CVG 3116 Fall 2017

Hydraulics – Tutorial 6
November 7, 2017

University of Ottawa

Lecture 9: Rapidly Varried Flows (Specific Energy)

RVFs occur when there is:

- sudden change in channel geometry
- sudden change in the regime of the flow (we will see this soon)

Specific Energy Equation – the energy of the flow referred to channel bed as datum

$$E_s = y + \alpha \frac{V^2}{2g} \qquad E_s = y + \alpha \frac{(Q/A)^2}{2g}$$

- Total discharge – **Q**
- Discharge per unit width – **q = Q/b**

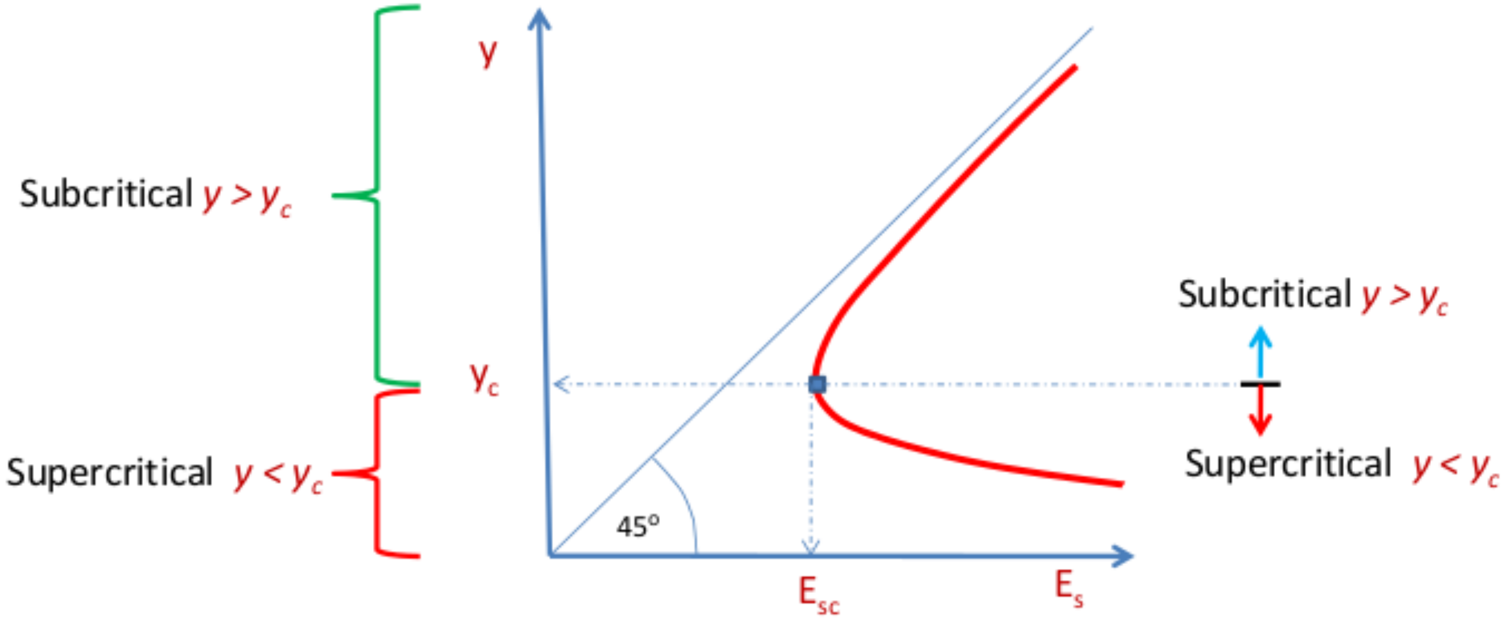
Consider a rectangular channel: $Q/A=(bq)/(by)=q/y$ where b is the channel width. Thus:

$$E_s = y + \alpha \frac{q^2}{2gy^2}$$

Lecture 9: The Critical Depth, Subcritical and Supercritical Flow

For a given discharge, the critical depth y_c is the depth corresponding to ***the minimum specific energy*** E_{sc}

For a given discharge,
If the flow is deeper than the critical depth y_c , it is called ***subcritical***. i.e. $y > y_c$
If the flow is shallower than the critical depth y_c , it is called ***supercritical***. i.e. $y < y_c$



The minimum specific energy (called critical specific energy)

Lecture 9: The Critical Depth

At the critical depth the following equation holds:

$$\frac{\alpha Q_{\max}^{2} B_c}{g A_c^{3}} = 1$$

where

B_c is the channel surface width at the critical depth

A_c is the channel cross section area at the critical depth

- **Special Case** – Rectangular Channel : $\alpha = 1 \quad , \quad B = b \quad , \quad Q = V_c A = V_c b y_c$

$$V_c = \sqrt{g y_c}$$

$$y_c = \frac{2}{3} E_c \qquad\qquad y_c = \sqrt[3]{\frac{q^2}{g}}$$

Lecture 9: Applications

➢ ***A rise in the channel bed***

A rise in the channel bed leads to changes in water depth and velocity.

Consider two points 1 (before the hump) and 2 (after the hump)

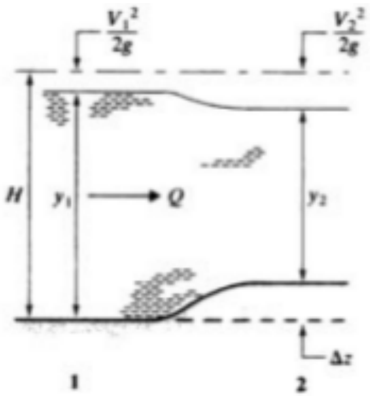


Figure 5.9 Flow transition problem.

1- Energy equation (taking α=1)

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + \Delta z \quad \text{or} \quad E_{s1} = E_{s2} + \Delta z$$

2- Continuity

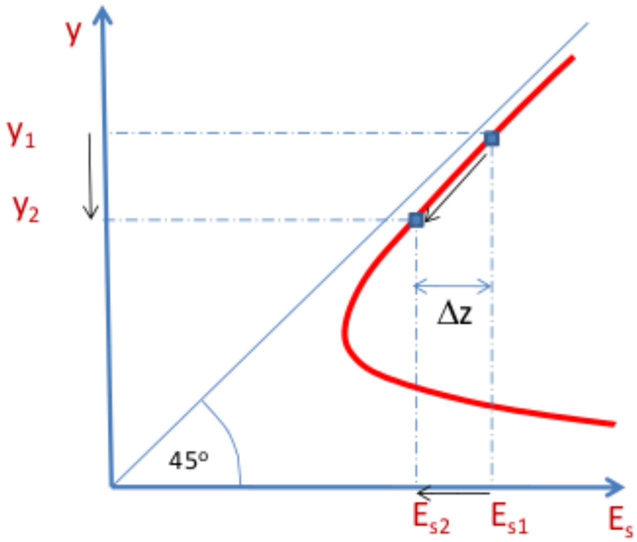
$$V_1 y_1 = V_2 y_2 = q$$

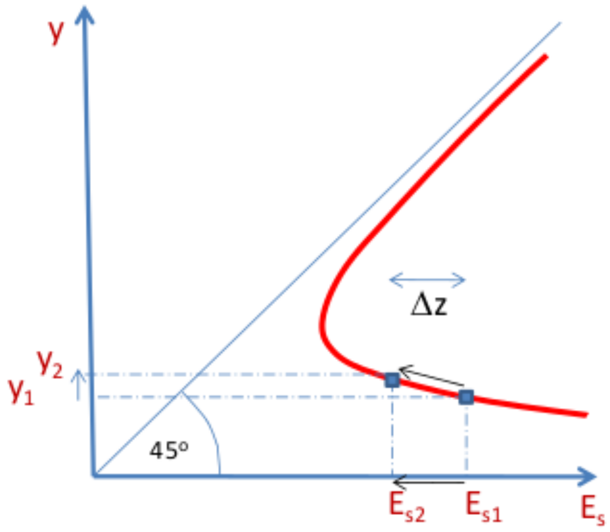
Combining the above equations we get

$$y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2} + \Delta z \qquad (*)$$

One has to solve the above cubic equation for y_2 which has three roots. Which one is correct? The solution is simple: Use the E_s curve and go from E_{s1} to E_{s2} (where $E_{s2} = E_{s1} - \Delta z$) You can then graphically find the range of y_2 using E_{s2}.

Lecture 9: Applications





- Subcritical Flow
- A local bed rise leads to a decrease in water depth
- $(y_2 < y_1)$

- Supercritical Flow
- A local bed rise leads to an increase in water depth
- $(y_2 > y_1)$

See also maximum local bed rise (dz) + choking– Slide 21 Lecture 9

Lecture 9: Summary

<u>Case 1: if</u> $E_{s1} - \Delta z = E_{sc}$

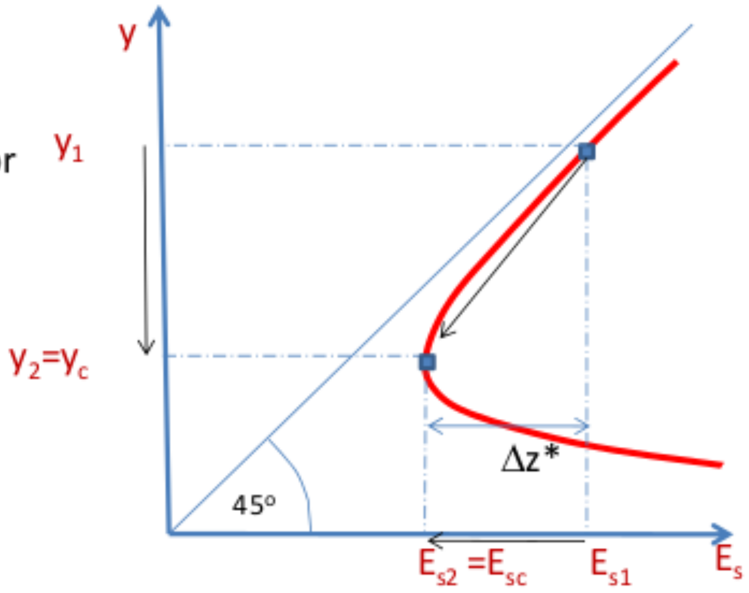
i.e. if bed rise is critical: $\Delta z = \Delta z^{*}$

In this case $y_2 = y_c$

and y_1 does not change.

This case is described as a control, because $y_2 = y_c$. A control point is a location where depth is known for a given discharge, and therefore "controlled".

$$y_c = \sqrt[3]{\frac{q^2}{g}}$$



Lecture 9: Summary

Case 2- Solution procedure if $E_{s1} - \Delta z > E_{sc}$

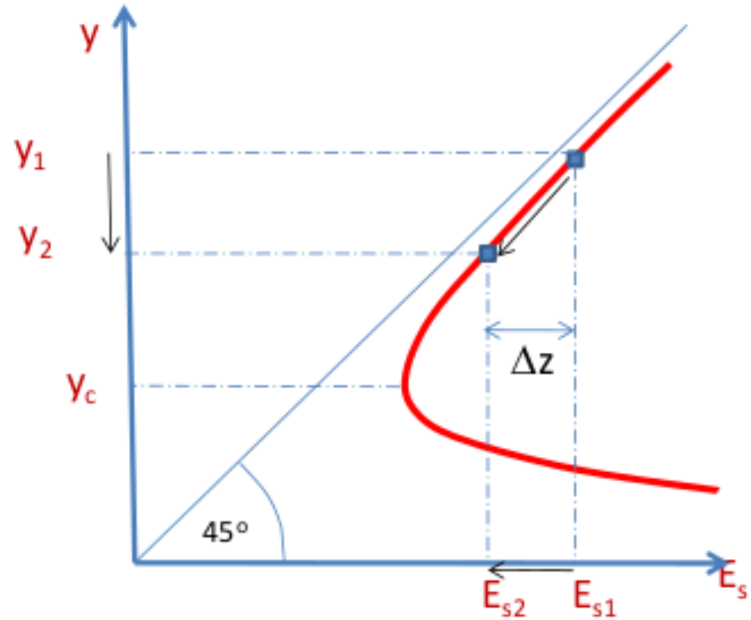
i.e. if bed rise is small: $\Delta z < \Delta z^{*}$

In this case y_1 does not change and

$$E_{s1} = E_{s2} + \Delta z$$

or

$$y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2} + \Delta z$$



Solve for y_2 by iteration

Note: in iterations, y_2 must be chosen between y_1 and y_c. (see the plot).

Lecture 9: Summary

Case 3- Solution procedure if $E_{s1} - \Delta z < E_{sc}$

i.e. if bed rise is large: $\Delta z > \Delta z^*$

***Choking** happens and upstream depth increases.*

In this case, water depth in the downstream is $y_2 = y_c$

1- Calculate E_{sc}

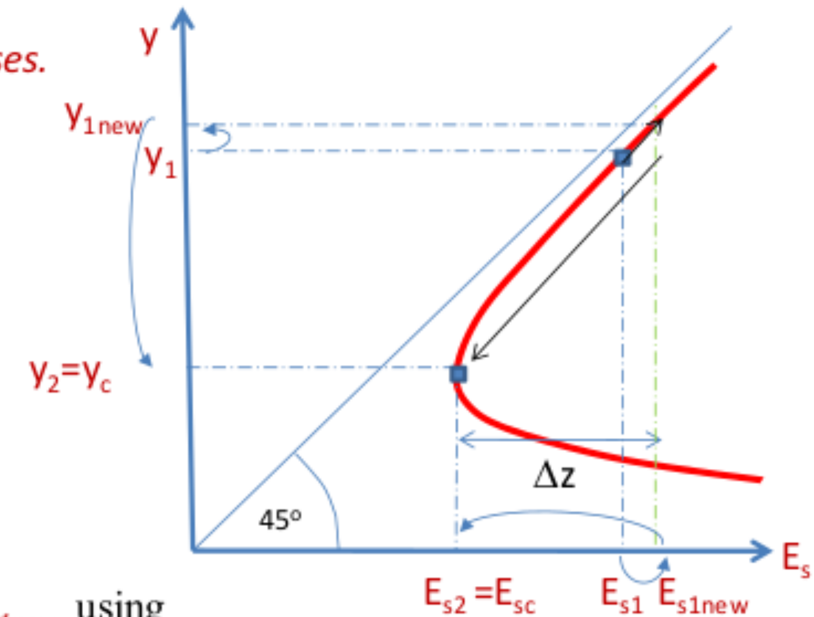
$$E_{sc} = \frac{3}{2} y_c = E_{s2}$$

2- The new value of E_{s1} *is*

$$E_{s1new} = E_{sc} + \Delta z$$

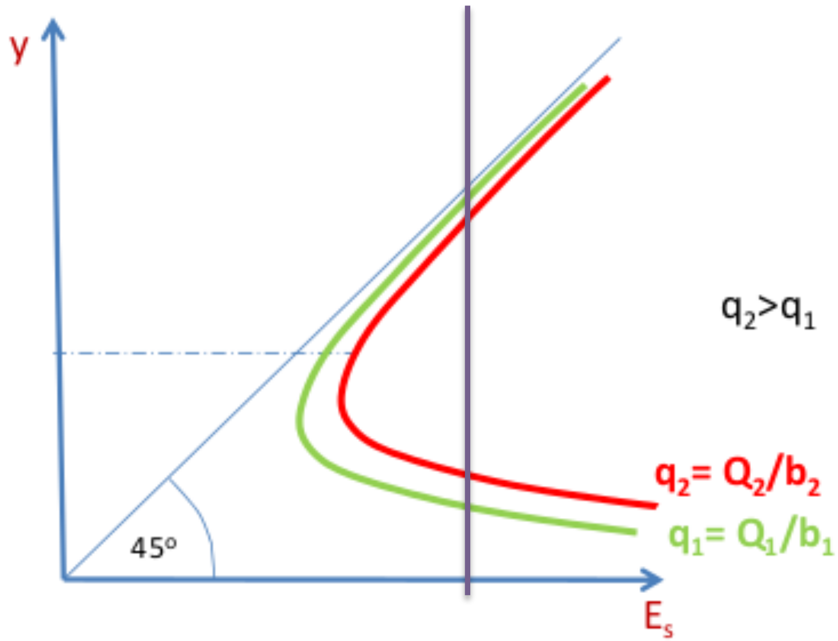
3- Calculate the new upstream water depth y_{1new} using

$$y_{1new} + \frac{q^2}{2 g y_{1new}^2} = E_{s1new}$$



Solution procedures are the same for subcritical and supercritical upstream flow.

Lecture 9: Specific Energy Curves for Various Discharges



$q_2 > q_1$ does not mean that $Q_2 > Q_1$

- **Subcritical Regime** – flow depth decreases ($y_2 < y_1$)
- **Supercritical Regime** – flow depth increases ($y_2 > y_1$)

See plot

Example – Channel contraction

- Assuming the bed elevation is constant, the specific energy remains constant

$$E_{s1} = E_{s2}$$

$$y_1 + \frac{q_1^2}{2gy_1^2} = y_2 + \frac{q_2^2}{2gy_2^2}$$

Lecture 10: RVFs - Momentum

- Froude Number

$$Fr = \frac{V}{\sqrt{gL}}$$

- L – length scale (depends on application).
- For rectangular channels L=y

$$Fr = \frac{V}{\sqrt{gy}}$$

We have seen that the water velocity in a **rectangular** channel for the critical ($y=y_c$) flow is given by

$$V_c = \sqrt{gy_c}$$

Let's calculate the Froude number for this flow:

$$Fr = \frac{V}{\sqrt{gy}} = \frac{\sqrt{gy_c}}{\sqrt{gy_c}} = 1$$

Subcritical flow: $y > y_c \quad and \quad V < V_c \quad \Rightarrow \quad Fr < 1$

Supercritical flow: $y < y_c \quad and \quad V > V_c \quad \Rightarrow \quad Fr > 1$

Lecture 10: Hydraulic Jump

- Occurs when a supercritical flow meets a subcritical flow.
- A large energy loss exists due to turbulence.

In a **rectangular** channel, the upstream and downstream depths are related via:

$$y_2 = (y_1/2)\left(\sqrt{1+8Fr_1^2}-1\right)$$

$$y_1 = (y_2/2)\left(\sqrt{1+8Fr_2^2}-1\right)$$

and the energy loss is given by:

$$\Delta E = \frac{(y_2-y_1)^3}{4y_1y_2}$$