CVG 3116 Assignment # 4 - Solutions

Q1.

$b_1 = 2.0\,m$, $b_2 = 1.2\,m$, $S_0 = 0.0015$, $n = 0.012$

$Q = 3\,m^3/s$

$y_n = ?$ $y_2 = ?$ $b_{2c} = ?$

Solution:

a) Calculate the depth of uniform flow (i.e., the normal depth) in the main channel using Manning's eq:

$$Q = \frac{1}{n}\frac{A^{5/3}}{P^{2/3}} S_0^{1/2}$$

$$A = b_1 y_n = 2y_n$$

$$P = b_1 + 2y_n = 2 + 2y_n$$

$$3.0 = \frac{1}{0.012} \times \frac{(2y_n)^{5/3}}{(2+2y_n)^{2/3}} \times 0.0015^{1/2}$$

Solve for y_n by trial & error:

$\underline{y_n = 0.80\,m}$

b) As the energy losses are neglected, the specific energy should remain constant:

E_{s1} (before contr.) = E_{s2} (in contr.)

$y_1 = y_n = 0.80\,m$ (from a)

$$q_1 = \frac{Q}{b_1} = \frac{3.0}{2.0} = 1.5\,m^3/s/m$$

$$q_2 = \frac{Q}{b_2} = \frac{3.0}{1.2} = 2.5\,m^3/s/m$$

$$E_{s1} = y_1 + \frac{q_1^2}{2gy_1^2} = 0.80 + \frac{(1.5)^2}{(2)(9.81)(0.8)^2} = 0.98\,m$$

Check if the contraction is too narrow and cause chocking of the flow:

$$y_{c2} = \sqrt[3]{\frac{q_2^2}{g}} = \sqrt[3]{\frac{(2.5)^2}{9.81}} = 0.86\,m$$

For rectangular channels:

$$E_{sc_2} = \frac{3}{2} y_c = \left(\frac{3}{2}\right)(0.86) = 1.29\,m$$

$E_{s1} = 0.98\,m < E_{sc_2} = 1.29\,m$, choking will happen!

Thus, the flow in the contraction will be critical and the upstream depth will increase!

Calculate $y_{1,new}$ using: $E_{s1,new} = E_{sc_2}$

$$1.29 = y_{1,new} + \frac{q_1^2}{2g\,y_{1,new}^2}$$

$$1.29 = y_{1,new} + \frac{(1.5)^2}{(2)(9.81)(y_{1,new})^2}$$

Solve for $y_{1,new} > y_{c2} > y_1$ by trial & error:

$y_{1,new} = \underline{1.22\,m}$

c) Let b_{2c} be the contraction width to create critical flow without choking.

$y_2 = y_{c2}$ and $E_{sc_2} = E_{s1} = 0.98\,m$

$$y_{c2} = \frac{2}{3} E_{sc_2} = \left(\frac{2}{3}\right)(0.98) = 0.65\,m$$

$$0.98 = y_{c2} + \frac{q_2^2}{2g\,y_{c2}^2}$$

$$0.98 = 0.65 + \frac{q_2^2}{(2)(9.81)(0.65)^2}$$

Solve for q_2 (no iterations needed)

$$q_2 = 1.65 \ m^3/s/m$$

$$q_2 = \frac{Q}{b_{2c}} \rightarrow b_{2c} = \frac{Q}{q_2} = \frac{3.0}{1.65} = \underline{1.81 \ m}$$

Q2.

$b = 4.0\,m$ (rectangular), $Q = 25\,m^3/s$, $y_n = 3.50\,m$

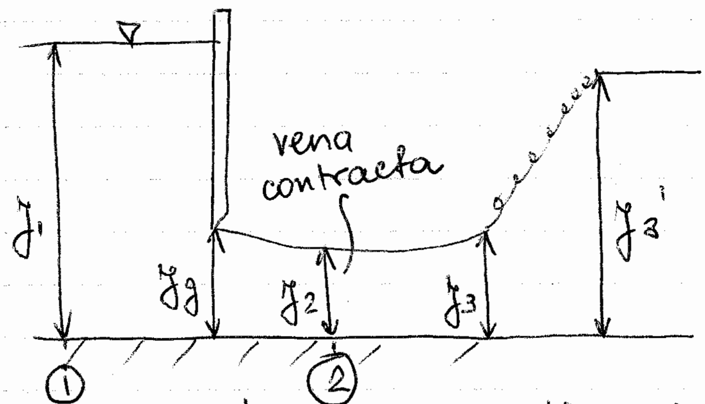
C_c(gate) $= 0.6$, $\alpha = 1.2$

$y_g = 0.9\,m$, $y_3 = ?$ $y_1 = ?$

$y_g = 1.5\,m$, $y_4 = ?$ $y_1 = ?$

Solution:

Diagram for a and b:



a) Hydraulic jump with $y_3' = y_n = 3.50\,m$

$y_g = 0.9\,m$ $\quad y_2 = C_c y_g = (0.6)(0.9) = 0.54\,m$

$$V_2 = \frac{Q}{A_2} = \frac{25.0}{(4.0)(0.54)} = 11.57\ m/s$$

$$Fr_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{11.57}{\sqrt{(9.81)(0.54)}} = 5.03 > 1.0 \rightarrow$$ supercritical flow under the gate!

Sequent depth $y_3' = y_n = 3.50\,m$

$$V_3' = \frac{Q}{A_3'} = \frac{25.0}{(4.0)(3.50)} = 1.79\ m/s$$

$$Fr_3' = \frac{V_3'}{\sqrt{g y_3'}} = \frac{1.79}{\sqrt{(9.81)(3.50)}} = 0.31 < 1 \rightarrow$$ subcritical flow at $y_3' = y_n = 3.50\,m$!

The transition from supercritical flow under the gate to subcritical flow at normal depth downstream of the gate can only happen through a hydraulic jump!

$$y_3 = \left(\frac{y_3'}{2}\right)\left(\sqrt{1+8Fr_{3'}^2}-1\right) = \left(\frac{3.50}{2}\right)\left(\sqrt{1+8(0.31)^2}-1\right) = 0.58m$$

$y_3 = 0.58m > y_2 = 0.54m \rightarrow$ hydraulic jump will form downstream of vena contracta!

b) Assuming no energy losses through the gate, apply the energy equation between ① and ②:

$$y_1 + \frac{\alpha V_1^2}{2g} = y_2 + \frac{\alpha V_2^2}{2g} \qquad \text{where } \alpha = 1.2$$

$$V_1 = \frac{Q}{A_1} = \frac{25.0}{4y_1} = \frac{6.25}{y_1}$$

$$y_1 + \frac{(1.2)(6.25)^2}{(2)(9.81)(y_1)^2} = 0.54 + \frac{(1.2)(11.57)^2}{(2)(9.81)}$$

Solve for y_1 by trial & error:

$y_1 = 8.69\ m$

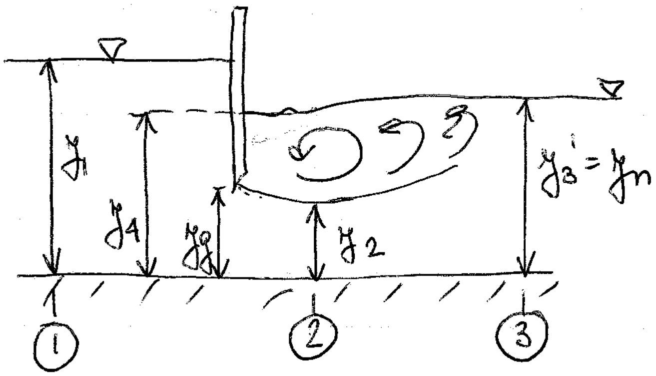
c) $y_g = 1.5m$

$y_2 = C_c y_g = (0.6)(1.5) = 0.9m$

$$V_2 = \frac{Q}{A_2} = \frac{25.0}{(4.0)(0.9)} = 6.94\ m/s$$

$$Fr_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{6.94}{\sqrt{(9.81)(0.9)}} = 2.34 > 1 \Rightarrow$$ supercritical flow under the gate!

Hydraulic jump will form downstream of the gate! As $y_2 = 0.9m > y_3 = 0.58m$, the jump will be submerged!



$y_2 = 0.9\,m$

$y_3' = y_n = 3.50\,m$

Apply the momentum equation between (2) and (3):

$$\frac{y_4^2}{2} + \frac{Q^2}{g b^2 y_2} = \frac{(y_3')^2}{2} + \frac{Q^2}{g b^2 y_3'}$$

$$\frac{y_4^2}{2} + \frac{(25.0)^2}{(9.81)(4.0)^2(0.9)} = \frac{(3.5)^2}{2} + \frac{(25.0)^2}{(9.81)(4.0)^2(3.50)}$$

$$y_4 = 2.38\,m$$

Apply the energy equation between (1) and (2)

$$y_1 + \frac{\alpha Q^2}{2 g b^2 y_1^2} = y_4 + \frac{\alpha Q^2}{2 g b^2 y_2^2}$$

$$y_1 + \frac{(1.2)(25.0)^2}{(2)(9.81)(4.0)^2 y_1^2} = 2.38 + \frac{(1.2)(25.0)^2}{(2)(9.81)(4.0)^2(0.9)^2}$$

Solve for y_1 by trial and error:

$$y_1 = 5.24\,m$$

Q3.

$b = B = 4.0\,m$, $Q = 25.0\,m^3/s$, $n = 0.015$, $S_0 = 0.001$

$y_{us} = 4.0\,m$

$y_c = ?$ $y_n = ?$ Water surface profile us?

$\Delta X = ?$ (for $y = 1.10\,y_n$)

Solution:

a) Find normal depth (y_n) from Manning's eq:

$$Q = \frac{1}{n} \times \frac{A^{5/3}}{P^{2/3}} \times S_0^{1/2}$$

$$A = b y_n = 4 y_n$$
$$P = b + 2 y_n = 4 + 2 y_n$$

$$25.0 = \frac{1}{0.015} \times \frac{(4 y_n)^{5/3}}{(4 + 2 y_n)^{2/3}} \times 0.001^{1/2}$$

Solve for y_n by trial & error: $\underline{y_n = 2.70\,m}$

Find the critical depth (y_c):

$$y_c = \sqrt[3]{\frac{q^2}{g}}$$, for a rectangular channel!

$$q = \frac{Q}{b} = \frac{25.0}{4.0} = 6.25\,m^3/s/m$$

$$y_c = \sqrt[3]{\frac{(6.25)^2}{9.81}} = \underline{1.59\,m}$$

b) As $y_n = 2.70\,m > y_c = 1.59\,m$, the channel is mild!

The given depth upstream from the dam (4.0m) is greater than both y_n and y_c. Therefore, we have region 1 and M-1 type curve. The water depth increases in the flow direction and tends to y_n as we move upstream.

c) The profile is found by starting at the dam (control point) and $y = 4.0\,m$, then proceeding upstream at small intervals of depth, Δy, until we reach the normal depth, y_n. Fr, S_o, S_f are evaluated at each intermediate depth.

Step 1 : Sample calculations

$y_1 = 4.0\,m \qquad \Delta y = 0.1\,m$

$y_2 = y_1 - \Delta y = 4.0 - 0.1 = 3.90\,m$

$A_1 = by = (4)(4.0) = 16.0\,m^2$

$P_1 = b + 2y_1 = 4 + 2(4.0) = 12.0\,m$

$$Fr_1^2 = \frac{q^2}{g y_1^3} = \frac{(25.0/4.0)^2}{(9.81)(4.0)^3} = 0.0622$$

$1 - Fr_1^2 = 1 - 0.0622 = 0.9378$

$$S_{f1} = \left(\frac{n Q P_1^{2/3}}{A_1^{5/3}}\right)^2 = \left[\frac{(0.015)(25.0)(12.0)^{2/3}}{(16.0)^{5/3}}\right]^2 = 3.74 \times 10^{-4}$$

$S_o - S_{f1} = 0.001 - 0.000374 = 0.000626 = 6.26 \times 10^{-4}$

$A_2 = b y_2 = (4)(3.9) = 15.6\,m^2$

$P_2 = b + 2y_2 = 4 + 2(3.9) = 11.8\,m$

$$Fr_2^2 = \frac{q^2}{g y_2^3} = \frac{(6.25)^2}{(9.81)(3.9)^3} = 0.0671$$

$1 - Fr_2^2 = 1 - 0.0671 = 0.9329$

$$S_{f2} = \left(\frac{n Q P_2^{2/3}}{A_2^{5/3}}\right)^2 = \left[\frac{(0.015)(25.0)(11.8)^{2/3}}{(15.6)^{5/3}}\right]^2 = 3.98 \times 10^{-4}$$

$S_o - S_{f2} = 0.001 - 0.000398 = 0.000602 = 6.02 \times 10^{-4}$

$$(S_o - S_f)_{mean} = \frac{(S_o - S_{f1}) + (S_o - S_{f2})}{2} = \frac{(6.26\times10^{-4}) + (6.02\times10^{-4})}{2} =$$

$$= 6.14 \times 10^{-4}$$

$$(1-Fr^2)_{mean} = \frac{(1-Fr_1^2)+(1-Fr_2^2)}{2} = \frac{0.9378 + 0.9329}{2} = 0.9354$$

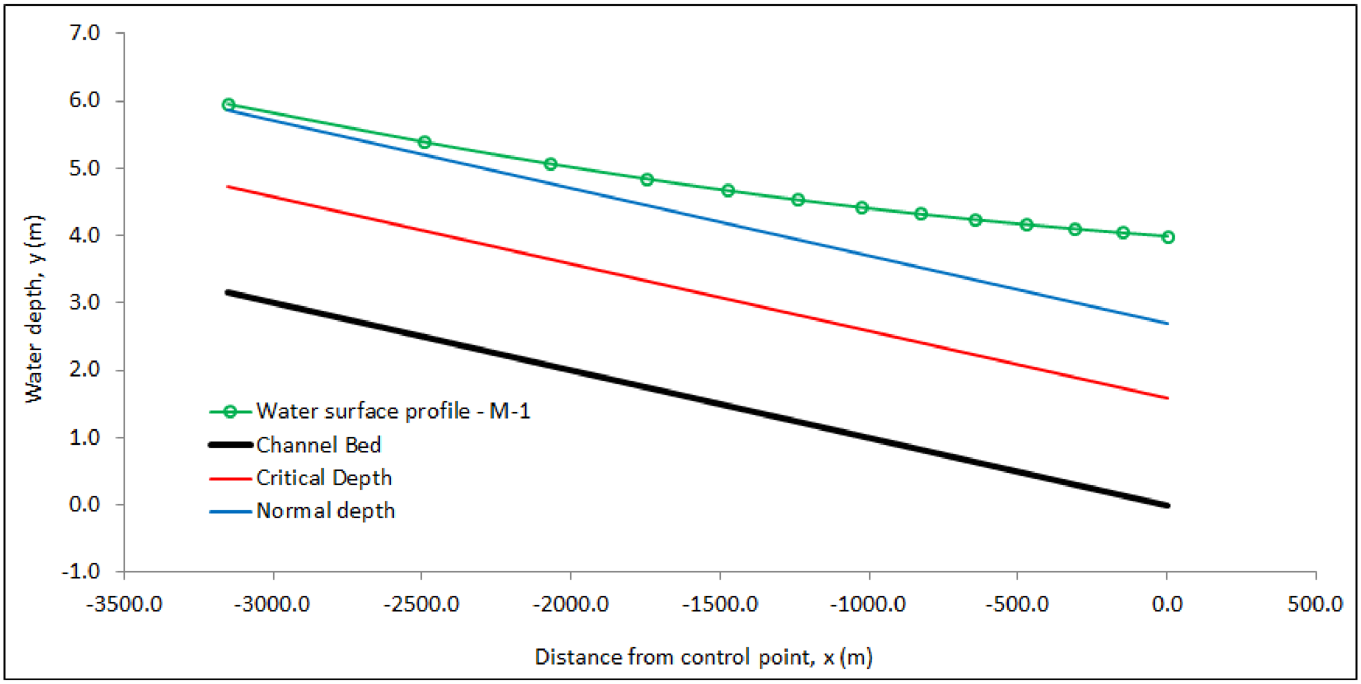
$$\Delta X_1 = \Delta y \left(\frac{1-Fr^2}{S_0 - S_f}\right)_{mean} = (-0.1) \times \left(\frac{0.9354}{0.000614}\right) = -152.4 \text{ m}$$

Continue upstream until $y = y_n = 2.70$ m

Delineating water surface profile - M-1 curve

y (m)	A (m^2)	P (m)	Fr^2 (-)	$1-Fr^2$ (-)	$1-Fr^2$ (mean)	S_f (m/m)	S_0-S_f (m/m)	S_0-S_f (mean)	x (m)	Bed (m)	y_c (m)	y_n (m)	y (vs bed) (m)
4.000	16.00	12.00	0.0622	0.938		3.74E-04	6.26E-04		0.00	0.00	1.59	2.70	4.00
3.900	15.60	11.80	0.0671	0.933	0.935	3.98E-04	6.02E-04	6.14E-04	-152.40	0.15	1.74	2.85	4.05
3.800	15.20	11.60	0.0726	0.927	0.930	4.24E-04	5.76E-04	5.89E-04	-310.42	0.31	1.90	3.01	4.11
3.700	14.80	11.40	0.0786	0.921	0.924	4.53E-04	5.47E-04	5.61E-04	-475.17	0.48	2.07	3.18	4.18
3.600	14.40	11.20	0.0853	0.915	0.918	4.85E-04	5.15E-04	5.31E-04	-648.12	0.65	2.24	3.35	4.25
3.500	14.00	11.00	0.0929	0.907	0.911	5.20E-04	4.80E-04	4.97E-04	-831.26	0.83	2.42	3.53	4.33
3.400	13.60	10.80	0.1013	0.899	0.903	5.59E-04	4.41E-04	4.60E-04	-1027.40	1.03	2.62	3.73	4.43
3.300	13.20	10.60	0.1108	0.889	0.894	6.02E-04	3.98E-04	4.19E-04	-1240.63	1.24	2.83	3.94	4.54
3.200	12.80	10.40	0.1215	0.878	0.884	6.51E-04	3.49E-04	3.73E-04	-1477.31	1.48	3.07	4.18	4.68
3.100	12.40	10.20	0.1337	0.866	0.872	7.05E-04	2.95E-04	3.22E-04	-1748.09	1.75	3.34	4.45	4.85
3.000	12.00	10.00	0.1475	0.853	0.859	7.66E-04	2.34E-04	2.65E-04	-2072.84	2.07	3.66	4.77	5.07
2.900	11.60	9.80	0.1633	0.837	0.845	8.35E-04	1.65E-04	2.00E-04	-2495.65	2.50	4.09	5.20	5.40
2.800	11.20	9.60	0.1814	0.819	0.828	9.13E-04	8.72E-05	1.26E-04	-3151.03	3.15	4.74	5.85	5.95
2.700	10.80	9.40	0.2023	0.798	0.808	1.00E-03	-1.89E-06	4.27E-05	-5045.15	5.05	6.64	7.75	7.75

Q = 25.00 m^3/s
S_0 = 0.001 m/m
n = 0.015
b = 4.00 m
y_c = 1.59 m
y_n = 2.70 m



e) $y = y_n + 0.3 = 2.7 + 0.3 = 3.0$ m

See water surface profile table in d. Distance from control point (dam) at which $y = 3.0$ m is

$X = 2072.84$ m from dam !