Assignment 2

(Due: 10 October, 2017 by 17:00)

Q1. In the system illustrated in Figure 1, a pump is installed in pipe *BC* to provide a flow of 40 L/s to reservoir *C*. Neglecting the velocity head at the junction and the local losses in the pipes, calculate the total head to be generated by pump and the input power assuming an overall efficiency of 60 percent. Also determine the flow rates in pipes *AB* and *BD*. Assume $\nu = 1 \times 10^{-6}$ m^2/s.

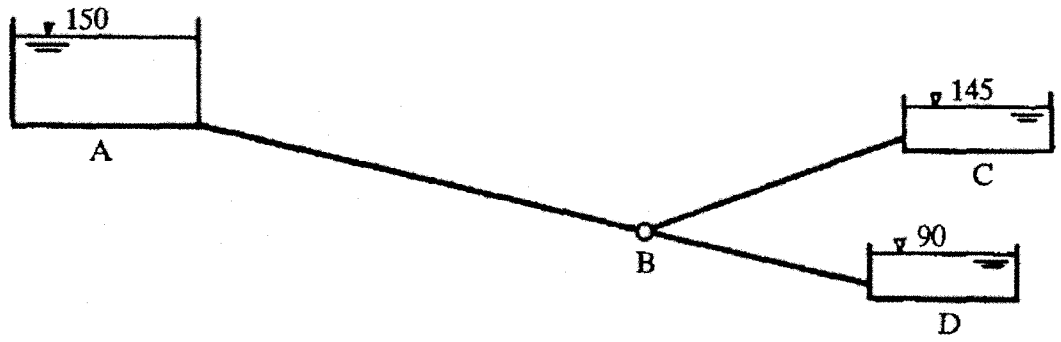


Figure 1

Pipe	AB	BC	BD
Roughness (mm)	0.06	0.06	0.06
Length (m)	10000	4000	5000
Diameter (mm)	400	250	250

Solution:

The flow in pipe BC is treated as external outflow of 40 L/s at junction B. For the first iteration assume pressure head elevation at junction B and calculate flow rates in pipes AB and BD so that: $q_{AB} - q_{BC} - q_{BD} = 0$ where $q_{BC} = 40$ L/s. Continue iterations (with $q_{BC} = 40$ L/s) until $\sum_{i=1}^{3} q_i = 0$.

Assume: H_B = 130.00 m

Pipe	L (m)	D (m)	ks/D (-)	ΔZ (m)	Initial λ (-)	V (m/s)	Re (-)	Revised λ (-)	Revised V (m/s)	New Re (-)	New λ (-)	q (m^3/s)
AB	10000	0.40	1.5E-04	20.00	0.0130	1.101	4.40E+05	0.0151	1.020	4.08E+05	0.0152	0.1282
BC	4000	0.25	2.4E-04	-15.00								-0.0400
BD	5000	0.25	2.4E-04	-40.00	0.0143	1.659	4.15E+05	0.0161	1.560	3.90E+05	0.0162	-0.0766
											δq =	0.0117

H_A = 150.00 m
H_C = 145.00 m
H_D = 90.00 m
ks = 6.00E-05 m
ν = 1.00E-06 m^2/s

Assume: H_B = 132.00 m

Pipe	L (m)	D (m)	ks/D (-)	ΔZ (m)	Initial λ (-)	V (m/s)	Re (-)	Revised λ (-)	Revised V (m/s)	New Re (-)	New λ (-)	q (m^3/s)
AB	10000	0.40	1.5E-04	18.00	0.0130	1.044	4.18E+05	0.0151	0.966	3.86E+05	0.0153	0.1214
BC	4000	0.25	2.4E-04	-13.00								-0.0400
BD	5000	0.25	2.4E-04	-42.00	0.0143	1.700	4.25E+05	0.0161	1.600	4.00E+05	0.0162	-0.0785
											δq =	0.002827

H_A = 150.00 m
H_C = 145.00 m
H_D = 90.00 m
ks = 6.00E-05 m
ν = 1.00E-06 m^2/s

Final: H_B = 132.62 m

Pipe	L (m)	D (m)	ks/D (-)	ΔZ (m)	Initial λ (-)	V (m/s)	Re (-)	Revised λ (-)	Revised V (m/s)	New Re (-)	New λ (-)	q (m^3/s)
AB	10000	0.40	1.5E-04	17.38	0.0130	1.026	4.10E+05	0.0152	0.948	3.79E+05	0.0153	0.1192
BC	4000	0.25	2.4E-04	-12.38					0.815	2.04E+05	0.0172	-0.0400
BD	5000	0.25	2.4E-04	-42.62	0.0143	1.713	4.28E+05	0.0161	1.612	4.03E+05	0.0162	-0.0791
											δq =	0.000021

H_A = 150.00 m
H_C = 145.00 m
H_D = 90.00 m
ks = 6.00E-05 m
ν = 1.00E-06 m^2/s

∴ The final result is: $H_B = 132.62\,m$ and flow rates: $q_{AB} = 119.2\,L/s$, $q_{BC} = 40\,L/s$, $q_{BD} = 79.1\,L/s$

The pump in pipe BC is required to overcome the difference in pressure head elevation between junction B and reservoir C ($\Delta Z_{BC} = 12.38m$) in addition to the frictional head loss along pipe BC.

$$V_{BC} = \frac{q_{BC}}{A_{BC}} = \frac{0.04}{\left(\frac{\pi \times 0.25^2}{4}\right)} = 0.815\,m/s$$

$$Re(BC) = \frac{V_{BC} \times D_{BC}}{\nu} = \frac{0.815 \times 0.25}{1.0 \times 10^{-6}} = 2.04 \times 10^5$$

From Moody diagram : $\lambda_{BC} = 0.0172$

D-W: $h_f(BC) = \frac{\lambda_{BC} L_{BC} V_{BC}^2}{2g D_{BC}} = \frac{0.0172 \times 4000 \times 0.815^2}{2 \times 9.81 \times 0.25} = 9.32 m$

Required $K_p = \Delta z_{BC} + h_f(BC) = 12.38 + 9.32 = 21.70 m$

$P_{out} = \rho g K_p q_{BC} = 1000 \times 9.81 \times 21.70 \times 0.04 = 8515.08 W$

If overall efficiency is 60%, than $\eta_\xi = 0.6$

$$P_{in}^{total} = \frac{P_{out}}{\eta_\xi} = \frac{8515.08}{0.6} = 14191.8 W = \underline{14.19 kW}$$

Q2. Determine the flow rate in the pipes (initial estimated flow rates are provided) and the pressure head elevations at the junctions of the closed-loop pipe network illustrated in Figure 2, neglecting local losses. Water enters the system at junction *A* from a storage tank (surface elevation of 60.0 m). All pipes have the same roughness size (k_s = 0.03 mm) with lengths and diameters provided in the table below. The outflows at the junctions are shown in L/s.

Hint: the solution is iterative. Continue iterations until $\Sigma h_f < 0.01$m for each loop.

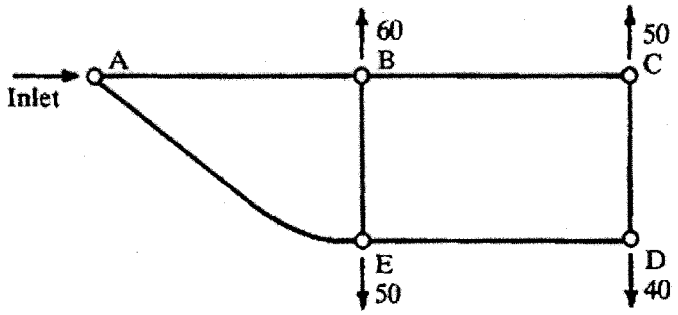
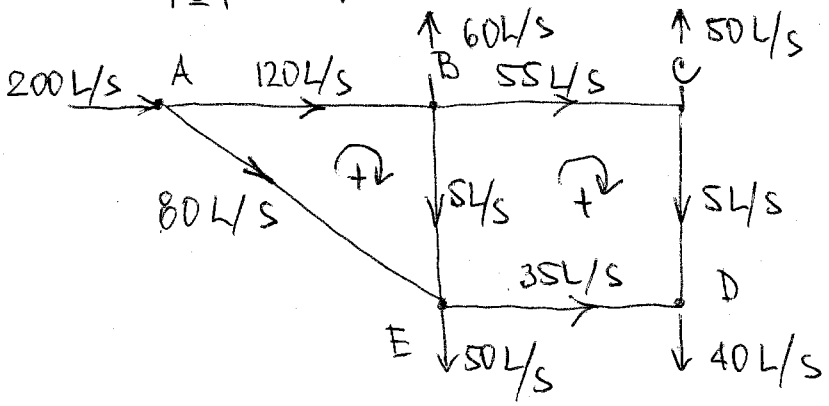


Figure 2

Pipe	AB	BC	CD	DE	EA	BE
Flow (L/s)	120	55	5	35	80	5
Length (m)	500	600	200	600	600	200
Diameter (mm)	200	150	100	150	200	100

Solution:

As inflow = outflow for continuity, the inflow at A = 60 + 50 + 50 + 40 = 200 L/s. Also at each junction, the inflow must be equal to the outflow. With the initial estimated flow rates in the pipes provided and $\sum_{i=1}^{n} = q$ we obtain:



Analysis is performed by Hardy-Cross method!

* Frictional head losses in pipes calculated from the Darcy-Weisbach equation with friction factor, λ determined from the Moody diagram:

D-W: $h_f = \frac{\lambda L V^2}{2gD}$

Moody diagram: $\lambda = 0.0055\left[1 + \left(\frac{20000 ks}{D} + \frac{10^6}{Re}\right)^{1/3}\right]$

The first two iterations are given below:

Pipe	q (m^3/s)	L (m)	D (m)	ks/D (-)	V (m/s)	Re (-)	λ* (-)	h_f (m)	h_f/q (-)	new q (m^3/s)	
AB	0.120	500	0.20	1.5E-04	3.820	6.76E+05	0.0146	27.080	225.668	0.1089	ks = 3.00E-05 m
BE	0.005	200	0.10	3.0E-04	0.637	5.63E+04	0.0213	0.880	176.073	-0.0061	ν = 1.13E-06 m^2/s
EA	-0.080	600	0.20	1.5E-04	2.546	4.51E+05	0.0150	-14.913	186.407	-0.0911	
							Σ	**13.048**	**588.149**		
							δq =	**-0.0111**			

Pipe	q (m^3/s)	L (m)	D (m)	ks/D (-)	V (m/s)	Re (-)	λ* (-)	h_f (m)	h_f/q (-)	new q (m^3/s)	
BC	0.0550	600	0.15	2.0E-04	3.112	4.13E+05	0.0157	31.050	564.543	0.0474	ks = 3.00E-05 m
CD	0.0050	200	0.10	3.0E-04	0.637	5.63E+04	0.0213	0.880	176.073	-0.0026	ν = 1.13E-06 m^2/s
DE	-0.0350	600	0.15	2.0E-04	1.981	2.63E+05	0.0164	-13.123	374.949	-0.0426	
BE	0.0061	200	0.10	3.0E-04	0.776	6.86E+04	0.0206	1.262	207.09	-0.0015	
							Σ	**20.069**	**1322.655**		
							δq =	**-0.0076**			

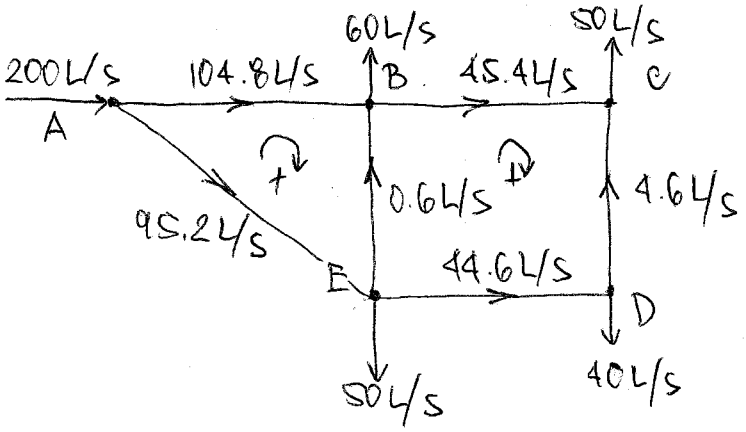
Pipe	q (m^3/s)	L (m)	D (m)	ks/D (-)	V (m/s)	Re (-)	λ* (-)	h_f (m)	h_f/q (-)	new q (m^3/s)	
AB	0.1089	500	0.20	1.5E-04	3.467	6.14E+05	0.0147	22.459	206.222	0.1054	ks = 3.00E-05 m
BE	0.0015	200	0.10	3.0E-04	0.190	1.68E+04	0.0277	0.102	68.2915	-0.0021	ν = 1.13E-06 m^2/s
EA	-0.0911	600	0.20	1.5E-04	2.900	5.13E+05	0.0149	-19.119	209.889	-0.0946	
							Σ	**3.442**	**484.402**		
							δq =	**-0.0036**			

Pipe	q (m^3/s)	L (m)	D (m)	ks/D (-)	V (m/s)	Re (-)	λ* (-)	h_f (m)	h_f/q (-)	new q (m^3/s)
BC	0.0474	600	0.15	2.0E-04	2.683	3.56E+05	0.0159	23.371	492.91	0.0455
CD	-0.0026	200	0.10	3.0E-04	0.329	2.91E+04	0.0244	-0.269	104.116	-0.0045
DE	-0.0426	600	0.15	2.0E-04	2.410	3.20E+05	0.0161	-19.044	447.176	-0.0445
BE	0.0021	200	0.10	3.0E-04	0.262	2.32E+04	0.0256	0.180	87.2308	0.0002
							Σ	**4.237**	**1131.433**	
							δq =	**-0.0019**		

ks = 3.00E-05 m

ν = 1.13E-06 m^2/s

After further iterations ($\Sigma h_f < 0.01$ m in any loop), the final flow rates and head losses in the pipes are:



Junction pressure head elevations:

$H_B = H_A - h_{AB} = 60 - 20.84 = 39.16$ m

$H_C = H_A - h_{AB} - h_{BC} = 60 - 20.84 - 21.52 = 17.64$ m

$H_D = H_A - h_{AB} - h_{BC} + h_{CD} = 60 - 20.84 - 21.52 + 0.75 = 18.39$ m

$H_E = H_A - h_{AE} = 60 - 20.82 = 39.18$ m

Final:

Pipe	AB	BC	CD	DE	EA	BE
Flow (m^3/s)	0.1048	0.0454	0.0046	0.0446	0.0952	0.0006
Flow (L/s)	104.8	45.4	4.6	44.6	95.2	0.6
Head Loss (m)	20.84	21.52	0.75	20.79	20.82	0.02

Junction	A	B	C	D	E
Pr. Head Elevation (m)	60.00	39.16	17.64	18.39	39.18