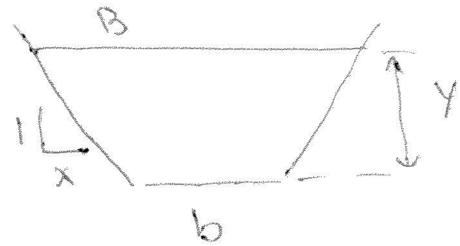
CVG 3116 ; Tutorial # 8 ; Solutions

1) Manning Eq: $$Q = \frac{1}{n}\,\frac{A^{5/3}}{P^{2/3}}\,S^{1/2}$$

if $S = S_o \Rightarrow y = y_n$

$A = (b + xy)y$, $P = b + 2y\sqrt{1+x^2}$



$B = b + 2xy$

$\Rightarrow$ part (A): $n_A = 0.015$, $S_{oA} = 0.0004$, $Q = 22.5\, m^3/s$, $b = 4^m$, $x =$

$\Rightarrow A_A = (4 + Y_A)\,Y_A$, $P = 4 + 2\sqrt{2}\,Y$

$$\Rightarrow Q = \frac{1}{n_A}\,\frac{(4+Y_A)^{5/3}\,Y_A^{5/3}}{(4+2\sqrt{2}\,Y_A)^{2/3}}\,S_{oA}^{1/2} \; ; \quad S_A = S_{oA} \Rightarrow Y_A = Y_{nA}$$

$\Rightarrow$ iteration $Y_{nA} = 2.264^m$

$\Rightarrow$ part (B) : $n_B = 0.012$, $S_{oB} = 0.009$, $Q = 22.5\, m^3/s$, $b = 4^m$, $x =$

$\Rightarrow Y_{nB} = 0.812\, m$,

part (C) : $Y_{nC} = 1.172^m$

part	Y_n	Y_c	
A	2.264		
B	0.812		
C	1.172		

at critical depth $\frac{\alpha Q^2 B}{g A^3} = 1$; $\alpha = 1$ (assuming)

$$\Rightarrow \frac{Q^2 B}{g A^3} = 1 \Rightarrow \frac{22.5^2 (4+2y_c)}{9.81 \, ((4+(1)yc)yc)^3} = 1$$

$$\Rightarrow y_c = 1.316^m$$

part	y_n	y_c	slope
A	2.264	1.316	M
B	0.812	1.316	S
C	1.172	1.316	S

$y_{nA} > y_{cA} \Rightarrow S_{oA} < S_{cA} \Rightarrow$ Mild slope

$y_{nB} < y_{cB} \Rightarrow S_{oB} > S_{cB} \Rightarrow$ steep slope

$y_{nc} < y_{cc} \Rightarrow S_{oB} > S_{cc} \Rightarrow$ steep slope

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2}$$

A: $y < y_n \Rightarrow S_f > S_0 \Rightarrow S_0 - S_f < 0 \Rightarrow \frac{dy}{dx} < 0 \Rightarrow M_2$

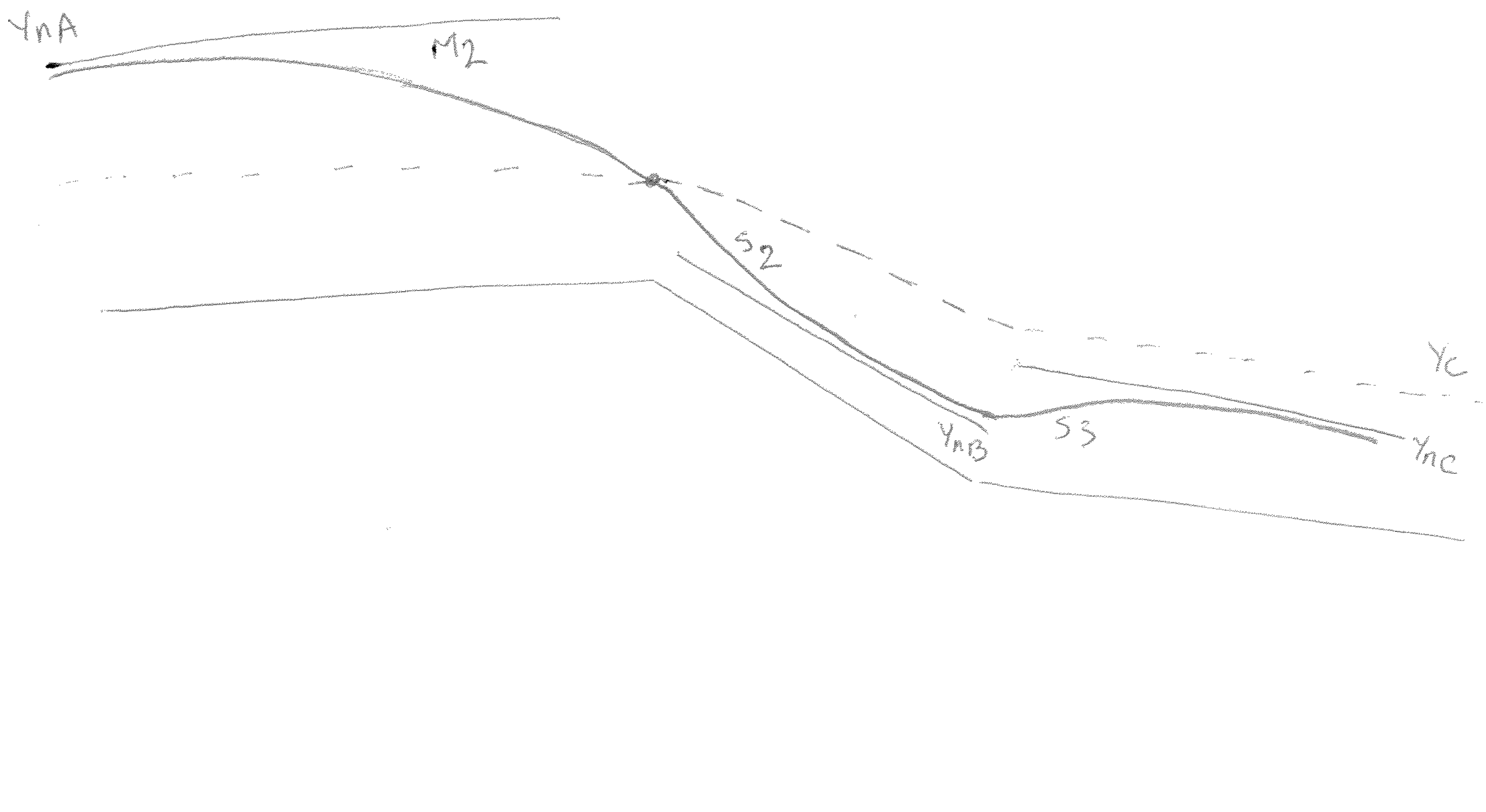
$y > y_c \Rightarrow Fr < 1 \Rightarrow 1 - Fr^2 > 0$

B: $y > y_n \Rightarrow S_f < S_0 \Rightarrow S_0 - S_f > 0 \Rightarrow \frac{dy}{dx} < 0 \Rightarrow S_2$

$y < y_c \Rightarrow Fr > 1 \Rightarrow 1 - Fr^2 < 0$

C: $y < y_n \Rightarrow S_f > S_0 \Rightarrow S_0 - S_f < 0 \Rightarrow \frac{dy}{dx} > 0 \Rightarrow S_3$

$y < y_c \Rightarrow Fr > 1 \Rightarrow 1 - Fr^2 < 0$



2) $y_c = \sqrt[3]{q^2/g}$; $q = \frac{Q}{b} = \frac{11.3}{3} = 3.77\ m^2/s$

$\Rightarrow y_c = 1.13^m$

y_n: $Q = \frac{1}{n}\frac{A^{5/3}}{P^{2/3}} S^{1/2}$; $S = S_0 \Rightarrow y = y_n$

$A = b y_n = 3 y_n$, $P = b + 2 y_n = 3 + 2 y_n$

$\Rightarrow 11.3 = \frac{1}{0.017}\frac{(3y_n)^{5/3}}{(3+2y_n)^{2/3}}(0.02)^{1/2} \Rightarrow y_n = 0.73^m$

$y_n < y_c \Rightarrow S_0 > S_c \Rightarrow$ steep slope

supercritical region ($y_1 < y_c$) reach subcritical region ($y_3 > y_c$) $\Rightarrow$ hydraulic jump happens

conjugate depth : $y_2 = \frac{y_1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right)$

$Fr_1^2 = \frac{q^2}{g y_1^3} = \frac{3.77^2}{9.81 \times 0.73^3} = 3.72 \Rightarrow y_2 = 1.66m$

$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2}$; $y_2 > y_n \Rightarrow S_f < S_0$ $\Rightarrow \frac{dy}{dx} > 0 \Rightarrow S_1$

$y_2 > y_c \Rightarrow Fr < 1$

$$\Delta x = \frac{E_s'' - E_s'}{S_0 - \bar{S_f}} \quad ; \text{ neglect } h_L$$

$$\Rightarrow \bar{S_f} = \frac{S_{f_2} + S_{f_3}}{2} \quad ; \quad S_f = \frac{Q^2 n^2 P^{4/3}}{A^{10/3}}$$

$$y_2 = 1.66 \Rightarrow S_f = \frac{11.3^2 \times 0.017^2 \times (3 + 2\times 1.66)^{4/3}}{(3\times 1.66)^{10/3}} = 0.002$$

$$y_3 = 2.9 \Rightarrow S_f = \frac{11.3^2 \times 0.017^2 \times (3 + 2\times 2.9)^{4/3}}{(3\times 2.9)^{10/3}} = 0.0005$$

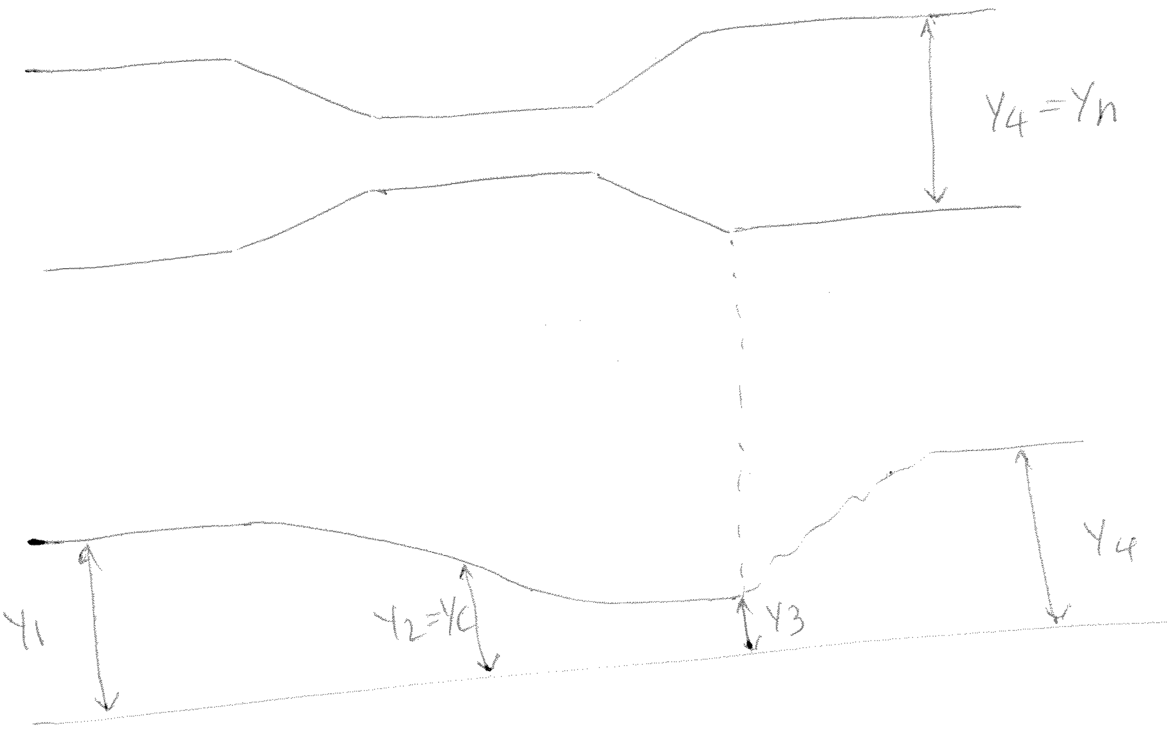
$$\Rightarrow \bar{S_f} = 0.00125$$

$$E_s = y + \frac{q^2}{2gy^2} : \quad E_{sup} = y_2 + \frac{q^2}{2g y_2^2} = 1.66 + \frac{3.77^2}{2\times 9.81\times 1.66^2} = 1.9$$

$$E_{sdn} = y_3 + \frac{q^2}{2g y_3^2} = 3$$

$$\Rightarrow \Delta x = \frac{3 - 1.92}{0.02 - 0.00125} = 57.6 \text{ m}$$

3)



$$Y_4 = Y_n : \quad Q = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} S_o^{1/2}$$

$$3 = \frac{1}{0.013} \frac{(2Y_4)^{5/3}}{(2+2Y_4)^{2/3}} (0.0018)^{1/2}$$

$\Rightarrow Y_4 = 0.8m$

$$\Rightarrow Y_3 = \frac{Y_4}{2}\left(\sqrt{1+8Fr_4^2} - 1\right) \quad ; \quad Fr_4 = \frac{V_4}{C_4} = \frac{Q/A_4}{\sqrt{gY_4}}$$

$$A_4 = bY_4 = 2 \times 0.8 = 1.6\,m^2 \Rightarrow Fr_4 = \frac{3/1.6}{\sqrt{9.81 \times 0.8}} = 0.67$$

$\Rightarrow Y_3 = 0.456\,m$

Energy Eq between 2 & 3

$$E_{s_2} = E_{s3}$$

2 is critical (we design it to be so)

$$E_{s3} = y_3 + \frac{q^2}{2g y_3^2} = 0.456 + \frac{(3/2)^2}{2 \times 9.81 \times 0.456^2} = 1.007 \text{ m}$$

$$\Rightarrow 1.007 = E_{c2} = \frac{3}{2} y_{c2} \Rightarrow y_{c2} = \begin{matrix}\mathbf{0.67}\\ \mathbf{1}\end{matrix} \text{ m}$$

$$y_{c2} = \sqrt[3]{q_2^2/g} = \sqrt[3]{\frac{Q^2/b_2^2}{g}} = \sqrt[3]{\frac{3^2}{9.81 b_2^2}}$$

$$\begin{matrix}\mathbf{0.67}\\ \mathbf{1}\end{matrix} = \sqrt[3]{\frac{3^2}{9.81 b_2^2}} \Rightarrow b_2 = \mathbf{1.74} \text{ m}$$

$$E_{s1} = E_{s3} \Rightarrow y_1 + \frac{q^2}{2g y_1^2} = 1.007 \text{ m}$$

$$\Rightarrow y_1 = \mathbf{0.85} \text{ m}$$