CVG3116 Tutorial 9 - Solutions

Q1.

$Q = 2 m^3/s$

$b = 6 m$, Slopes 2H : 1V $(x = m = 2)$

$S_0 = 0.005$, $n = 0.03$

$d_{50} = ?$

$\rho_s = 2650 kg/m^3$, $\rho = 1000 kg/m^3$

$\nu = 1.306 \times 10^{-6} m^2/s$

Solution:

Determine the normal depth in the channel from Manning's eq:

$$Q = \frac{1}{n} \times \frac{A^{5/3}}{P^{2/3}} \times S_0^{1/2}$$

$$A = (b + x y_n) y_n = (6 + 2 y_n) y_n$$

$$P = b + 2 y_n \sqrt{1 + x^2} = 6 + 2 y_n \sqrt{1 + 2^2} = 6 + 4.47 y_n$$

$$2 = \frac{1}{0.03} \times \frac{[(6 + 2 y_n) y_n]^{5/3}}{(6 + 4.47 y_n)^{2/3}} \times 0.005^{1/2}$$

Solve for y_n by trial & error:

$y_n = 0.305 m$

$$R = \frac{A}{P} = \frac{[6 + (2)(0.305)](0.305)}{6 + (4.47)(0.305)} = 0.274 m$$

The bed shear stress at uniform flow is:

$$\tau_0 = \gamma R S_0 = (9.81)(1000)(0.274)(0.005) = 13.44 N/m^2$$

For the sediment particles to be on the verge of motion $\tau_o = \tau_{cr}$.

To determine τ_{cr} we need $F_s \rightarrow D_{gr} \rightarrow D$ and D is unknown. Assume $D_{gr} > 150$ and $F_s = 0.056$ (see Van Rijn table in lecture notes)

$$F_s = \frac{\tau_{cr}}{(\rho_s - \rho) g D} = 0.056$$

$$\tau_{cr} = (0.056)(2650 - 1000)(9.81) D = 906.44 D$$

From $\tau_o = \tau_{cr}$:

$$13.44 = 906.44 D \rightarrow D = 0.0148 \text{ m}$$

Check if $D_{gr} = 150$

$$D_{gr} = D \sqrt[3]{\frac{g[(\rho_s/\rho) - 1]}{\nu^2}} = 0.0148 \sqrt[3]{\frac{[(2650/1000) - 1]}{(1.306 \times 10^{-6})^2}} = 146.4$$

For the second iteration: $20 < D_{gr} < 150$

$$F_s = 0.013 D_{gr}^{0.29} = (0.013)(146.4)^{0.29} = 0.055$$

$$\tau_{cr} = (0.055)(2650 - 1000)(9.81) D = 893.47 D$$

$$13.44 = 893.47 D \rightarrow \underline{D = 0.015 \text{ m}}$$

Remark: τ_o used in the above example is a representative average value of shear stress across the channel wetted perimeter. In truth, shear stress varies over the wetted perimeter.

Q2.

Given: See Q1

Find d_{50} for $\tau_{b,max}$

d_{50} for $\tau_{s,max}$

Solution:

For the channel bed:

$\tau_{b,max} = \gamma R S_0 = 13.44 \ N/m^2$ - same as Ex. 1

The mean sediment particle diameter will be the same as in Ex. 1, $d_{50} = 0.015 m = 15 mm$

For the side slopes of the channel:

$\tau_{s,max} = 0.75 \ \tau_{b,max} = (0.75)(13.44) = 10.08 \ N/m^2$

The solution is again iterative. Assume

$20 < D_{gr} < 150$ and take $D_{gr} = 146.4$ (same as bed)

$F_s = 0.055$ } see Ex. 1

$\tau_{cr} = 893.47D$ }

Set $\tau_{s,max} = \tau_{cr}$

$10.08 = 893.47D \rightarrow D = 0.011 \ m = 11 \ mm$

Recalculate D_{gr}.

$$D_{gr} = D\sqrt[3]{\frac{g[(\rho_s/\rho) - 1]}{\nu^2}} = (0.011)\sqrt[3]{\frac{[(2650/1000) - 1]}{(1.306 \times 10^{-6})^2}} = 108.79$$

$$F_s = 0.013 \ D_{gr}^{0.29} = (0.013)(108.79)^{0.29} = 0.051$$

$$\tau_{cr} = F_s(\rho_s - \rho)gD = (0.051)(2650 - 1000)(9.81)D = 819.8D$$

$$10.08 = 819.8D \rightarrow \underline{D = 0.012 m = 12 mm}$$

Q3.

$Q = 50\ m^3/s$

$D = 2\ cm$ (very rounded)

$S_0 = 5 \times 10^{-4}$

$\rho_w = 1000\ kg/m^3$

$\rho_s = 2650\ kg/m^3$

$\nu = 10^{-6}$

Solution:

$D = 2\ cm = 0.787\ in \rightarrow \phi$ (angle of repose) $= 34^\circ$ (graph)

$$D_{gr} = D \sqrt[3]{\frac{g[(\rho_s/\rho_w)-1]}{\nu^2}} = 0.02 \sqrt[3]{\frac{9.81[(2650/1000)-1]}{(10^{-6})^2}} = 506$$

For $D_{gr} = 506 > 150 \rightarrow F_s = 0.056$

The critical bed shear stress is:

$$\tau_{b,cr} = \tau_{cR} = F_s(\rho_s - \rho_w) g D = 0.056(2650-1000)9.81 \times 0.02$$

$\tau_{b,cr} = 18.13\ N/m^2$

$\alpha < \phi$ but $m = \cot \alpha \geq \cot \phi$

$m \geq \cot 34^\circ = 1.48$ take $m = 1.75$ (conservative)

Check $\alpha = \cot^{-1}(m) = \cot^{-1}(1.75) = 29.7^\circ < 34^\circ$

$$K = \frac{\tau_{s,cR}}{\tau_{b,cR}} \qquad K = \sqrt{1 - \frac{\sin^2\alpha}{\sin^2\phi}} = \sqrt{1 - \frac{\sin^2(29.7^\circ)}{\sin^2(34^\circ)}} = 0.464$$

$\tau_{s,cR} = 0.464\, \tau_{b,cR} = 0.464 \times 18.13 = 8.41\ kN/m^2$

$$y = \min\left(\frac{\tau_{b,cR}}{\gamma S_0}, \frac{\tau_{s,cR}}{0.75 \gamma S_0}\right)$$

$$y = \min\left(\frac{18.13}{9810 \times 5 \times 10^{-4}}, \frac{8.41}{0.75 \times 9810 \times 5 \times 10^{-4}}\right)$$

$y = \min(3.70, 2.29) = 2.29\ m$

Calculate Manning's coefficient from Strickler's f-la:

$$n = 0.042\, d_{50}^{1/6} = 0.042 \times (0.02)^{1/6} = 0.022$$

$$A = by + my^2 = 2.29b + 1.75 \times 2.29^2 = 2.29b + 9.18$$

$$P = b + 2y\sqrt{1+m^2} = b + 2 \times 2.29\sqrt{1+1.75^2} = b + 9.23$$

$$Q = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} S_0^{1/2} = \frac{1}{0.022} \times \frac{(2.29b + 9.18)^{5/3}}{(b + 9.23)^{2/3}} \times (0.0005)^{1/2} = 50$$

$b = 11.1$ m

Design parameters: $\begin{cases} m = 1.75 \\ y = 2.29 \text{ m} \\ b = 11.1 \text{ m} \end{cases}$

Q4.

$L = 26\ m$, $D = 0.75m$, $S_0 = 2.5\% = 0.025$, $n = 0.014$

$HW = 2.5m$

$TW = 1.0\ m$

$k_L = 0.5$ (entrance), $k_2 = 1.0$ (exit); $C_d = 0.6$

$Q = ?$

Solution:

$HW > 1.2D = (1.2)(0.75) = 0.9\,m$

$TW > D$

Inlet and outlet are both submerged.

First, assume full pipe flow through the culvert and outlet control. Use energy eq.:

$$HW + S_0 L = TW + \left(k_L + 1 + \frac{2gn^2}{R^{4/3}} \times L\right)\frac{V^2}{2g}$$

$$A = \frac{\pi D^2}{4} = \frac{(\pi)(0.75)^2}{4} = 0.442\,m^2$$

$$R = \frac{D}{4} = \frac{0.75}{4} = 0.19\,m \quad (\text{only for circular sections!})$$

$$2.5 + (0.025)(26) = 1.0 + \left(0.5 + 1 + \frac{(2)(9.81)(0.014)^2}{(0.19)^{4/3}} \times 26\right) \times \frac{V^2}{(2)(9.81)}$$

$V = 4.179\ m/s$

$Q = VA = (4.179)(0.442) = 1.847\ m^3/s$

Next, assume inlet control and orifice flow at the inlet. Use orifice flow formula:

$$Q = C_d A_0 \sqrt{2g\,(HW - D/2)}$$

$$HW - \frac{D}{2} = 2.5 - \frac{0.75}{2} = 2.125\,m$$

$Q = (0.6)(0.442)\sqrt{(2)(9.81)(2.125)} = 1.712\ m^3/s$

The smallest discharge is the design discharge.

$\underline{Q = 1.712\ m^3/s}$ with inlet control.

Lastly, check if the culvert is flowing full by calculating y_n from given nomographs:

For $\dfrac{nQ}{S_o^{1/2}\, d_o^{8/3}} = \dfrac{(0.014)(1.712)}{(0.025)^{1/2}(0.75)^{8/3}} = 0.33 \rightarrow \dfrac{y_n}{d_o} = 0.9$

Note: $d_o = D$ - diameter of the culvert

$y_n = 0.9\, d_o = (0.9)(0.75) = 0.68\ m < D = 0.75m$, the culvert is not flowing full!