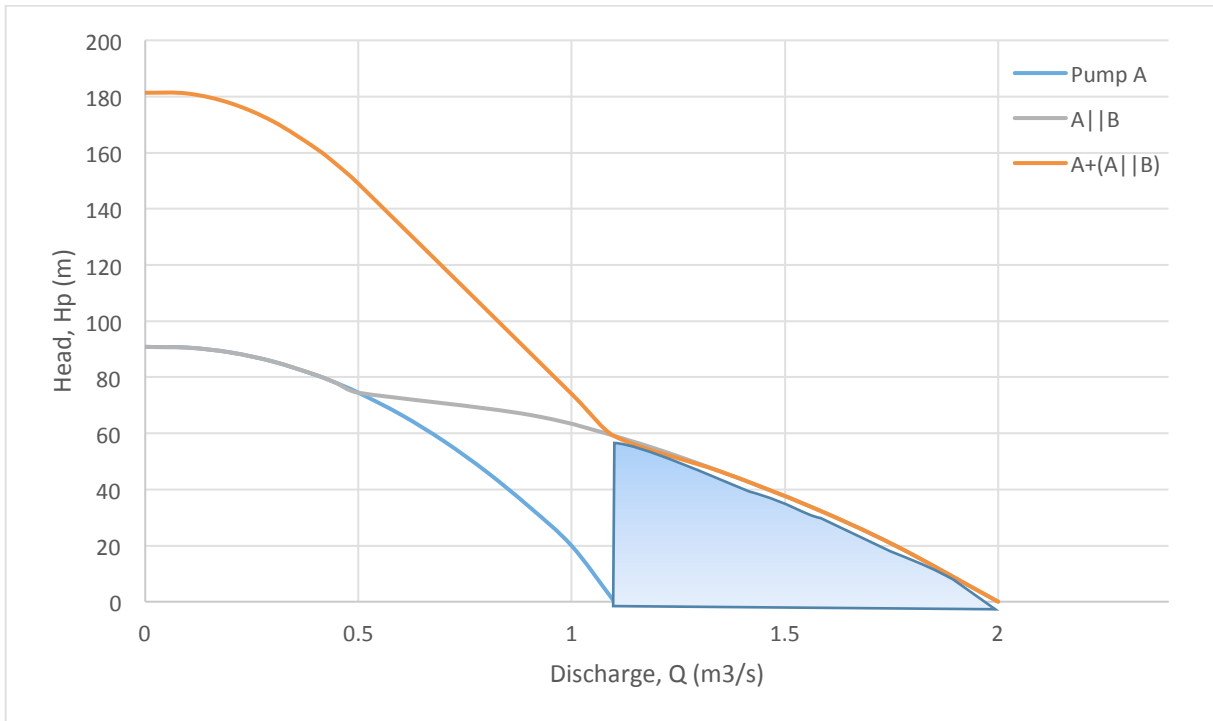
To generate a higher discharge, two of the three pumps are operated in parallel. Thus, the discharges of the individual pumps are summed in order to produce the performance curve for Pumps A and B in parallel; however, in the shaded region, pump B does not work, as its head is lower. Therefore, in this region, the performance curve for pumps A and B working in parallel is the same as for pump A. Afterwards, both discharges are considered.



Performance curve for pumps A and B combined in parallel

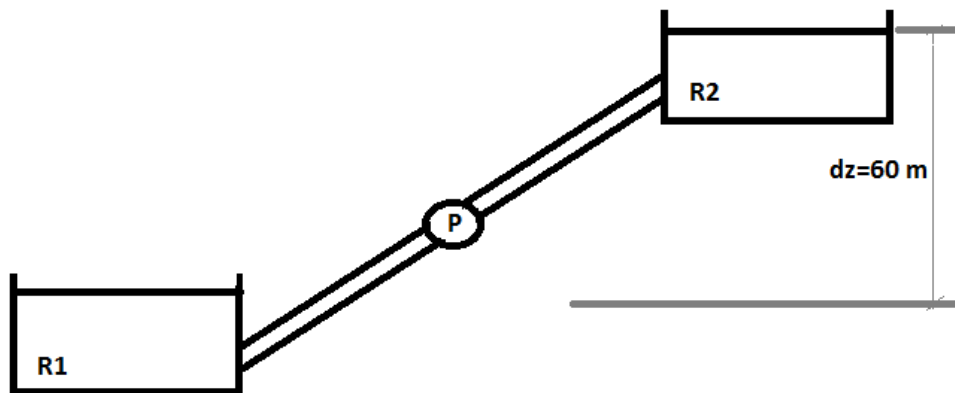


To generate higher head, pump A is connected in series to both pumps A and B. In this case, the heads of the pumps in series must be summed. However, in the shaded region, pump A does not work as its discharge is lower.



b) Plot the system curve (H_p vs Q)

First, write the energy equation between the two reservoirs and calculate Q for different H_p values.



Bernoulli Eq. from R_1 to R_2 :

$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} + H_p = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g} + h_L + h_f$$

$$\Delta z + H_p = h_L + h_f = 10 \frac{V^2}{2g} + \frac{\lambda L V^2}{2gD}$$

$$-60 + H_p = 10 \frac{V^2}{2g} + \frac{0.02 \times 1000}{0.4} \frac{V^2}{2g} = 60 \frac{V^2}{2g} = 3.1 V^2$$

$$V = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}} = \frac{Q}{\frac{\pi (0.4)^2}{4}} \rightarrow V = 7.96 Q$$

Therefore,

$$-60 + H_p = 3.1 (7.96 Q)^2$$

$$-60 + H_p = 196.5 Q^2$$

$$H_p = 196.5 Q^2 + 60$$

$$H_p = 60 \rightarrow Q = 0 \text{ m}^3/\text{s}$$

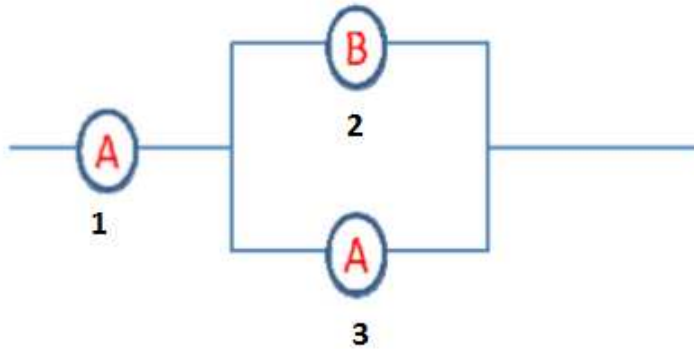
$$H_p = 80 \rightarrow Q = 0.32 \text{ m}^3/\text{s}$$

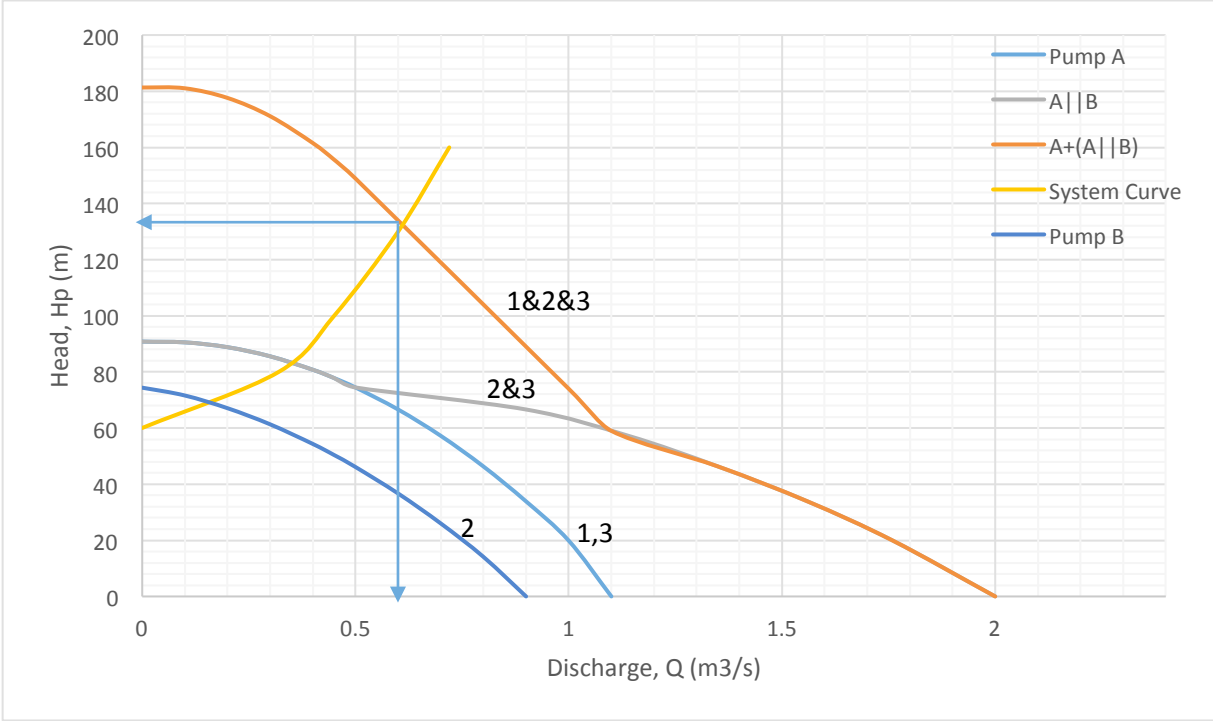
$$H_p = 100 \rightarrow Q = 0.45 \text{ m}^3/\text{s}$$

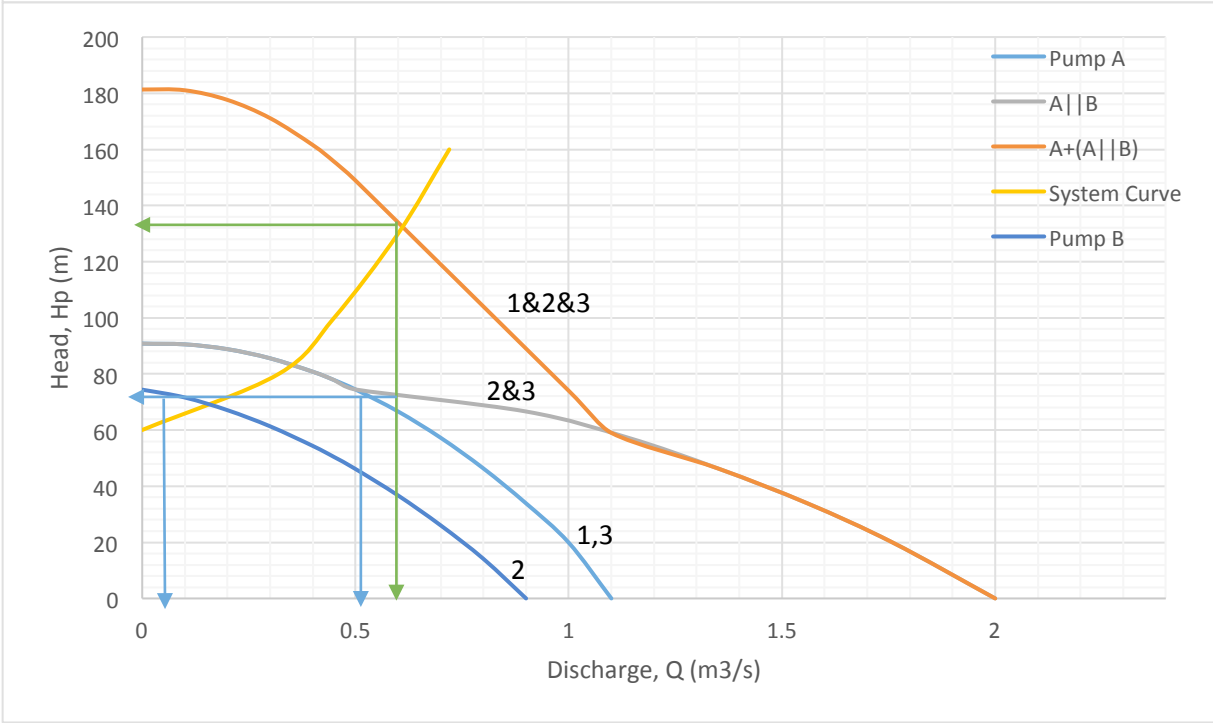
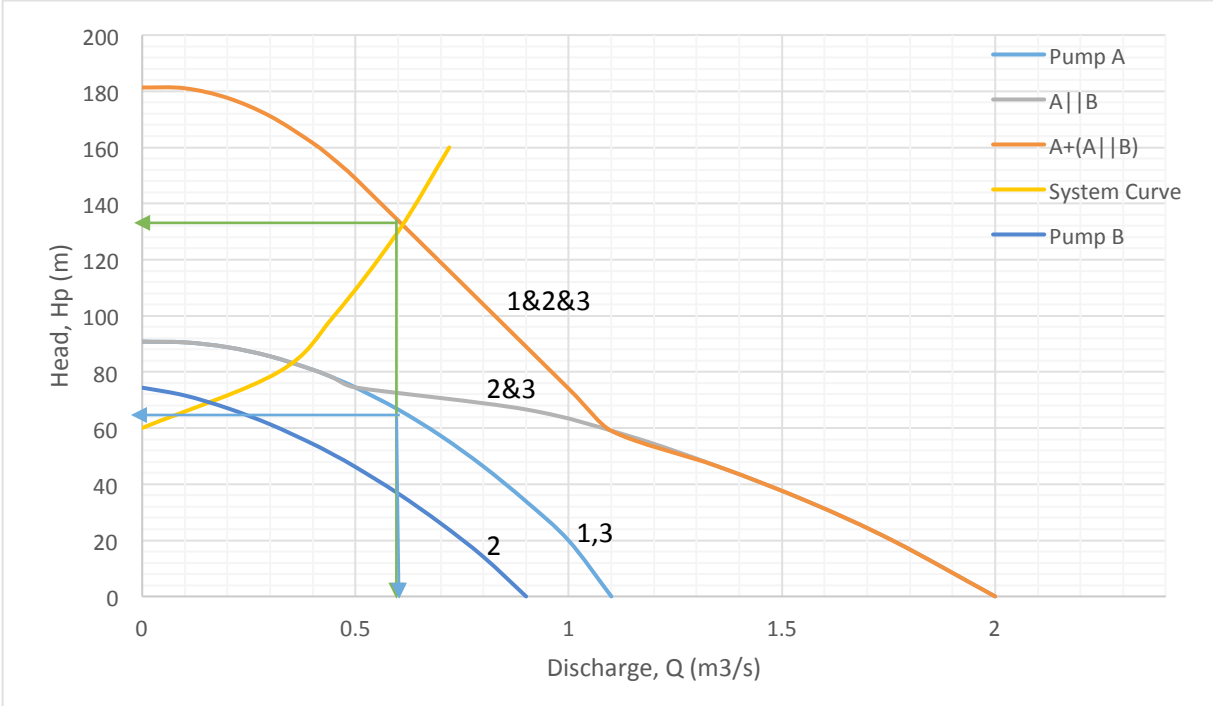
$$H_p = 130 \rightarrow Q = 0.60 \text{ m}^3/\text{s}$$

$$H_p = 160 \rightarrow Q = 0.72 \text{ m}^3/\text{s}$$

- c) Find the discharge and the head in the system and define the operating conditions for each of the three pumps.







$$\text{Duty point of the system} \begin{cases} H_{pt} = 134 \text{ m} \\ Q_t = 0.61 \text{ m}^3/\text{s} \end{cases}$$

$$\text{Pump 1} \begin{cases} H_{p1} = 64 \text{ m} \\ Q_1 = 0.61 \text{ m}^3/\text{s} \\ \eta_1 = 69 \% \end{cases}$$

$$\text{Pump 2} \begin{cases} H_{p2} = 70 \text{ m} \\ Q_{t2} = 0.08 \text{ m}^3/\text{s} \\ \eta_2 = 15 \% \end{cases}$$

$$\text{Pump 3} \begin{cases} H_{p3} = 70 \text{ m} \\ Q_{t3} = 0.53 \text{ m}^3/\text{s} \\ \eta_3 = 66 \% \end{cases}$$

Note:

$$Q_1 = Q_2 + Q_3 = Q_t = 0.61 \text{ m}^3/\text{s}$$

$$H_{p2} = H_{p3}$$

$$H_{pt} = H_{p1} + H_{p2} = H_{p1} + H_{p3} \cong 134 \text{ m}$$

