CVG3116 Assignment 3 - Solutions

Q1:

Discharge line: $L = 140\,m$; $D = 0.35\,m$; $k_L = 0.5$

Suction line: $L = 10\,m$; $D = 0.35\,m$, $h_L = 3.7\frac{V^2}{2g}$

$Q = 120\,L/s = 0.12\,m^3/s$

$H_s = 45\,m$ $\qquad$ $P_{atm} = 101.4\ kN/m^2$

$k_s = 0.12\,mm = 0.12 \times 10^{-3}\,m$ $\qquad$ $P_{vap} = 2.37\ kN/m^2$

$\nu = 1 \times 10^{-6}\ m^2/s$ $\qquad$ $\sigma_{crit} = 0.13$

Solution:

To deliver the discharge required, the pump must provide a total energy head of:

$$H_p = H_s + \left(\frac{\lambda L}{D} + \Sigma k_L\right)\frac{V^2}{2g}$$, where

the frictional and local losses are considered on both the suction and the discharge lines:

$$\frac{k_s}{D} = \frac{0.12 \times 10^{-3}}{0.35} = 0.00034$$

$$V = \frac{Q}{A} = \frac{0.12}{\left(\frac{\pi \times 0.35^2}{4}\right)} = 1.247\,m/s$$

$$Re = \frac{DV}{\nu} = \frac{0.35 \times 1.247}{1 \times 10^{-6}} = 4.36 \times 10^5$$

From Moody diagram: $\lambda = 0.017$

$L = L_{suction} + L_{discharge} = 140 + 10 = 150\,m$

$\Sigma k_L = \Sigma k_L(\text{suction}) + \Sigma k_L(\text{discharge}) = 3.7 + 0.5 = 4.2$

$$H_p = 45.0\,m + \left(\frac{0.017 \times 150}{0.35} + 4.2\right) \times \frac{1.247^2}{2 \times 9.81} = 45.91\ m$$

$NPSH\ crit = H_p \times \sigma\ crit = 45.91 \times 0.13 = 5.97\,m$

Calculate losses on the suction line only:

$$H_L = \left(\frac{\lambda L}{D} + \Sigma k_L\right)\frac{V^2}{2g} = \left(\frac{0.017 \times 10.0}{0.35} + 3.7\right) \times \frac{1.247^2}{2 \times 9.81} = 0.33\ m$$

The maximum elevation of the pump is:

$$z < \frac{P_{atm} - P_{vap}}{\rho g} - \frac{V^2}{2g} - H_L(\text{suction line}) - NPSH\ crit$$

$$z < \frac{(101.4 - 2.37) \times 10^3}{1000 \times 9.81} - \frac{1.247^2}{2 \times 9.81} - 0.33 - 5.97$$

$z < 3.72\ m$

As the pump is to be installed 1m to 3m above the supply reservoir, cavitation is not a problem!

Q2. Two identical pumps having the tabulated characteristics are to be installed in a pumping station to deliver sewage to a settling tank through a 200 mm in diameter pipeline that is 2.5 km long. The elevation difference between the reservoirs is 15 m. Allowing for minor head losses of 10 $v^2/2g$ and assuming an effective roughness of 0.15 mm calculate the discharge and power consumption if:

a) a single pump is used;

b) the two identical pumps are used is series;

c) the two identical pumps are used in parallel.

Q (L/s)	H_p (m)	η (%)
0	30.0	-
10	27.5	44
20	23.5	58
30	17.0	50
40	7.5	18

Solution:

Develop system curve:

$$H_{SH} = H_s + \left(\frac{\lambda L}{D} + \Sigma k_L\right)\frac{v^2}{2g} = 15.0 + \left(\frac{\lambda \times 2500}{0.2} + 10\right)\frac{v^2}{2g}$$

$$\frac{k_s}{D} = \frac{0.15\times10^{-3}}{0.2} = 0.00075$$; $H_s = 15.0$ m - static head

Assume values of Q and calculate system head, H_{SH}:

$Q = 0$

$V = 0$, $\lambda = 0$, $Re = 0$, $h_f = 0$, $h_L = 0$

$Q = 10$ L/s $= 0.010$ m^3/s

$$V = \frac{Q}{A} = \frac{0.01}{\left(\frac{\pi\times0.2^2}{4}\right)} = 0.318 \text{ m/s}$$

$$Re = \frac{DV}{\nu} = \frac{0.2\times0.318}{1.13\times10^{-6}} = 5.63\times10^4$$

λ is calculated from the Moody formula:

$$\lambda = 0.0055\left[1+\left(\frac{20000\,k_s}{D}+\frac{10^6}{Re}\right)^{1/3}\right] =$$

$$= 0.0055\left[1+\left(\frac{20000\times 0.15\times 10^{-3}}{0.2}+\frac{10^6}{5.63\times 10^4}\right)^{1/3}\right] = 0.0231$$

$$h_f = \frac{\lambda L V^2}{2gD} = \frac{0.0231\times 2500\times 0.318^2}{2\times 9.81\times 0.2} = 1.49\text{ m}$$

$$h_L = \Sigma k_L \frac{V^2}{2g} = (10)\left(\frac{0.318^2}{2\times 9.81}\right) = 0.05\text{ m}$$

$$H_{SH} = 15.0 + 1.49 + 0.05 = \underline{16.54\text{ m}}$$

Repeat above steps for $Q = 20, 30, 40$ L/s.

a) single pump operation: plot the given Q vs H_p, and Q vs η. Obtain the coordinates of the duty point ($Q, H_p; \eta$) as the point of intersection of the pump curve and the system curve. See plot:

$Q = 22.5$ L/s ; $H_p = 22$ m ; $\eta = 58\%$

$$P_{in}\,\text{total} = \frac{P_{out}}{\eta} = \frac{\rho g H_p Q}{\eta} = \frac{1000\times 9.81\times 22\times 0.0225}{0.58} =$$

$$= \underline{8.37\text{ kW}}$$

b) two identical pumps in series operation: plot given Q vs double the pump head (Q vs $2H_p$) and Q vs η.

Duty point coordinates:

$Q = 33$ L/s ; $H_p = 29$ m ; $\eta = 42\%$

$$P_{in}\,\text{total} = \frac{1000\times 9.81\times 29\times 0.033}{0.42} = \underline{22.35\text{ kW}}$$

c) two identical pumps in parallel operation: plot double the given Q vs the pump head (2Q vs Hp), and 2Q vs η.
Duty point coordinates:

$Q = 0.0285 \, m^3/s = 28.5 \, L/s$; $H_p = 27 \, m$; $\eta = 51\%$

$$P_{in} \text{ total} = \frac{1000 \times 9.81 \times 27 \times 0.0285}{0.51} = \underline{14.80 \text{ kW}}$$

Two pumps in series

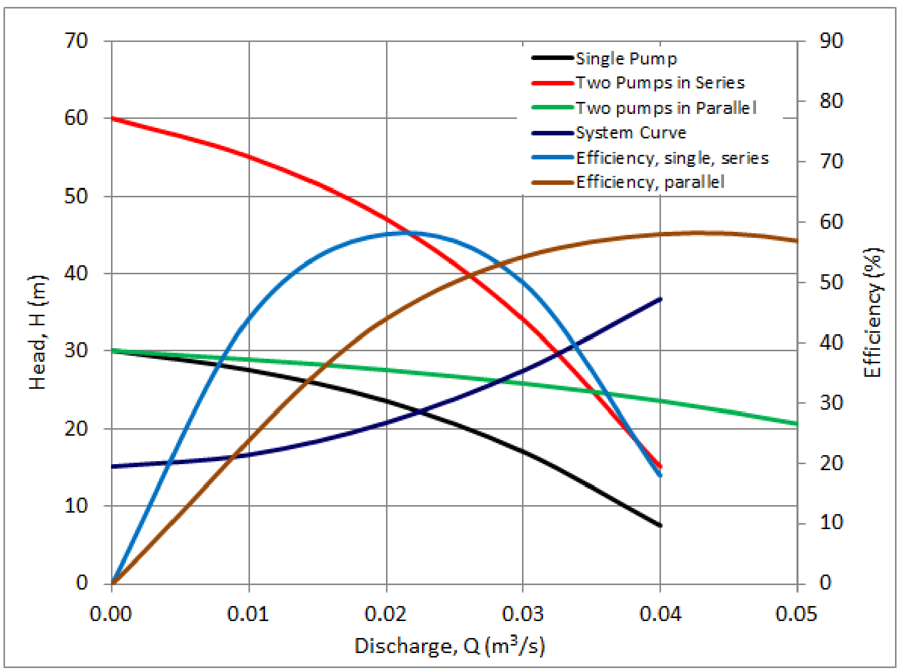
Q (m^3/s)	H_p (m)	η (%)	$2H_p$ (m)	v (m/s)	Re (-)	λ (-)	h_f (m)	h_L (m)	H_s (m)	H_{SH} (m)
0.00	30.00	-	60.00	0.000	0.00E+00	0.0000	0.00	0.00	15.00	15.00
0.01	27.50	44.00	55.00	0.318	5.63E+04	0.0231	1.49	0.05	15.00	16.54
0.02	23.50	58.00	47.00	0.637	1.13E+05	0.0213	5.51	0.21	15.00	20.72
0.03	17.00	50.00	34.00	0.955	1.69E+05	0.0207	12.00	0.46	15.00	27.46
0.04	7.50	18.00	15.00	1.273	2.25E+05	0.0203	20.95	0.83	15.00	36.78

ν = 1.13E-06 m^2/s
L = 2500 m
D = 0.2 m
Σk_L = 10
ks = 1.50E-04 m

Two pumps in parallel

Q (m^3/s)	H_p (m)	η (%)	2Q (m^3/s)	v (m/s)	Re (-)	λ (-)	h_f (m)	h_L (m)	H_s (m)	H_{SH} (m)
0.00	30.00	-	0.00	0.00	0.00E+00	0.0000	0.00	0.00	15.00	15.00
0.01	27.50	44.00	0.02	0.32	5.63E+04	0.0231	1.49	0.05	15.00	16.54
0.02	23.50	58.00	0.04	0.64	1.13E+05	0.0213	5.51	0.21	15.00	20.72
0.03	17.00	50.00	0.06	0.95	1.69E+05	0.0207	12.00	0.46	15.00	27.46
0.04	7.50	18.00	0.08	1.27	2.25E+05	0.0203	20.95	0.83	15.00	36.78

ν = 1.13E-06 m^2/s
L = 2500 m
D = 0.2 m
Σk_L = 10
ks = 1.50E-04 m



Q3. A pump is required to deliver water to an elevated storage tank. The water must be raised 44 m, and a 150 m-long ductile-iron pipe (k_s = 0.12 mm), 35 cm in diameter is to be used. Determine the appropriate pump speed (based on the highest efficiency obtainable) and the operating conditions if **Pump III** in Figure 5.24 is used. The suction line is 10 m of the 150 m length, minor loss coefficients in the suction line total 3.7, and the discharge line contains only an exit loss (k_L = 1.0).

Assume ν = 1 x 10^{-6} m^2/s.

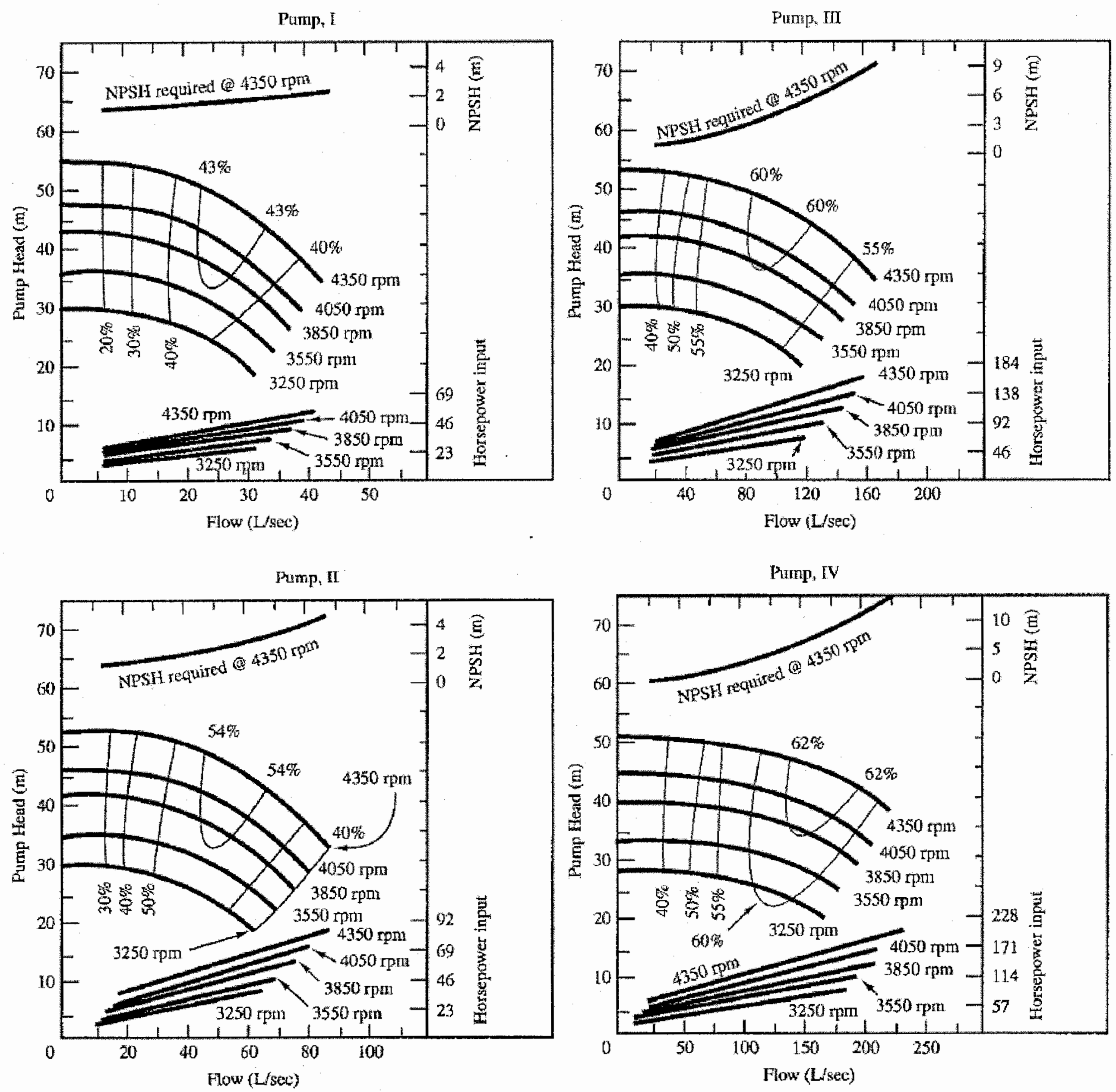


Figure 5.24 Characteristic curves for several pump models

Solution:

The pump must generate a total head equal to the static head (elevation difference) plus the pipeline friction and local head losses, including those on the suction line. Develop system curve:

$$H_{SH} = H_s + \left(\frac{\lambda L}{D} + \Sigma k_L\right)\frac{V^2}{2g}\ ,\ \text{where}$$

$\Sigma k_L = 3.7 + 1.0 = 4.7$ - for the discharge and suction lines

$$H_{SH} = 44.0 + \left(\frac{\lambda \times 150}{0.35} + 4.7\right)\frac{V^2}{2g}$$

Assume values of Q and calculate system head, H_{SH}:

$Q = 40 L/s = 0.04\ m^3/s$

$$V = \frac{Q}{A} = \frac{0.04}{\left(\frac{\pi \times 0.35^2}{4}\right)} = 0.416\ m/s$$

$$Re = \frac{DV}{\nu} = \frac{0.35 \times 0.416}{1.0 \times 10^{-6}} = 1.46 \times 10^5$$

$$\frac{k_s}{D} = \frac{0.12 \times 10^{-3}}{0.35} = 0.000343$$

$$\lambda = 0.0055\left[1 + \left(\frac{20000 k_s}{D} + \frac{10^6}{Re}\right)^{1/3}\right] =$$

$$= 0.0055\left[1 + \left(\frac{20000 \times 0.12 \times 10^{-3}}{0.35} + \frac{10^6}{1.46 \times 10^5}\right)^{1/3}\right] = \underline{0.0187}$$

$$h_f = \frac{\lambda L V^2}{2gD} = \frac{0.0187 \times 150 \times 0.416^2}{2 \times 9.81 \times 0.35} = 0.07\ m$$

$$h_L = (3.7 + 1.0)\frac{V^2}{2g} = 4.7 \times \frac{0.416^2}{2 \times 9.81} = 0.04\ m$$

$$H_{SH} = 44.0 + 0.07 + 0.04 = 44.11\ m$$

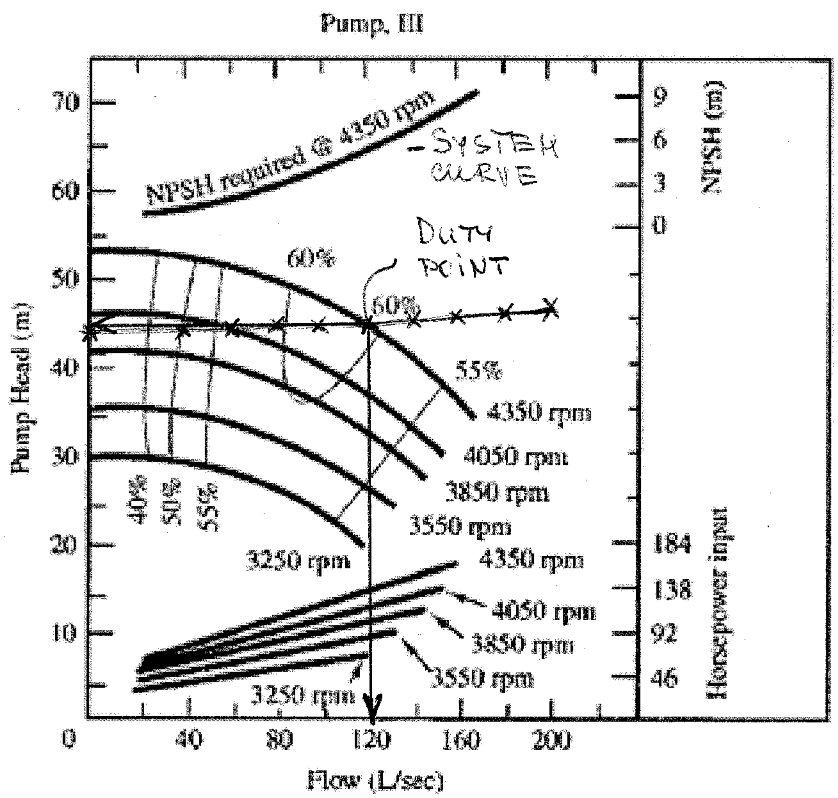
Repeat previous steps until enough points on the system curve are obtained.

System Curve

Q (L/s)	Q (m^3/s)	v (m/s)	R_e (-)	λ (-)	h_f (m)	h_L (m)	H_s (m)	H_{SH} (m)
0.00	0.00	0.000	0.00E+00	0.0000	0.00	0.00	44.00	44.00
40.00	0.04	0.416	1.46E+05	0.0187	0.07	0.04	44.00	44.11
60.00	0.06	0.624	2.18E+05	0.0179	0.15	0.09	44.00	44.25
80.00	0.08	0.832	2.91E+05	0.0175	0.26	0.17	44.00	44.43
100.00	0.10	1.039	3.64E+05	0.0172	0.41	0.26	44.00	44.66
120.00	0.12	1.247	4.37E+05	0.0170	0.58	0.37	44.00	44.95
140.00	0.14	1.455	5.09E+05	0.0169	0.78	0.51	44.00	45.29
160.00	0.16	1.663	5.82E+05	0.0168	1.01	0.66	44.00	45.67
180.00	0.18	1.871	6.55E+05	0.0167	1.27	0.84	44.00	46.11
200.00	0.20	2.079	7.28E+05	0.0166	1.57	1.04	44.00	46.60

ν = 1.00E-06 m^2/s
L = 150 m
D = 0.35 m
Σk_L = 4.7
k_s = 0.00012 m
k_s/D = 3.43E-04

Plot the system curve on the chart for Pump III:



The highest efficiency is obtained if the pump operates at a speed of: 4350 rpm. At that speed the coordinates of the duty point are:

$\omega = 4350$ rpm $\rightarrow \eta = 61\%$, $Q = 120 L/s$, $H_p \approx 45m$

Q4. A 0.5 m–diameter concrete (k_s = 0.36 mm) pipe has 5 cm-thick rigid walls and carries water 600 m before discharging it into another reservoir. The surface elevation of the downstream reservoir is 55 m lower than the supply reservoir. A gate valve (k_L = 0.15) just upstream of the lower reservoir controls flow rate. Minor losses on the pipe also include a simple entrance (k_L = 0.5). Calculate the maximum water hammer pressure that can be expected on the valve if it closes in 0.65 sec. Also determine the total pressure the pipeline will be exposed to during the water hammer phenomenon.

Assume: $K = 2.2 \times 10^9$ N/m², $\nu = 1.0 \times 10^{-6}$ m²/s

Solution:

Apply the energy equation between the surface of the upper (supply) and that of the downstream reservoir:

$$z_1 + \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_f + h_L$$

but $p_1 = p_2 = 0$ and $V_1 = V_2 = 0$ $\quad \Sigma k_L = 0.15 + 0.5 = 0.65$

$$z_1 - z_2 = h_f + h_L$$

$$55.0 = \left(\frac{\lambda L}{D} + \Sigma k_L\right)\frac{V^2}{2g}$$ where V is the velocity in the pipe.

$$\frac{k_s}{D} = \frac{0.36 \times 10^{-3}}{0.5} = 0.00072$$

Assuming rough turbulence and using C-W f-la:

$$\lambda = \frac{1}{4\left[\log\left(\frac{k_s}{3.7D}\right)\right]^2} = \frac{1}{4\left[\log\left(\frac{0.00072}{3.7}\right)\right]^2} = 0.0182$$

$$55.0 = \left(\frac{0.0182 \times 600}{0.5} + 0.65\right)\frac{V^2}{2 \times 9.81}$$

$V = 6.927$ m/s

$$Re = \frac{DV}{\nu} = \frac{(0.5)(6.927)}{1.0 \times 10^{-6}} = 3.46 \times 10^6$$

Using Moody's equation:

$$\lambda = 0.0055\left[1+\left(\frac{20\,000\,k_s}{D}+\frac{10^6}{Re}\right)^{1/3}\right] =$$

$$= 0.0055\left[1+\left(\frac{20000\times 0.36\times 10^{-3}}{0.5}+\frac{10^6}{3.46\times 10^6}\right)^{1/3}\right] \doteq 0.019$$

Recalculate V_{pipe}:

$$55.0 = \left(\frac{0.019\times 600}{0.5}+0.65\right)\frac{V^2}{2\times 9.81}$$

$$V = 6.789 \text{ m/s}$$

$$Q = V\times A = (6.789)\left(\frac{\pi\times 0.5^2}{4}\right) = 1.33 \text{ m}^3/\text{s}$$

Calculate celerity of shock wave:

For rigid pipe: $c = \sqrt{K/\rho} = \sqrt{\frac{2.2\times 10^9}{1000}} = 1483$ m/s

$$\frac{2L}{c} = \frac{2\times 600}{1483} = 0.81 \text{ s}$$

$t_c = 0.65 \text{ s} < \frac{2L}{c} = 0.81 \text{ s}$ - instantaneous closure.

$\Delta p = \rho c u_0 \qquad u_0 = V = 6.789$ m/s

$\Delta p = 1000\times 1483\times 6.789 = 10\,068\,087 \text{ N/m}^2 = \underline{10.07 \text{ MPA}}$

The total pressure in the pipeline during water hammer is:

$$P_t = P_{static} + \Delta P = \rho g \Delta z + \Delta p =$$

$$= 1000\times 9.81\times 55.0 + 10\,068\,087 = 10\,607\,637 \text{ N/m}^2$$

$\underline{P_t = 10.61 \text{ MPA}}$