Tutorial # 7 - Solutions

1. In a rectangular channel, the bed width is equal to 4 m and the discharge is 22.5 m^3/s. The channel has two long sections with slopes S_1=0.0004 and S_2=0.009 and Manning's coefficients n_1=0.015 and n_2=0.012, respectively. Identify the type of water surface profile along the channel.

To plot the water surface profile, we need the actual water depth and critical water depth.

We can calculate the normal water depth from Manning's equation:

$$Q = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} S_o^{1/2}$$

For section 1:

$$22.5\ m^3/s = \frac{1}{0.015} \frac{(4 \cdot y_1)^{5/3}}{(4+2y_1)^{2/3}} 0.0004^{1/2}$$

using Excel: $y_1 = 3.57\ m$

For section 2:

$$22.5\ m^3/s = \frac{1}{0.012} \frac{(4 \cdot y_2)^{5/3}}{(4+2y_2)^{2/3}} 0.009^{1/2}$$

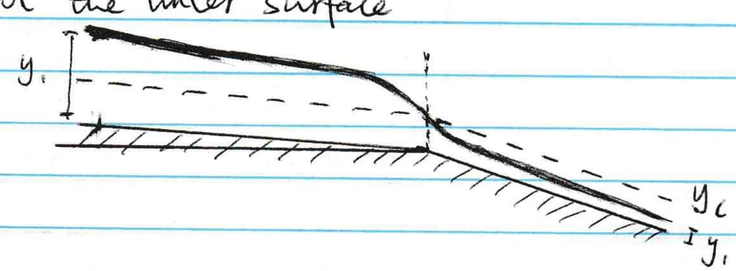
using Excel: $y_2 = 0.95\ m$

Find critical depth of the channel

In a rectangular channel: $q = \frac{Q}{b}$

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(22.5/4)^2}{9.81}} = 1.48\ m$$

Plot the water surface



For section 1:

$y_n > y_c$: Mild **M2**

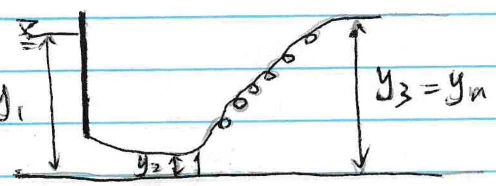
For section 2:

$y_n < y_c$: Steep **S2**

2. A vertical sluice gate with an opening of 0.67 m produces a downstream jet when installed in a long concrete rectangular channel conveying a discharge of 20 m^3/s. The channel width is equal to 5 m, the Manning's coefficient of the channel is n = 0.013 and the channel slope is 0.00032. The contraction coefficient of the sluice gate is 0.6. Assuming that the flow downstream of the gate eventually returns to the uniform flow depth:

a. Verify that a hydraulic jump occurs downstream of the gate.

b. Assuming no energy losses through the gate, calculate the depth upstream of the gate.



a) Hydraulic jump occurs when a supercritical flow loses enough energy, meaning it can happen when a supercritical flow meets a subcritical flow. Hence, we need Fr after the gate and after the jump.

First, we need water depth y_2 and y_3 ($y_3 = y_n$)

$$Q = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} S_0^{1/2} \Rightarrow$$

$$20 = \frac{1}{0.013} \frac{(5y_n)^{\frac{5}{3}}}{(5+2y_n)^{\frac{2}{3}}} (0.00032)^{1/2}$$

$y_n = y_3 = 2.5m$

y_2 = gate opening x Contraction coefficient
= 0.67 m x 0.6 = 0.4 m

Find Fr_2 and Fr_3

$$Fr_2 = \frac{Q/A_2}{\sqrt{g\, y_2}} = \frac{20/(5\times0.4)}{\sqrt{9.81\times0.4}} = 5.05 > 1$$

$$Fr_3 = \frac{Q/A_3}{\sqrt{g y_3}} = \frac{20/(5\times 2.5)}{\sqrt{9.81\times 2.5}} = 0.323 < 1$$

Hydraulic jump occurs when a supercritical flow meet a subcritical flow.

b) Find the water depth before the gate

use the energy equation before and after the gate

$$y_1 + \frac{q^2}{2g y_1^2} = y_2 + \frac{q^2}{2g y_2^2}$$

$$q = \frac{Q}{b} = 4 \ m^2/s$$

$$y_1 + \frac{4^2}{2\times 9.81\times y_1^2} = 0.4 + \frac{4^2}{2\times 9.81\times 0.4^2}$$

$\therefore \ y_1 =$ **5.47 m**

3. A trapezoidal channel has a bottom width b = 8 m and a side slope ratio of 2:1 (m=2). The Manning's coefficient of the channel is n = 0.025 and it is laid on a slope of 0.001. If the channel ends in a free overfall, compute the water surface profile for a discharge of 30 m^3/s using the direct step method.

Direct Step Method – Solution

1. Find the normal and critical depths.
   Normal flow and depth from Manning's Equation

$$Q = \frac{1}{n}\frac{A^{5/3}}{P^{2/3}}S_o^{1/2}$$

   For a Trapezoidal channel:

$$A = (b + my)y$$

$$P = \left(b + 2y\sqrt{1 + m^2}\right)$$

   From Excel, Normal Depth y_n = **1.754 m**

   Find critical depth using:

$$\frac{Q^2 B_c}{gA_c^3} = 1$$

   From Excel, Critical Depth y_c = **1.03 m**

   Since normal depth is higher than the critical depth, the water surface profile is mild. Since the profile will de decreasing as it goes downstream and approaches the overfall (where the depth will equal critical depth) this is an M-2 profile.
   We will use the direct step method to calculate the water surface profile moving upstream from the overfall where y = yc to the normal depth. We will assume values for depth and calculate the corresponding upstream distance where the higher depth occurs using the direct step method.

2. Direct Step Method
   a. Assume a flow depth near the control point (critical depth)
      Use y = 1.05 m (even increments are more strsightforward)
   b. Calculate the area for the given depth:

$$A = (b + my)y = (8m + 2 * 1.05m) * 1.05m = \mathbf{10.605\ m^2}$$

   c. Calculate wetted perimeter *P:*

$$P = \left(b + 2\,y\sqrt{1 + m^2}\right)$$
$$= 8m + 2 * 1.05m\sqrt{1 + 2^2} = \mathbf{12.696\ m}$$

   d. Calculate mean depth D_m:

$$D_m = A/B = 10.605/(8m + 2*2*1.05m) = \mathbf{0.869\ m}$$

   e. Calculate the velocity:

$$v = \frac{Q}{A} = \frac{30\ m^3/s}{10.605m^2} = \mathbf{2.83\ m/s}$$

   f. Calculate the Froude Number:

$$Fr = \frac{v}{\sqrt{gD_{\mathrm{m}}}}$$

$$Fr = \frac{2.83\frac{m}{s}}{\sqrt{\frac{9.81m}{s^2} * 0.869m}} = \mathbf{0.969}$$

   g. Calculate the friction slope by rearranging Manning's equation

$$S_f = \left(\frac{Q * n * P^{2/3}}{A^{5/3}}\right)^2 = \left(\frac{30m^3/s * 0.025 * (12.696m)^{2/3}}{(10.605m^2)^{5/3}}\right)^2 = 0.00636$$

   h. Prepare a table to calculate steps a) through g) for several rows. Move forward in increments of **Δy = 0.05 m** [This increment is selected by the user], so complete the calculations for y_{i+1} = 1.05m+0.05m = 1.10 m.

   i. Calculate the Mean of (1-Fr^2) between y = 1.05 and y = 1.10 m = **0.1281**

   j. Calculate the Mean of (S_o-S_f) between y = 1.05 and y = 1.10 m = **-0.0049**

   k. Calculate distance at which depth y_{i+1} (i.e y = 1.10 m) occurs. (We set distance x = 0 for the first row.) The direct step equation provides the distance between y_i and y_{i+1}

$$\Delta x = \Delta y \left(\frac{1 - Fr^2}{S_o - S_f} \right)_{mean}$$

And

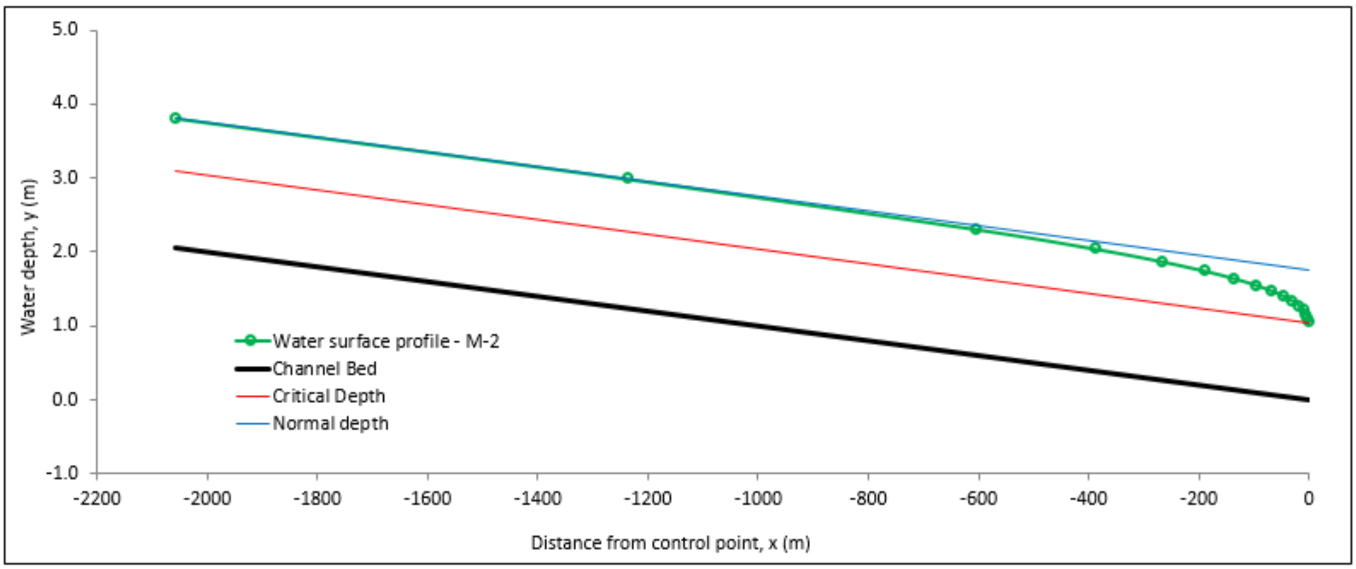
$$x_{i+1} = x_i + \Delta x$$

$$x_{i+1} = 0m + 0.05m * \left(\frac{0.1281}{-0.0049} \right) = -1.31m$$

Continue the calculations for a sufficient number of steps to reach the normal depth $y_{n.}$ See the Excel table results. Note that the x value will be negative as you are moving upstream. Conceptually, x is the distance from the downstream control point to the depth (y_i) being calculated.

See from the Excel Table that normal depth occurs at approximately -2056 m (i.e.2056 m upstream from the free outfall).

The water surface profile from the direct step method is:



Delineating water surface profile - M-2 curve

y (m)	A (m^2)	P (m)	B (m)	Dm (m)	v (m/s)	Fr^2 (-)	$1-Fr^2$ (-)	$1-Fr^2$ (mean)	S_f (m/m)	S_0-S_f (m/m)	S_0-S_f (mean)	x (m)	Bed (m)	y_c (m)	y_n (m)	y (vs bed) (m)
1.050	10.605	12.696	12.200	0.869	2.829	0.9384	0.0616		6.36E-03	-5.36E-03		0.00	0.00	1.03	1.75	1.05
1.100	11.220	12.919	12.400	0.905	2.674	0.8054	0.1946	0.1281	5.39E-03	-4.39E-03	-4.88E-03	-1.31	0.00	1.03	1.76	1.10
1.150	11.845	13.143	12.600	0.940	2.533	0.6956	0.3044	0.2495	4.61E-03	-3.61E-03	-4.00E-03	-4.43	0.00	1.03	1.76	1.15
1.200	12.480	13.367	12.800	0.975	2.404	0.6041	0.3959	0.3501	3.96E-03	-2.96E-03	-3.28E-03	-9.77	0.01	1.04	1.76	1.21
1.250	13.125	13.590	13.000	1.010	2.286	0.5275	0.4725	0.4342	3.42E-03	-2.42E-03	-2.69E-03	-17.84	0.02	1.05	1.77	1.27
1.300	13.780	13.814	13.200	1.044	2.177	0.4628	0.5372	0.5048	2.97E-03	-1.97E-03	-2.20E-03	-29.34	0.03	1.06	1.78	1.33
1.350	14.445	14.037	13.400	1.078	2.077	0.4079	0.5921	0.5647	2.59E-03	-1.59E-03	-1.78E-03	-45.17	0.05	1.08	1.80	1.40
1.400	15.120	14.261	13.600	1.112	1.984	0.3610	0.6390	0.6156	2.28E-03	-1.28E-03	-1.44E-03	-66.61	0.07	1.10	1.82	1.47
1.450	15.805	14.485	13.800	1.145	1.898	0.3207	0.6793	0.6592	2.00E-03	-1.00E-03	-1.14E-03	-95.52	0.10	1.13	1.85	1.55
1.500	16.500	14.708	14.000	1.179	1.818	0.2859	0.7141	0.6967	1.77E-03	-7.73E-04	-8.89E-04	-134.72	0.13	1.16	1.89	1.63
1.550	17.205	14.932	14.200	1.212	1.744	0.2558	0.7442	0.7291	1.57E-03	-5.73E-04	-6.73E-04	-188.91	0.19	1.22	1.94	1.74
1.600	17.920	15.155	14.400	1.244	1.674	0.2296	0.7704	0.7573	1.40E-03	-4.01E-04	-4.87E-04	-266.66	0.27	1.30	2.02	1.87
1.650	18.645	15.379	14.600	1.277	1.609	0.2067	0.7933	0.7819	1.25E-03	-2.52E-04	-3.26E-04	-386.47	0.39	1.42	2.14	2.04
1.700	19.380	15.603	14.800	1.309	1.548	0.1865	0.8135	0.8034	1.12E-03	-1.22E-04	-1.87E-04	-601.67	0.60	1.63	2.36	2.30
1.750	20.125	15.826	15.000	1.342	1.491	0.1688	0.8312	0.8223	1.01E-03	-8.10E-06	-6.49E-05	-1235.25	1.24	2.27	2.99	2.99
1.754	20.185	15.844	15.016	1.344	1.486	0.1675	0.8325	0.8318	1.00E-03	0.00E+00	-4.05E-06	-2056.68	2.06	3.09	3.81	3.81

Q =	30.00	m^3/s
S_0 =	0.001	m/m
n =	0.025	
b =	8.00	m
y_c =	1.030	m
y_n =	1.754	m