CVG 3116 Fall 2017

Hydraulics – Tutorial 7
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Lecture 10: RVFs - Momentum

- Froude Number

$$Fr = \frac{V}{\sqrt{gL}}$$

Where L is a length scale and depends on application. For general open channels, we may take the mean depth D_m=A/B (the ratio of area to the surface width):

- For rectangular channels L=y

$$Fr = \frac{V}{\sqrt{gy}}$$

We have seen that the water velocity in a **rectangular** channel for the critical ($y=y_c$) flow is given by

$$V_c = \sqrt{gy_c}$$

Let's calculate the Froude number for this flow:

$$Fr = \frac{V}{\sqrt{gy}} = \frac{\sqrt{gy_c}}{\sqrt{gy_c}} = 1$$

Subcritical flow: $y > y_c \quad and \quad V < V_c \quad \Rightarrow \quad Fr < 1$

Supercritical flow: $y < y_c \quad and \quad V > V_c \quad \Rightarrow \quad Fr > 1$

Lecture 10: Hydraulic Jump

- **Occurs when a supercritical flow meets a subcritical flow.**
- **A large energy loss exists due to turbulence.**

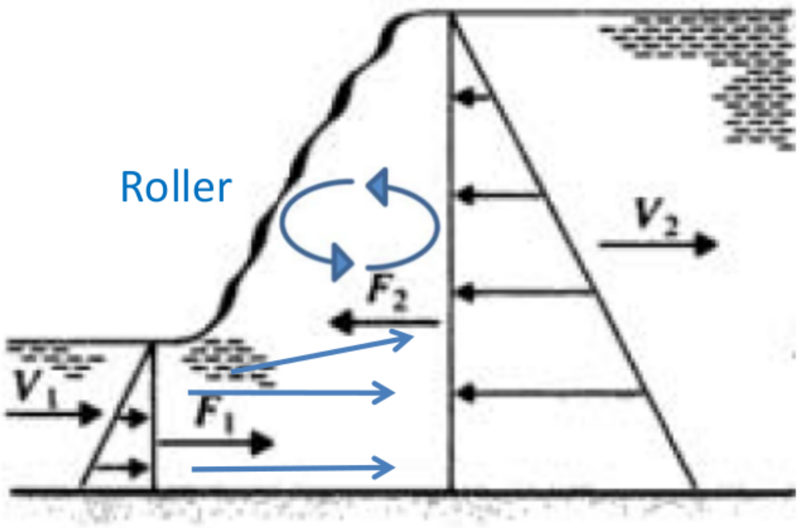
In a **rectangular** channel, the upstream and downstream depths are related via:

$$y_2 = (y_1/2)\left(\sqrt{1+8Fr_1^2}-1\right)$$

$$y_1 = (y_2/2)\left(\sqrt{1+8Fr_2^2}-1\right)$$

and the energy loss is given by:

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}$$



Lecture 11: Gradually varied non-uniform flow

- *Normal depth y_n (for a given discharge):*
  - Is the depth of uniform flow.
  - Can be calculated using the Manning Equation

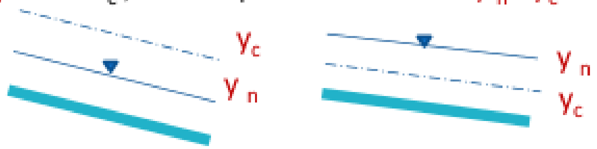
$$Q = \frac{1}{n}\frac{A^{5/3}}{P^{2/3}}S_0^{1/2}$$

The *bed friction* is ignored in rapidly varied flows e.g., in hydraulic jump, because the changes happen over a short distance. But in gradually varied flows, the bed friction becomes important.

The normal depth can be calculated using the Manning formula with $S=S_0$

The critical slope S_c :
For a given discharge Q, S_c is the slope for which, the normal depth is equal to the critical depth.
i.e. Setting $y=y_c$ in the Manning formula leads to $S=S_c$.

If the channel slope S_0 is *steeper* than S_c , the slope is called *steep* slope and $y_n < y_c$
If the channel slope S_0 is *milder* than S_c , the slope is called *mild* slope and $y_n > y_c$
If the channel slope S_0 is *equal* to S_c , the slope is critical and $y_n = y_c$



Steep slope: $S_0 > S_c$
$y_n < y_c$

Mild slope: $S_0 < S_c$
$y_n > y_c$

Lecture 11: Critical Slope, Sc

Manning formula:

$$Q = \frac{1}{n}\frac{A^{5/3}}{P^{2/3}}S_0^{1/2}$$

For critical flow, (assuming α=1), we already know that:

$$\frac{Q^2 B}{gA^3} = 1$$

Eliminating Q:

$$\frac{1}{n}\frac{A^{5/3}}{P^{2/3}}S_c^{1/2} = \sqrt{\frac{gA^3}{B}}$$

$$S_c = \left(\frac{nQP^{2/3}}{A^{5/3}}\right)^2$$

Where A and P are calculated based on the critical depth.

- Special Case – wide rectangular channel (b >> y)

$$A = by$$

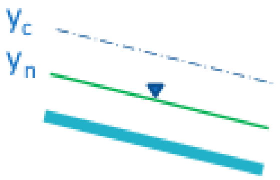
$$P = b + 2y \approx b \quad because \quad y << b$$

$$S_c = gn^2 / y_c^{1/3}$$

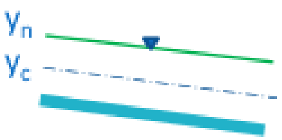
Lecture 11: Summary – mild and steep slope

- ***Critical slope:***
  S_c is the slope for which normal depth is equal to critical depth, i.e. if S_c is used in the Manning formula, it gives the critical depth ($y_n = y_c$).

- ***Mild slope:*** The channel slope is less than the critical slope
- ***Steep slope:*** The channel slope is greater than the critical slope



Steep slope: $S_0 > S_c$
$y_n < y_c$

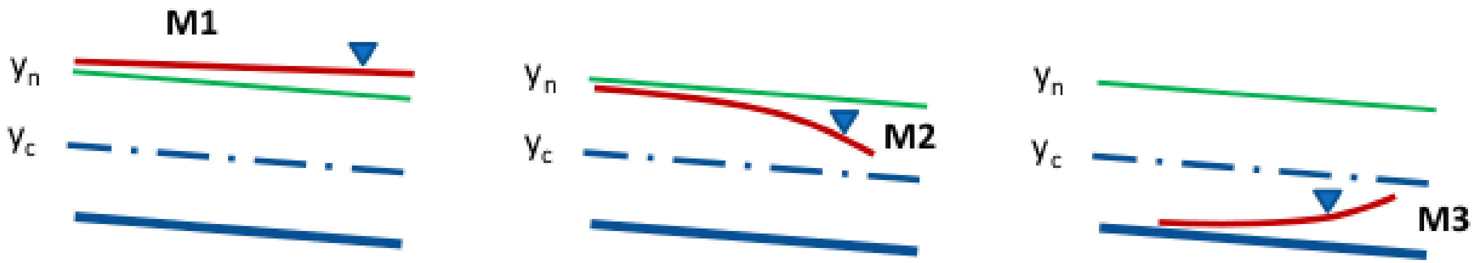


Mild slope: $S_0 < S_c$
$y_n > y_c$

Note: A channel may be steep for one discharge and mild for another.

Lecture 11: Summary – mild and steep slope

- Mild Slope (yn > yc)

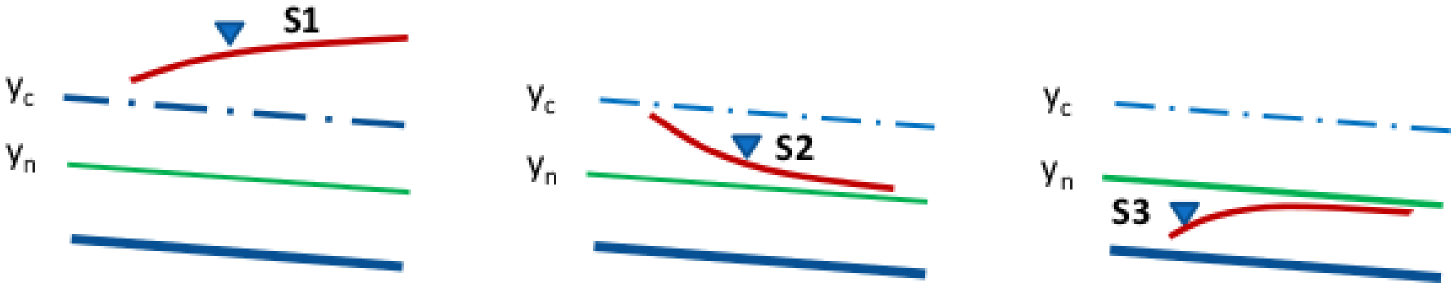


Water depth increases in region 1
Water depth decreases in region 2
Water depth increases in region 3

❑ Water surface
- is asymptotic to $y=y_n$
- has a large angle with $y=y_c$

Lecture 11: Summary – mild and steep slope

- Steep Slope (yn < yc)



Water depth increases in region 1
Water depth decreases in region 2
Water depth increases in region 3

Water surface

- is asymptotic to $y=y_n$
- has a large angle with $y=y_c$

Lecture 12: Control Points and Non-uniform flwo profiles (review)

1 – Control points:

- Bed rise (or hump) - if flow becomes critical $\Delta z \geq \Delta z^*$
- Channel contraction - if flow becomes critical $(b < b_c)$
- Change in the slope from mild to steep
- Free overfall (flow becomes critical to maximize the discharge) (Tutorial 7)

2 - Normal depth (i.e. flow is controlled by channel friction).

3 - Sluice gate: water depth after the gate ≈ 0.6 x gate opening.

We saw that variations of water depth in channels in gradually varied flows can be calculated using:

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2}$$

We can find out if the water depth increases or decreases using the above equation.

Lecture 12: Water depth in channels and rivers

We have already shown that the energy equation leads to:

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2}$$

We can numerically integrate the above equation to calculate *y* at each point.
The derivative dy / dx is approximated between two points as $\Delta y / \Delta x$.

$$\frac{\Delta y}{\Delta x} = \left(\frac{S_0 - S_f}{1 - Fr^2} \right)_{mean}$$

where "mean" is the average value between the two points.

- **The Direct Step Method:** Δy is assumed, Δx is calculated.
- **The Standard Step Method:** Δx is assumed, Δy is calculated (iteration required).

Lecture 12: Water depth in channels and rivers

The Direct Step Method:

Δy is assumed, Δx is calculated:

$$\Delta x = \Delta y\, K_{mean} \qquad where \qquad K = \frac{1 - Fr^2}{S_0 - S_f}$$

Procedure: (Discharge is given)

1- Start from point 1 where you know the water depth y_1 (control point).
For supercritical flow, we go from upstream to downstream.
For subcritical flow , we go from downstream to upstream.

2- Assume a value for y_2 **based on the profile type.**

3- $\Delta y = y_2 - y_1$

4- Calculate $K_{mean} = (K_1 + K_2)/2$

5- Calculate $\Delta x = \Delta y\, K_{mean}$

6- Repeat from step 2.