Tutorial #6 Solutions

Problem 1.

a)

Given:

n = 0.01
b = 0.4 m
Q = 150 L/s = 0.150 m^3/s

Solution:

In rectangular channels:

$$y_C = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$$

Where:

$$q = \frac{Q}{b} = \frac{0.150\,\frac{m^3}{s}}{0.4\,m} = 0.375\,\frac{m^2}{s}$$

$$y_C = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{\left(0.375\,\frac{m^2}{s}\right)^2}{\frac{9.81m}{s^2}}\right)^{\frac{1}{3}} = 0.243\,m$$

b)

Given:

$S_0 = 0.003$
n = 0.035
$Q = 75\ m^3/s$

Stage (m)	Area (m^2)	Perimeter (m)	Surface Width (m)
0.5	3.4	9.0	8.1
1.0	10.1	12.2	11.0
1.5	18.2	18.6	16.5
2.0	31.3	26.2	22.0

Solution:

At critical depth in natural channels:

$$\frac{\alpha Q^2 B_c}{g A_c^3} = 1$$

Thus:

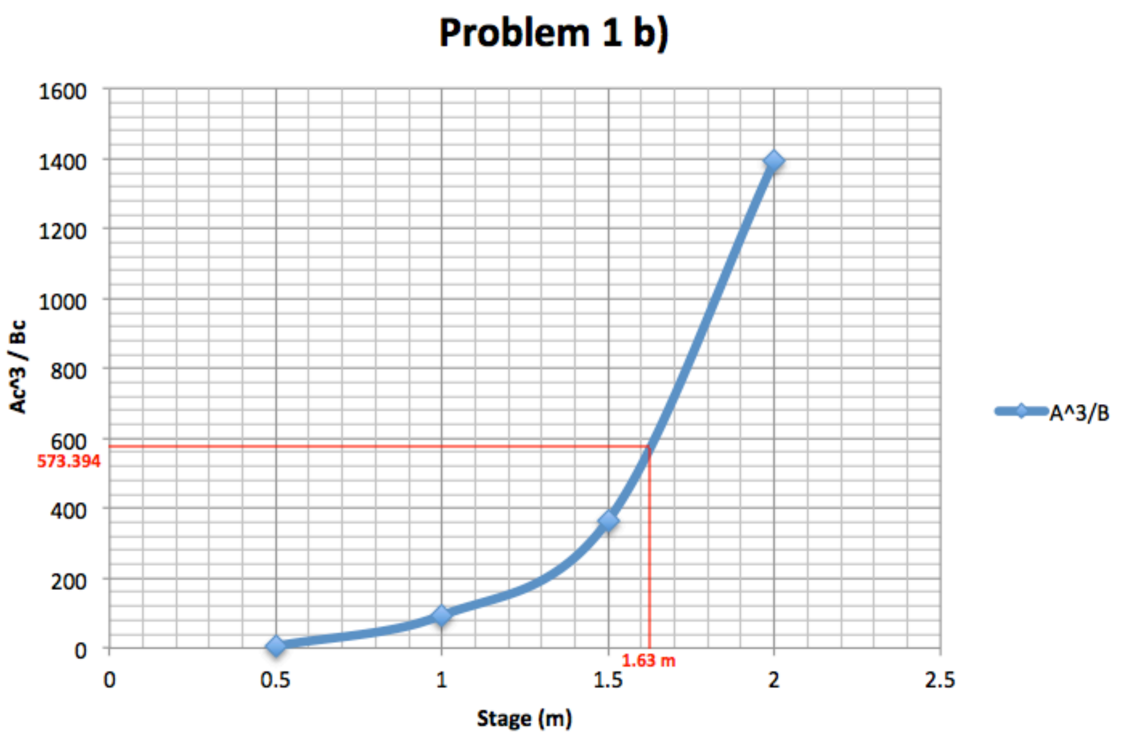
$$\frac{\alpha Q^2}{g} = \frac{A_c^3}{B_c}$$

Assuming $\alpha = 1$:

$$\frac{\alpha Q^2}{g} = \frac{1 * \left(75 \frac{m^3}{s}\right)^2}{\frac{9.81m}{s^2}} = 573.394\ m^5$$

Now plot:

$$\frac{A_c^3}{B_c} \; vs \; Stage$$



Using excel solver, or graphically:

$$\frac{\alpha Q^2}{g} = \frac{A_c^3}{B_c} \quad at \; y \approx 1.63 \; m$$

Thus:

$$y_c = 1.63 \; m$$

Problem 2.

Given:

Q = 5 m^3/s
y = 0.4 m
b = 4 m

Solution:

a)

Compare to critical depth:

$$y_C = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{\left(\frac{5\frac{m^3}{s}}{4\,m}\right)^2}{9.81\frac{m}{s^2}}\right)^{\frac{1}{3}} = 0.543\,m$$

$$y_C > y$$

Thus flow is supercritical.

Alternatively, check the Froude number:

$$Fr = \frac{v}{\sqrt{gd}} = \frac{\frac{Q}{A}}{\sqrt{gd}} = \frac{\frac{5\frac{m^3}{s}}{0.4\,m * 4\,m}}{\sqrt{\frac{9.81m}{s^2} * 0.4\,m}} = 1.578 > 1$$

Thus flow is supercritical.

b)
Determine y_2 in hydraulic jump:

$$y_2 = \left(\frac{y_1}{2}\right)\left(\sqrt{1 + 8\,Fr_1^2} - 1\right)$$

$$y_2 = \left(\frac{y_1}{2}\right)\left(\sqrt{1 + 8\,Fr_1^2} - 1\right)$$

$$y_2 = \left(\frac{0.4\,m}{2}\right)\left(\sqrt{1 + 8 * 1.578^2} - 1\right)$$

$$y_2 = 0.715\,m$$

Calculate energy lost in the hydraulic jump:

Since:

$$\Delta E = E_2 - E_1$$

And:

$$E = y + \frac{v^2}{2g}$$

Then:

$$\Delta E = \left(y_2 + \frac{v_2^2}{2g}\right) - \left(y_1 + \frac{v_1^2}{2g}\right)$$

$$\Delta E = \left(0.715 + \frac{\left(\frac{5\,\frac{m^3}{s}}{0.715\,m * 4\,m}\right)^2}{2 * 9.81\,\frac{m}{s^2}}\right) - \left(0.4 + \frac{\left(\frac{5\,\frac{m^3}{s}}{0.4\,m * 4\,m}\right)^2}{2 * 9.81\,\frac{m}{s^2}}\right)$$

$$\Delta E = 0.898\,m - 0.871\,m = 0.027\,m$$

Problem 3.

<u>Given:</u>

Q = 6.5 m^3/s
b_1 = 3 m
y_1 = 0.4 m

<u>Solution:</u>

a) b_2 = 2.5 m , Δz = 0 m

Calculate specific energy:

$$E_1 = y_1 + \frac{v_1^2}{2g} = 0.4\,m + \frac{\left(\frac{6.5\,\frac{m^3}{s}}{3\,m * 0.4\,m}\right)^2}{2 * 9.81\,\frac{m}{s^2}} = 1.895\,m$$

Calculate critical energy of b_2:

$$y_{C2} = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{\left(\frac{6.5\,\frac{m^3}{s}}{2.5\,m}\right)^2}{9.81\,\frac{m}{s^2}}\right)^{\frac{1}{3}} = 0.883\,m$$

$$E_{c2} = \frac{3}{2} y_{c2} = \frac{3}{2} * 0.883\,m = 1.324\,m$$

Check for choking:

$$E_{s2} = E_{s1} - \Delta z = 1.895\text{ m} - 0\text{ m} = 1.895 > \mathrm{E_{c2}}$$

Thus, no choking.

Determine y_2:

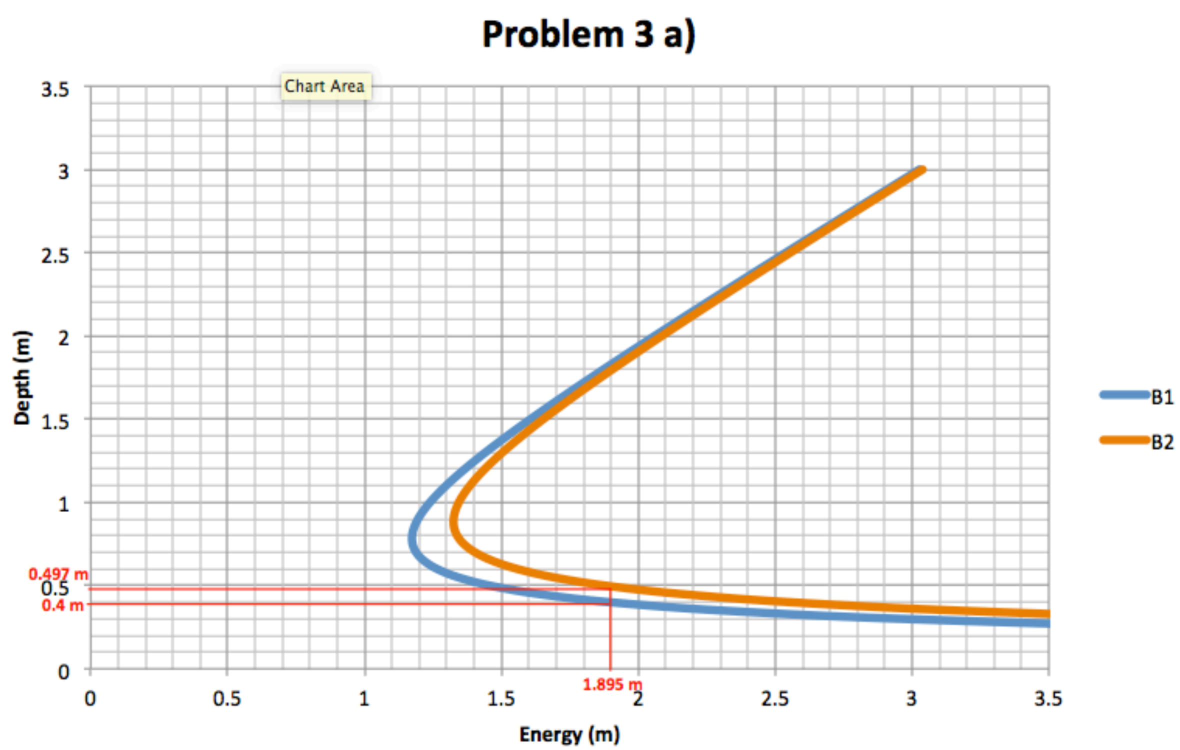
$$E_2 = y_2 + \frac{v_2^2}{2g} = y_2 + \frac{\left(\frac{Q}{b * y_2}\right)^2}{2g} = 1.895\ m$$

$$y_2 + \frac{\left(\frac{Q}{b_2 * y_2}\right)^2}{2g} = y_2 + \frac{\left(\frac{6.5\frac{m^3}{s}}{2.5\ m * y_2}\right)^2}{2 * 9.81\ m/s^2} = y_2 + \frac{0.345}{y_2^2} = 1.895\ m$$

Using excel solver, or trial and error:

$$y_2 = 0.497\ m$$

Graphically:



b) b_2 = 1.2 m , Δz = 0 m

Calculate critical energy of b_2:

$$y_{C2} = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{\left(\frac{6.5\,\frac{m^3}{s}}{1.2\ m}\right)^2}{9.81\,\frac{m}{s^2}}\right)^{\frac{1}{3}} = 1.441\ m$$

$$E_{c2} = \frac{3}{2}y_{c2} = \frac{3}{2} * 1.441\ m = 2.162\ m$$

Check for choking:

$$E_{s2} = E_{s1} - \Delta\text{z} = 1.895\ \text{m} - 0\ \text{m} = 1.895 < \text{E}_{\text{c2}}$$

Thus, choking occurs.

Downstream of the contraction will now be at critical conditions, and the upstream flow will increase to the same energy as the downstream energy.

$$E_{s2} = E_{c2} = 2.162\ m$$

$$y_2 = y_{c2} = 1.441\ m$$

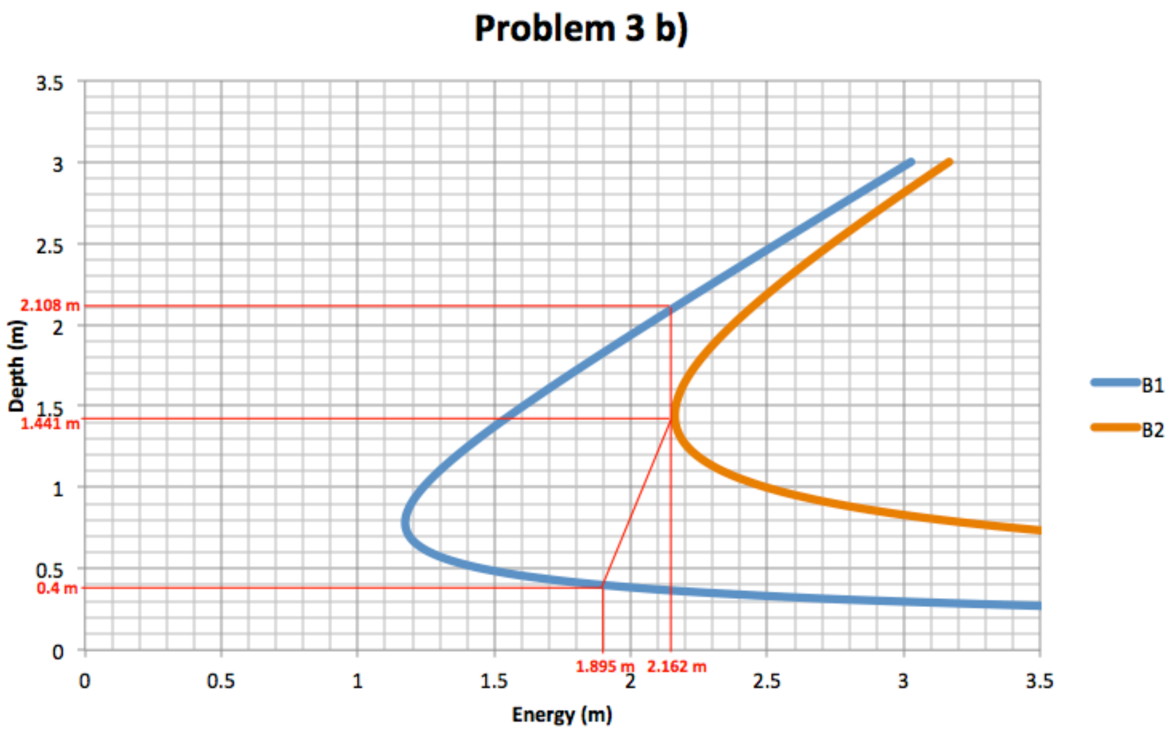
Now:

$$E_{s1} = y_1 + \frac{\left(\frac{Q}{b_1 * y_1}\right)^2}{2g} = y_1 + \frac{\left(\frac{6.5\,\frac{m^3}{s}}{3\ m * y_1}\right)^2}{2 * 9.81\ m/s^2} = y_1 + \frac{0.239}{y_1^2} = E_{s2} = 2.162\ m$$

Using excel solver, or trial and error:

$$y_1 = 2.108\ m$$

Graphically:



c) b_2 = 2.5 m , Δz = -0.3 m (downward step)

Check for choking:

From part a)

$$E_{s2} = E_{s1} - \Delta z = 1.895\text{ m} + 0.3\text{ m} = 2.195 > E_{c2}$$

Thus, no choking.

Determine y_2:

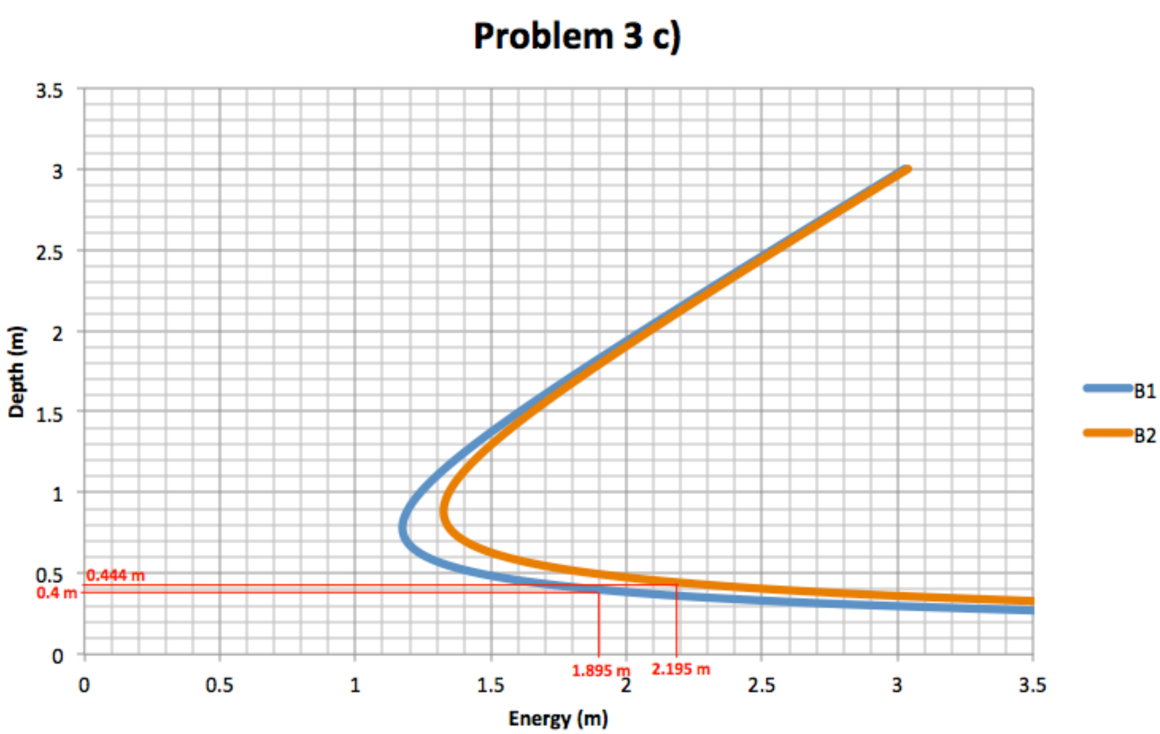
$$E_2 = y_2 + \frac{v_2^2}{2g} = y_2 + \frac{\left(\frac{Q}{b * y_2}\right)^2}{2g} = 2.195m$$

$$y_2 + \frac{0.345}{y_2^2} = 2.195\ m$$

Using excel solver, or trial and error:

$$y_2 = 0.444\ m$$

Graphically:



c) $b_2 = 2.5$ m , $\Delta z = 0.7$ m (upward step)

Check for choking:

From part a)

$$E_{s2} = E_{s1} - \Delta z = 1.895 \text{ m} - 0.7 \text{ m} = 1.195 < \text{E}_{c2}$$

Thus, choking occurs.

Downstream of the contraction will now be at critical conditions, and the upstream flow will increase to the same energy as the downstream energy.

From part a)

$$y_2 = y_{C2} = 0.883\ m$$

$$E_{s2} = E_{c2} = 1.324\ m$$

Now:

$$E_{s1} = E_{s2} + \Delta z = 2.024 \text{ m}$$

From part b)

$$y_1 + \frac{0.239}{y_1^2} = E_{s1} = 2.024\ m$$

Using excel solver, or trial and error:

$$y_1 = 1.962\ m$$

Graphically: