Tutorial #5 Solutions

Problem 1.

Given:

$A_1 = 930\ m^2$
$A_2 = 185\ m^2$
$K_s = 4.6 \times 10^{-5}$ m
L = 3050 m
$H_{initial} = 30$ m
$Q_1 = Q_2 = 0\ m^3/s$
D = 0.3 m
$\nu = 1.007 \times 10^{-6}\ m^2/s$
$\Delta H_1 = 3$ m
$K_{Li} = 1.0$

Solution:

Since no inflow or outflow, volume leaving reservoir 1 must equal volume entering reservoir 2.

$$\Delta V_1 = \Delta V_2$$

$$\Delta H_1 A_1 = \Delta H_2 A_2$$

$$3\ \mathrm{m} * 930\ \mathrm{m}^2 = \Delta H_2 * 185 m^2$$

$$\Delta H_2 = 15.08\ m$$

*Therefore, the difference in reservoir levels at time **t** will be:*

$$H_t = 30\ m - 3m - 15.08m$$

$$H_t = 11.92\ m$$

Determine average value of λ:

Assuming fully rough turbulent

$$\frac{K_s}{D} = \frac{4.6x10^{-5}\ m}{0.3\ m} = 0.00015 \rightarrow \lambda = 0.0145$$

At initial conditions H = 30 m

$$30\ m = \left(\frac{0.0145 * 3050\ m}{0.3\ m} + 1\right)\frac{V^2}{2 * 9.81\ m/s^2} \rightarrow V = 1.99\ m/s$$

$$Re = \frac{vD}{\nu} = \frac{1.99\frac{m}{s} * 0.3\ m}{1.007\ x\ 10^{-6}\ m^2/s} = 5.9\ x\ 10^5 \rightarrow \lambda = 0.015$$

At final conditions H = 11.92 m

$$11.92\ m = \left(\frac{0.0145 * 3050\ m}{0.3\ m} + 1\right)\frac{V^2}{2 * 9.81\ m/s^2} \rightarrow V = 1.26\ m/s$$

$$Re = \frac{vD}{\nu} = \frac{1.26\frac{m}{s} * 0.3\ m}{1.007\ x\ 10^{-6}\ m^2/s} = 3.75\ x\ 10^5 \rightarrow \lambda = 0.0155$$

Thus

$$\lambda = \lambda_{avg} = \frac{\lambda_1 + \lambda_2}{2} = \frac{0.015 + 0.0155}{2} = 0.0153$$

Determine value of K:

$$K = \frac{A\sqrt{2g}}{\sqrt{\frac{\lambda L}{D} + \sum K_{Li}}} = \frac{\frac{\pi}{4}(0.3\ m)^2\sqrt{2 * 9.81\ m/s^2}}{\sqrt{\frac{0.0153 * 3050\ m}{0.3\ m} + 1}} = 0.025$$

Set up differential equation:

$$\mathrm{dt} = \frac{\mathrm{dH}}{-KH^{0.5}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)} = \frac{dH}{-0.025 * H^{0.5} * \left(\frac{1}{930\ m^2}+\frac{1}{185\ m^2}\right)}$$

$$\mathrm{dt} = \frac{\mathrm{dH}}{-0.00016 * H^{0.5}} = \frac{-6172 * dH}{H^{0.5}}$$

$$\int \mathrm{dt} = \int \frac{-6172 * dH}{H^{0.5}}$$

$$\int \mathrm{dt} = \int_{30}^{11.92} \frac{-6172 * dH}{H^{0.5}}$$

$$\mathrm{t} = |_{30}^{11.92}(-12344 * H^{0.5})$$

$$\mathrm{t} = (-12344 * 11.92\ m^{0.5}) - (-12344 * 30\ m^{0.5}) = 24993\ s = 6.94\ hrs$$

Problem 2.

Given:

L = 3000 m
U_0 = 2.5 m/s
C = 1500 m/s

Solution:

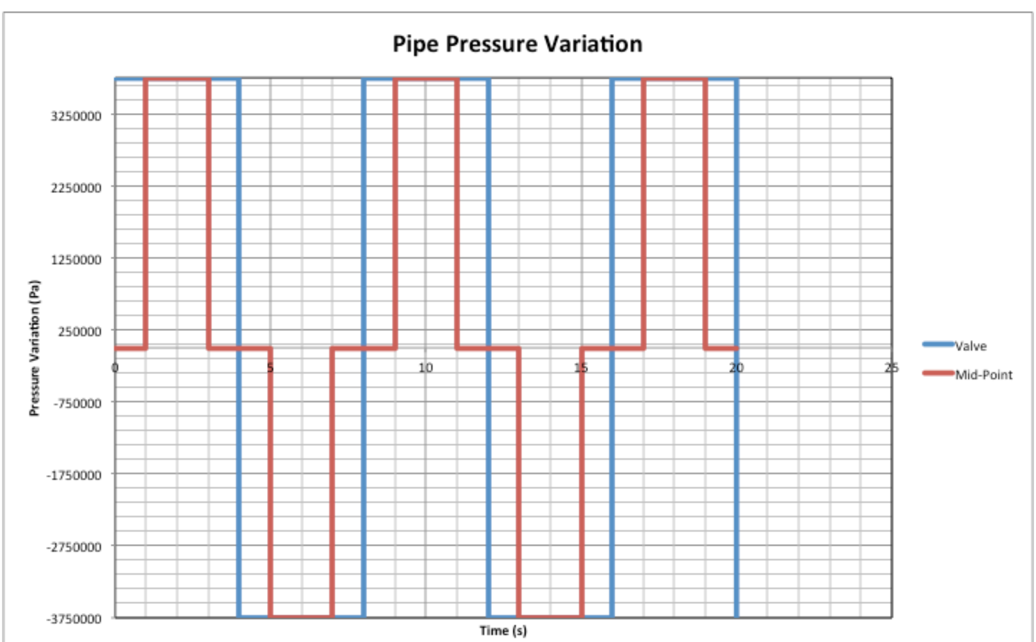
Estimate the increase is pressure due to the valve closure:

$$\Delta P = \rho C U_0 = 1000\frac{kg}{m^3} * 1500\frac{m}{s} * 2.5\frac{m}{s} = 3.75x10^6\ Pa$$

Plot the variation in pressure at the valve and mid-point of the pipe:

$$t = \frac{L}{C} = \frac{3000\ m}{1500\ m/s} = 2\ s$$

The increase in pressure at the valve will be maintained while the shock wave travels to the reservoir and the reflected decompression wave returns, i.e. 4 s. Similarly the pressure will be negative for another 4 s following this, and will continue to oscillate. Pressure at the midpoint takes 1 s to arrive; the initial increase is maintained while the wave reaches the reservoir and the reflected decompression wave returns, this takes 2 s. Similarly, this process will continue to oscillate between positive and negative.



Problem 3.

Given:

$n = 0.013$
$S_0 = 0.001$
$W_{bottom} = 3.0$ m
$m = 3H : V1$
$D_n = 2.1$ m

Solution:

Determine hydraulic radius:

$$R_h = \frac{A}{P_w} = \frac{(B + my)y}{B + 2y\sqrt{1 + m^2}} \ (for\ trapezoidal\ channels)$$

$$R_h = \frac{(3.0\ m + 3 * 2.1\ m)2.1\ m}{3.0\ m + 2 * 2.1\ m\sqrt{1 + 3^2}} = \frac{19.53\ m^2}{16.28\ m} = 1.20\ m$$

Determine normal flow velocity using Manning's equation:

$$V = \frac{1}{n} * R_h^{\frac{2}{3}} * S_0^{\frac{1}{2}}$$

$$V = \frac{1}{0.013} * 1.20\ m^{\frac{2}{3}} * 0.001^{\frac{1}{2}} = 2.75\ m/s$$

Determine channel discharge:

$$Q = VA = 2.75\frac{m}{s} * 19.53\ m^2 = 53.7\frac{m^3}{s}$$