CVG 3116 Fall 2017

Hydraulics – Tutorial 9
November 28, 2017

University of Ottawa

Lecture 14: Sediment Transport – Initiation of Motion of Sediments

We have already seen that the bed shear stress is: $\tau_0 = \gamma R S_0$ (in uniform flow)

When this shear stress is higher than a critical shear stress, τ_{CR} , particles start to move, i.e., if $\tau_0 > \tau_{CR}$, then particles move and the channel (or river) is not stable.

At the threshold of movement, $\tau_0 = \tau_{CR}$

It is possible to show that initiation of motion depends on a parameter called entrainment function, F_s

$$F_S = \tau^*_{CR} = \frac{\tau_{CR}}{(\rho_s - \rho) g D}$$

τ_{CR}= critical shear stress
ρ_s= density of sediment
ρ= density of water
D= mean diameter of particles (or d_{50})

Shields (1936) obtained the value of F_S in a series of experiments, and plotted results as a function of particle Reynolds number (Re_*)

Van Rijn (1983) fitted the following formula on the Shields results:

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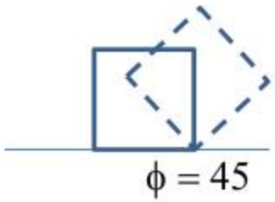
$$
\begin{cases}
For\ D_{gr} < 4 \quad , \quad F_S = 0.24 / D_{gr} \\
4 < D_{gr} < 10 \quad , \quad F_S = 0.14 / D_{gr}^{0.64} \\
10 < D_{gr} < 20 \quad , \quad F_S = 0.04 / D_{gr}^{0.1} \\
20 < D_{gr} < 150 \quad , \quad F_S = 0.013 D_{gr}^{0.29} \\
D_{gr} > 150 \quad , \quad F_S = 0.056
\end{cases}
$$

Where: $D_{gr} = D\left(g\left[\left(\rho_s / \rho\right) - 1\right] / \nu^2\right)^{1/3}$, D_{gr} is dimensionless particle size parameter and v is the kinematic viscosity of water (m^2/s).

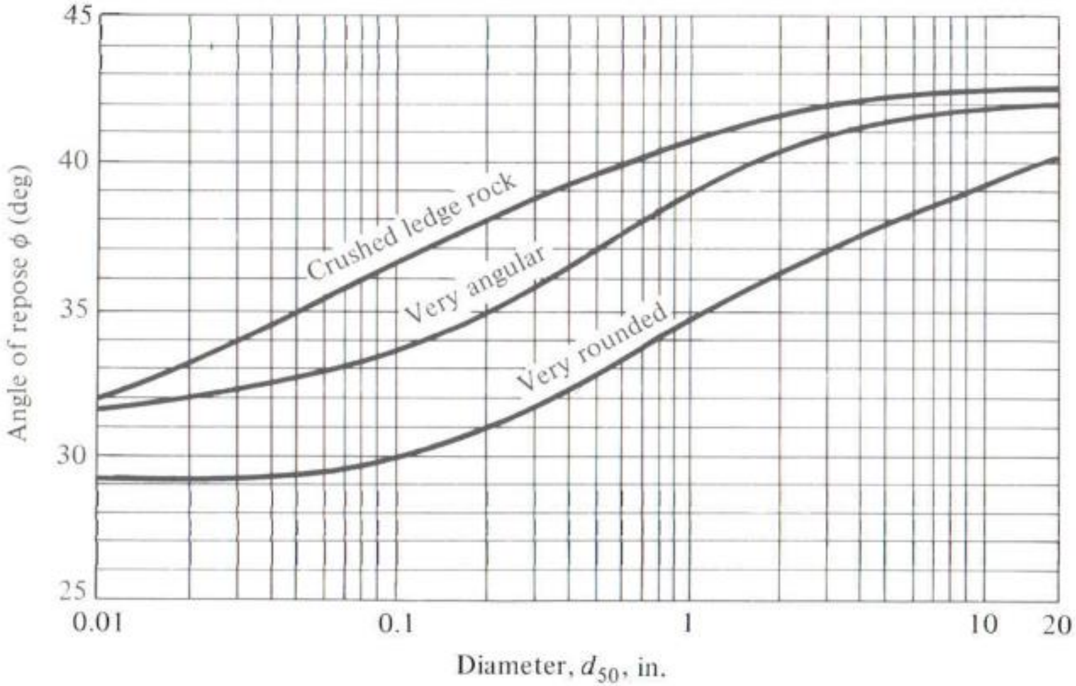
For a given sediment size D, calculate D_{gr}, then F_s and then τ_{CR}

Lecture 14: Design of Stable Trapezoidal Channels

1- Calculate Angle of repose, ϕ: Consider the stability of a particle. The angle of repose, is the angle that particle must be rotated such that its center of gravity lies on a vertical line with the point of contact.

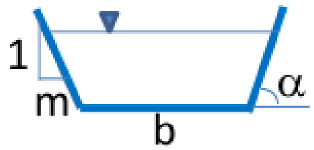


For sediments, the angle of repose may be found from the following diagram:



Lecture 14: Design of Stable Trapezoidal Channels (cont.)

2- Calculate side slope: Side angle, α must be less than the angle of repose, ϕ



3- Calculate critical shear stresses:

Bed shear stress: Critical shear stress $\tau_{b,CR}$ is found from the Shields diagram (or the Van Rijn formula).

Side slope shear stress: It could be shown that

$$\tau_{s,CR} = K\tau_{b,CR} \quad \text{where} \quad K = \sqrt{1 - \frac{\sin^2 \alpha}{\sin^2 \varphi}}$$

$\tau_{b,CR} = \textit{Critical bed shear stress}$

$\tau_{s,CR} = \textit{Critical side slope shear stress}$

α : *channel side angle*

φ : *angle of repose*

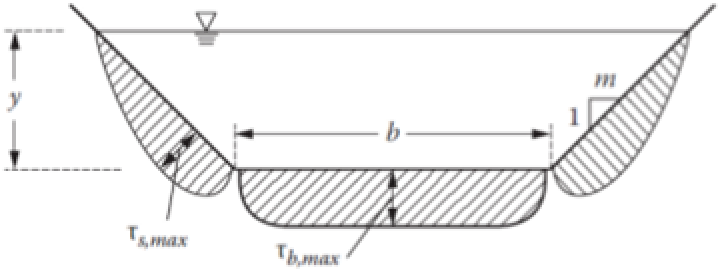
Note that gravity reduces critical shear stress on side slope

Lecture 14: Design of Stable Trapezoidal Channels (cont.)

4- Calculate actual shear stresses : Shear stress is not equal on the bottom and side slopes. It is conservative to assume that:

$$\tau_{b,\max} = \gamma \, y \, S_0$$

$$\tau_{s,\max} = 0.75\gamma \, y \, S_0$$



$$\tau_{b,\max} = Maximum \;\; bed \; shear \; stress$$

$$\tau_{s,\max} = Maximum \;\; side \; slope \; shear \; stress$$

NOTE: this is not conservative in outer bends of channels, where bank shear can exceed bed shear.

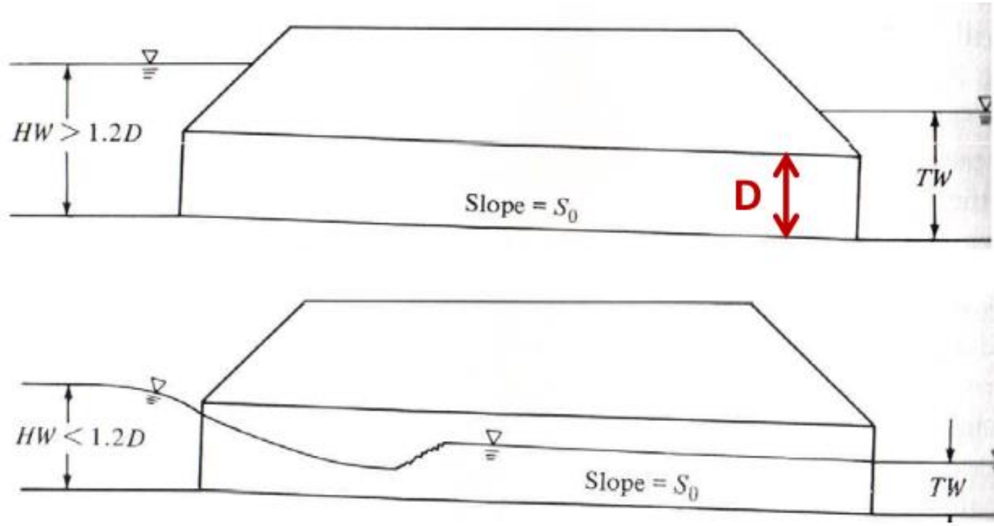
5- Channel water depth: actual shear stresses must be less than the critical value. This gives the water depth:

$$y = \min\left(\frac{\tau_{b,CR}}{\gamma S_0}, \frac{\tau_{s,CR}}{0.75\gamma S_0} \right)$$

5- Calculate Channel width, b from the Manning formula.

The Strickler formula for the Manning coefficient $n = 0.042 d_{50}^{1/6}$

Lecture 15: Hydraulics Design of Culverts



Flow in culverts depends on HW and TW:

- If HW > 1.2 D , inlet is submerged
- If TW > D , outlet is submerged

HW: head water
TW: tail water
D: Culvert size

Inlet control: characteristics of inlet and the headwater elevation controls the discharge. If inlet is submerged - use orifice equation, if inlet not submerged - use weir equation.

Outlet control: characteristics of pipe barrel (roughness and/or slope) and/or tailwater elevation control the discharge. Use energy equation to estimate discharge.

Lecture 15: Hydraulics Design of Culverts

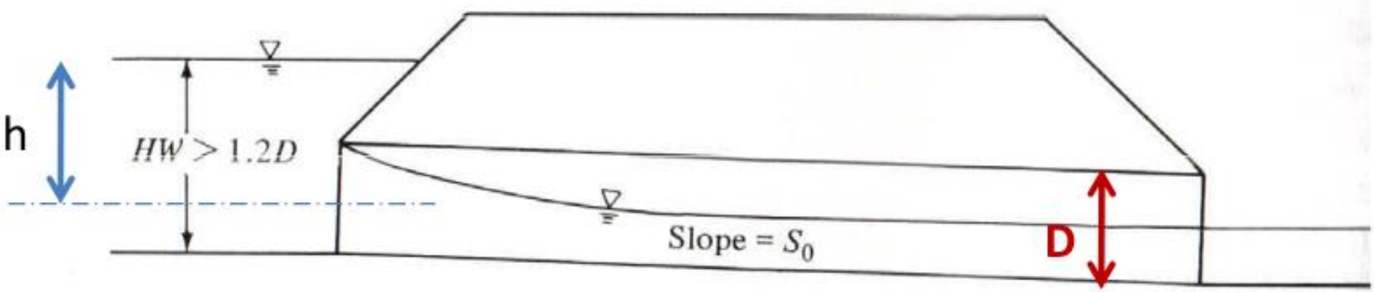
- Inlet control with entrance submerged:

Use the Orifice flow formula

$$Q = C_d A \sqrt{2gh}$$

where

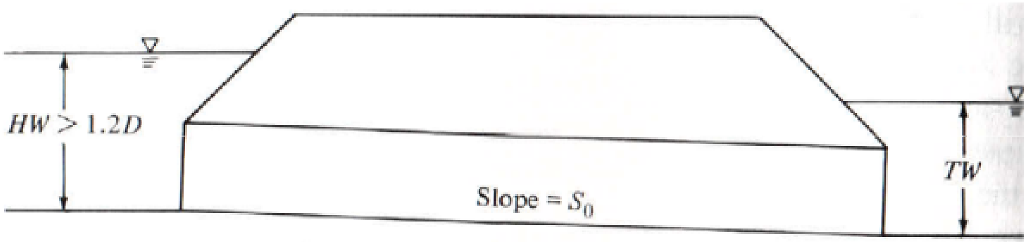
$$h = HW - D/2$$



(a) Inlet control, submerged inlet

- Outlet control:

Use the energy equation



(b) Outlet control, barrel flowing full

$$E_1 = E_2 + h_f + h_L$$

or

$$y_1 + \frac{V_1^2}{2g} + z_1 = y_2 + \frac{V_2^2}{2g} + z_2 + h_f + h_L$$

$$HW + S_0 L = TW + \left(k_L + 1 + \frac{2gn^2}{R^{4/3}} L \right) \frac{V^2}{2g}$$

Lecture 15: Hydraulics Design of Culverts - Summary

- Inlet control, inlet submerged:

use the Orifice flow formula

$$Q = C_d A_o \sqrt{2\,g\,(HW - D/2)}$$

- Inlet control, inlet not submerged, thus have y_c at inlet:

use the Weir formula

$$Q = C_d A_c \sqrt{2g\left(HW - y_c\right)}$$

- Outlet control: use the Energy equation

$$HW + S_0 L = TW + \left(k_L + 1 + \frac{2gn^2}{R^{4/3}} L\right)\frac{V^2}{2g}$$