Assignment 1 - Solutions

(Due: 25 September, 2017 by 17:00)

Q1. Figure 1 below shows a conical vessel having its outlet at *A* to which a *U*-tube manometer is connected. The manometer gauge fluid is mercury (ρ_m = 13600 kg/m^3). The reading of the manometer given in the figure corresponds to an empty vessel. What will be the reading of the manometer when the vessel is completely filled with water (ρ_w = 1000 kg/m^3).

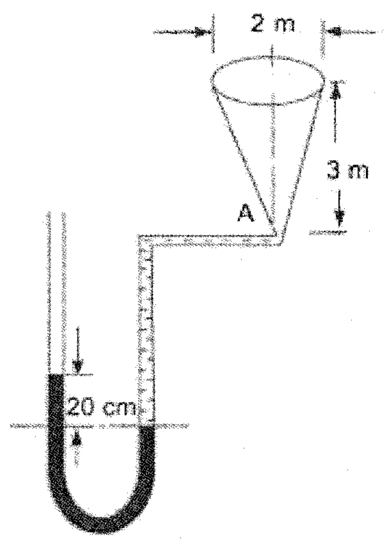
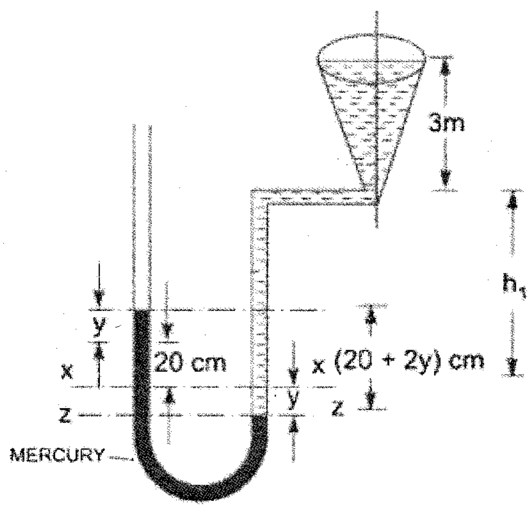


Figure 1

Solution:



Assume that vessel is empty:

Let the difference of mercury level is $h_2 = 20$ cm.
Equating the pressure above datum X-X gives:

$$\rho_m \times g \times h_2 = \rho_w \times g \times h_1$$

$$(13.6 \times 1000) \times (9.81) \times (0.2) = (1000) \times (9.81) \times h_1$$

$h_1 = 2.72$ m water

Assume that vessel is full of water:

The pressure in the right manometer limb will increase and the mercury level in that limb will go down by y cm. Correspondingly, the mercury in the left limb will rise by distance y cm. Let the new datum line be Z-Z. Equating the pressure about the datum line Z-Z gives:

$$(13.6 \times 1000) \times (9.81) \times \left(0.2 + \underbrace{\frac{2y}{100}}_{\text{as } y \text{ is in cm}}\right) = (1000) \times (9.81) \times \left(3 + 2.72 + \frac{y}{100}\right)$$

Solving for y gives: $y = 11.45$ cm

The reading is given by the difference of mercury level in the limbs:

Reading $= 20 + 2y = 20 + 2 \times 11.45 = 42.90$ cm

Q2. A syphon of diameter 200 mm connects two reservoirs having a difference in elevation of 20 m (Figure 2). The length of the syphon is 500 m and the summit is 3 m above the water level in the upper reservoir. The length of the pipe from the upper reservoir to the summit is 100 m. Determine the discharge through the syphon and the pressure at the summit. Neglect minor losses. Assume coefficient of friction for the pipe, $\lambda = 0.02$.

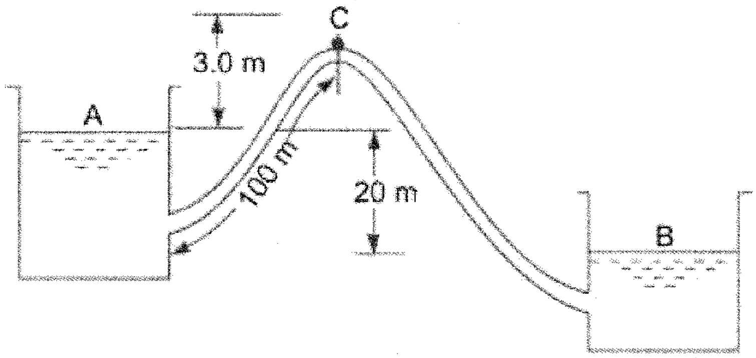


Figure 2

Solution:

If minor losses are neglected, then the entire head loss between reservoirs A and B is due to friction.
Applying Bernoulli's eq. between A and B:

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + h_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + h_B + \Delta z$$

But $p_A = p_B = 0$ (atmospheric pressure)

$V_A = V_B = 0$ at the surface of reservoirs

$h_A - h_B = \Delta z$

$$\Delta z = h_f(\text{pipe}) = \frac{\lambda L V^2}{2gD} \quad (\text{Darcy-Weisbach eq.})$$

$\Delta z = 20$ m (given), $\lambda = 0.02$, $D = 200\,mm = 0.2\,m$

$L = 500\,m$

$$20 = \frac{(0.02)(500)(V^2)}{(2\times 9.81)(0.2)}$$

$V_{pipe} = 2.80$ m/s

$Q = A_{pipe} \times V_{pipe}$

$$Q = \left(\frac{\pi \times 0.2^2}{4}\right)(2.80) = 0.0880 \ m^3/s = 88 \ L/s$$

Pressure at the summit:

Applying Bernoulli's eq. between points A and C:

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + h_A = \frac{P_C}{\rho g} + \frac{V_{pipe}^2}{2g} + h_C + h_{f,A-C}$$

where $h_{f,A-C}$ is loss of head due to friction between points A and C.

Set datum at A level: $h_A = 0 m$

$$0 + 0 + 0 = \frac{P_C}{(1000)(9.81)} + \frac{(2.80)^2}{(2)\times(9.81)} + 3.0 + h_{f,A-C}$$

Calculate $h_{f,A-C}$ using Darcy-Weisbach eq:

$$h_{f,A-C} = \frac{\lambda L V_{pipe}^2}{2 g D} = \frac{(0.02)(100)(2.80^2)}{(2)(9.81)(0.2)} = 4.00 \ m$$

$$\frac{P_C}{9810} + 0.40 + 3.0 + 4.0 = 0$$

$$\frac{P_C}{9810} = -7.40 \ m \text{ of water or } P_C = -72\,594 \ N/m^2$$

As expected for a syphon, the pressure at the summit is negative, i.e, less than atmospheric pressure.

Q3. Determine the discharge of water through a pipe of diameter 20 cm and length 50 m when one end of the pipe is connected to a tank and the other end of the pipe is open to the atmosphere (see Figure 3). The pipe is horizontal and the height of water in the tank is 4 m above the centerline of the pipe. Consider all losses. Assume a square-edge entrance and take $\lambda = 0.036$.

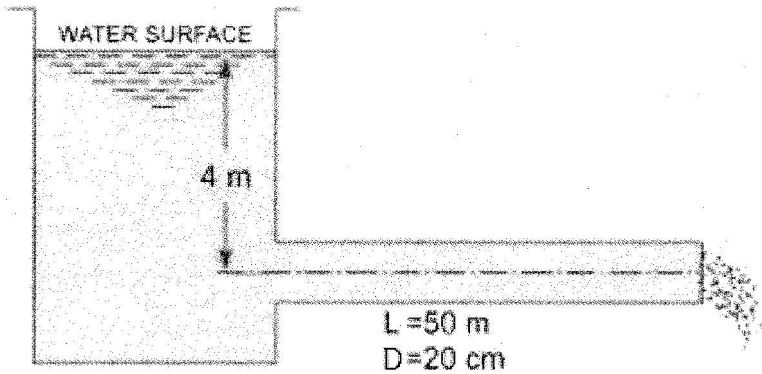


Figure 3

Solution:

For the pipe: $D = 20\,cm = 0.2\,m$ $\quad L = 50\,m$

Applying Bernoulli's eq. between water surface in the tank (point 1) and the outlet of the pipe (point 2):

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_2 + h_{L,2} + h_{f,2}$$

where $h_{L,2}$ and $h_{f,2}$ are the local and frictional losses in the pipe.

For a square-edge (simple) entrance: $k_L = 0.5$

Set datum at pipe's centerline.

$P_1 = 0$ and $V_1 = 0$; $P_2 = 0$ (outlet at atm. pressure)

$$4.0 = 0 + \frac{V_2^2}{2g} + h_{L,2} + h_{f2}$$

$$h_{L,2} = k_L \frac{V_2^2}{2g} = \frac{0.5V_2^2}{2g}$$

$$h_{f,2} = \frac{\lambda L V_2^2}{D\,2g} = \frac{(0.036)(50)V_2^2}{(0.2)(2g)} = \frac{9V_2^2}{2g}$$

$$4.0 = \frac{V_2^2}{2g}\left(1 + 0.5 + 9\right)$$

$$V_2 = 2.734 \text{ m/s}$$

$$Q = V_2 \times A_{pipe} = (2.734) \times \left(\frac{\pi \times 0.2^2}{4}\right) = \underline{0.0859 \text{ m}^3/\text{s} = 85.9 \text{ L/s}}$$

Q4. In Figure 4 below, when a sudden contraction from 50 cm to 25 cm is introduced in a horizontal pipeline, the pressure changes from 103005 N/m^2 to 67689 N/m^2. Determine the discharge through the pipeline. Following this, if there is a sudden expansion from 25 cm to 50 cm and if the pressure at the 25 cm section is 67689 N/m^2, determine the pressure at the 50 cm enlarged section.

Assume $K_L = 0.29$ at the sudden contraction and also consider the local losses due to sudden expansion.

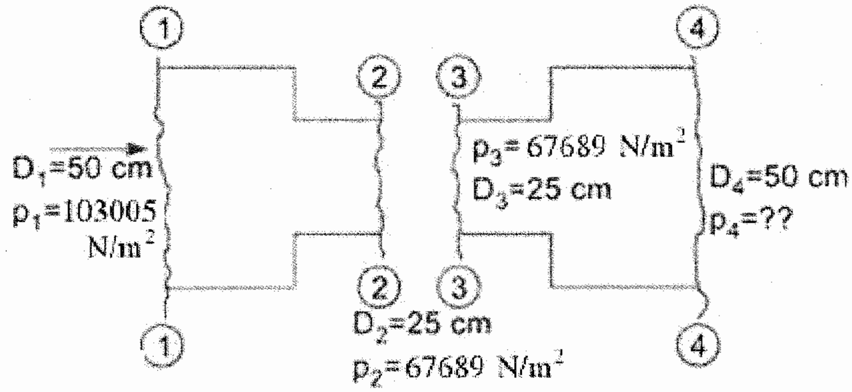


Figure 4

Solution:

For the larger section:

$D_1 = 50\,cm = 0.5\,m$

$$A_1 = \frac{\pi \times D_1^2}{4} = \frac{\pi \times 0.5^2}{4} = 0.196\,m^2$$

$p_1 = 103\,005\,N/m^2$

For the smaller section:

$D_2 = 25\,cm = 0.25\,m$

$$A_2 = \frac{\pi \times D_2^2}{4} = \frac{\pi \times 0.25^2}{4} = 0.049\,m^2$$

$p_2 = 67\,689\,N/m^2$

From continuity equation:

$Q = A_1 V_1 = A_2 V_2$

$$V_1 = \frac{A_2 V_2}{A_1} = \left(\frac{0.049}{0.196}\right) V_2 = \frac{V_2}{4} = 0.25\,V_2$$

Applying Bernoulli's eq. to sections 1 an 2 :

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_2 + h_{L,1-2}$$

$h_{L,1-2}$ = local losses due to the sudden contraction

$$h_{L,1-2} = k_L \frac{V_2^2}{2g} = 0.29 \frac{V_2^2}{2g}$$

$$\frac{103\,005}{1000 \times 9.81} + \frac{(0.25 V_2)^2}{2 \times 9.81} + 0 = \frac{67\,689}{1000 \times 9.81} + \frac{V_2^2}{2 \times 9.81} + 0 + 0.29 \frac{V_2^2}{2 \times 9.81}$$

$V_2 = 7.586$ m/s

$Q = A_2 V_2 = (0.049)(7.586) = 0.372\ m^3/s = 372$ L/s

Applying Bernoulli's eq. to sections 3 and 4 :

$$\frac{P_3}{\rho g} + \frac{V_3^2}{2g} + h_3 = \frac{P_4}{\rho g} + \frac{V_4^2}{2g} + h_4 + h_{L,3-4}$$

$h_{L,3-4}$ = local losses doe to sudden expansion :

$$h_{L,3-4} = \left(1 - \frac{A_3}{A_4}\right)^2 \frac{V_3^2}{2g} \quad \text{but} \quad A_3 = A_2 \quad A_4 = A_1$$

$$V_3 = V_2 \quad V_4 = V_1$$

$$h_{L,3-4} = \left(1 - \frac{0.049}{0.196}\right)^2 \times \frac{(7.586)^2}{(2 \times 9.81)} = 1.65m$$

$$\frac{67689}{(1000 \times 9.81)} + \frac{7.586^2}{(2 \times 9.81)} = \frac{P_4}{(1000 \times 9.81)} + \frac{(0.25 \times 7.586)^2}{(2 \times 9.81)} + 1.65$$

$P_4 = 78478\ N/m^2$