CVG3116 Assignment 5 - Solutions

Q1.

$b = B = 5.0\,m$ (rectangular)

$Q = 8\,m^3/s$

$y_n = 1.25\,m$

$y_c = ?$

$\Delta z = ?$ (no choking)

$\Delta z = ?$ (hydraulic jump)

Solution:

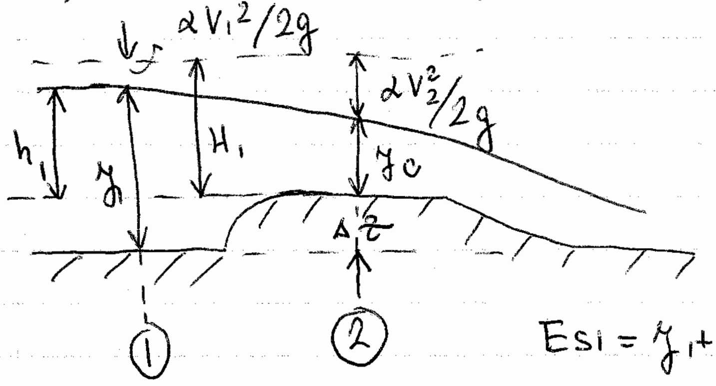
a) $y_c = \sqrt[3]{\frac{q^2}{g}}$ - for rectangular channel

$$q = \frac{Q}{b} = \frac{8.0}{5.0} = 1.6\ m^3/s/m$$

$$y_c = \sqrt[3]{\frac{(1.6)^2}{9.81}} = 0.639\,m$$

$y_c = 0.639\,m < y_n = 1.25\,m \Rightarrow$ Mild channel!

b) For critical flow over the weir:



$E_{S1} = E_{SC_2} + \Delta z$ (No choking!)

$$E_{SC_2} = \frac{3}{2} y_c = \left(\frac{3}{2}\right)(0.639) = 0.96\,m$$

$$E_{S1} = y_1 + \alpha \frac{q^2}{2g y_1^2} = 1.25 + (1.0) \times \frac{(1.6)^2}{(2 \times 9.81)(1.25^2)} = 1.33\,m$$

$1.33 = 0.96 + \Delta z$

$\Delta z = 0.37\,m$ for critical flow without choking!

c) $E_{S1} = E_{SC_2} + \Delta z$

$$y_1 + \frac{V_1^2}{2g} = \frac{3}{2} y_c + \Delta z \quad (\text{assuming } \alpha = 1.0)$$

$$y_1 + \frac{V_1^2}{2g} - \Delta z = H_1 = \frac{3}{2} y_c$$

$$y_c = \frac{2}{3} H_1$$

but $Q = VA = \sqrt{g y_c}\,(b y_c) = \sqrt{g}\, b\, y_c^{3/2} = \sqrt{g}\, b \left(\frac{2}{3} H_1\right)^{3/2}$

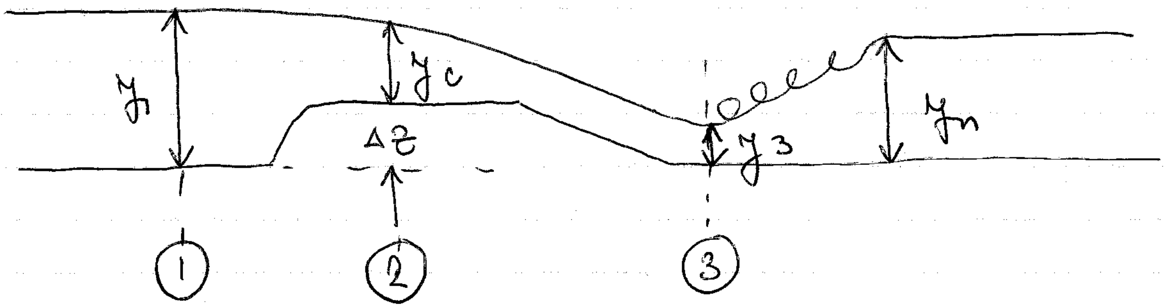
Replacing H with h and introducing a coefficient C_v to account for the velocity head in ①:

$$Q = C_v \sqrt{g}\, b \left(\frac{2}{3} h_1\right)^{3/2}$$

If we consider energy losses by introducing a coefficient, C_d :

$$Q = C_d C_v \sqrt{g}\, b \left(\frac{2}{3} h\right)^{3/2}$$

d) For $y_n = 1.25\,m$



$y_3 = \frac{y_n}{2}\left(\sqrt{1 + 8 Fr_n^2} - 1\right)$ where y_3 is the required depth at ③ to sustain a hydraulic jump with a sequent depth of $y_n = 1.25\,m$.

$$Fr_n^2 = \frac{q^2}{g y_n^3} = \frac{(1.6)^2}{(9.81)(1.25)^3} = 0.134 \rightarrow Fr_n = 0.366 < 1.0$$

The flow at normal depth ($y_n = 1.25\,m$) is subcritical.

$$y_3 = \frac{1.25}{2}\left(\sqrt{1+8(0.134)} - 1\right) = 0.27\,m$$

$$E_{S_3} = y_3 + \frac{V_3^2}{2g} = y_3 + \frac{q^2}{2g\,y_3^2} = 0.27 + \frac{(1.6)^2}{(2\times 9.81)(0.27)^2} = 2.06\,m$$

$E_{S3} = E_{SC_2} + \Delta z$ where $E_{SC_2} = 0.96\,m$ (from b)

$$2.06 = 0.96 + \Delta z$$

$$\underline{\Delta z = 1.10\,m}$$

e) The upstream depth can be calculated by equating E_{S1} to E_{S3} neglecting losses:

$$y_1 + \frac{q^2}{2g\,y_1^2} = 2.06$$

$$y_1 + \frac{(1.6)^2}{(2\times 9.81)\,y_1^2} = 2.06$$

Solve for y_1 by trial & error:

$$\underline{y_1 = 2.03\,m}$$

Q2.

$Q = 10\ m^3/s$

$b = B = 10.5\,m$ (rectangular spillway)

$V_2 = 9.1\ m/s$

$y_2' = ?$

$L = ?$

$\Delta E = ?$

$E_{s_2} / E_{s_2}' = ?$ $(\alpha = 1.0)$

Solution:

a) $V_2 = Q/A_2$ $\quad A_2 = b y_2$

$$y_2 = \frac{Q}{b V_2} = \frac{10.0}{(10.5)(9.1)} = 0.10\ m$$

$$Fr_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{9.1}{\sqrt{(9.81)(0.10)}} = 9.19$$

For $Fr_2 = 9.19 > 4.5$ and $V_2 = 9.1\ m/s < 20\ m/s$, Select USBR stilling basin Type III.

$$y_2' = \left(\frac{y_2}{2}\right)\left(\sqrt{1+8Fr_2^2} - 1\right) = \left(\frac{0.10}{2}\right)\left(\sqrt{1+8(9.19)^2} - 1\right) = \underline{1.25\,m}$$

b) $L_{type\ III} = 2.7 y_3$ $\quad$ where $y_3 = y_2'$

$$L = (2.7)(1.25) = \underline{3.38\,m}$$

c) $$\Delta E = \frac{(y_2' - y_2)^3}{4 y_2 y_2'} = \frac{(1.25 - 0.10)^3}{(4)(0.10)(1.25)} = \underline{3.04\,m}$$

d) Before jump: $y_2 = 0.10\ m$, $V_2 = 9.1\ m/s$, $\alpha = 1.0$

$$E_{s_1} = y_2 + \alpha \frac{V_2^2}{2g} = 0.10 + (1.0)\frac{(9.1)^2}{(2)(9.81)} = 4.32\ m$$

After jump: $y_2' = 1.25\,m$, $\alpha = 1.0$

$$V_2' = \frac{Q}{b y_2'} = \frac{10.0}{(10.5)(1.25)} = 0.76\ m/s$$

$$E_{s_2}' = y_2' + \alpha \frac{V_2'^2}{2g} = 1.25 + (1.0)\frac{(0.76)^2}{(2)(9.81)} = 1.28\,m$$

$$\text{Efficiency} = \frac{E_{s_2}'}{E_{s_2}} = \frac{1.28}{4.32} = 0.30 = \underline{30\,\%}$$

Q3.

$S_0 = 0.0001$

$\rho_s = 2650\ kg/m^3$, $D = 0.4\ mm$

$\rho\ (water) = 1000\ kg/m^3$, $\nu = 10^{-6}\ m^2/s$

$y_0 = ?$

$V = ?$

Solution:

a) Calculate the critical shear stress for erosion

$$D_{gr.} = D \sqrt[3]{\frac{g[(\rho_s/\rho_w)-1]}{\nu^2}}$$

$$D_{gr} = (0.0004) \sqrt[3]{\frac{9.81 \times [(2650/1000)-1]}{(10^{-6})^2}} = 10.12$$

For $10 < D_{gr} = 10.12 < 20$, calculate $F_s = 0.04/D_{gr}^{0.1}$

$$F_s = \frac{0.04}{(10.12)^{0.1}} = 0.032$$

$$F_s = \frac{\tau_{cr}}{(\rho_s - \rho)gD}$$

$\tau_{cr} = (0.032)(2650-1000)(9.81)(0.0004) = 0.207\ N/m^2$

Calculate the bed shear stress:

$\tau_0 = \gamma R S_0$ but $R \approx y$ (wide rectangular channel)

set:

$\tau_0 = \gamma y S_0 = \tau_{cr}$ – for critical condition

$(9810)(y_0)(0.0001) = 0.207$

$\underline{y_0 = 0.211\ m}$

b) The velocity is found from Manning's eq.:

$V = \frac{1}{n} R_h^{2/3} S_0^{1/2}$ where $R_h \approx y$

$n = 0.042\, d_{50}^{1/6}$ where $d_{50} = D = 0.0004\ m$

$n = (0.042)(0.0004)^{1/6} = 0.0114$

$V = \frac{1}{0.0114} \times (0.211)^{2/3} \times (0.0001)^{1/2} = 0.311\ m/s$

Q4.

$D = 0.5\,m$ (circular) , $n = 0.013$, $S_o = 3\% = 0.03$, $L = 12\,m$

$HW = 1.5\,m$, $TW = 1.0\,m$, $C_d = 0.62$

$Q = ?$

Solution:

a) As both the inlet and the outlet are sumberged, we have either outlet control (Case 1-3) or inlet control (Case 1-4). First check outlet control and Case 1-3:

$HW = 1.5\,m > 1.2D = (1.2)(0.5) = 0.6\,m$, OK

Use energy equation to calculate discharge:

$$HW + S_o L = TW + \Big(\underbrace{k_e + 1}_{\Sigma k_L} + \frac{2gn^2}{R^{4/3}} \times L\Big)\frac{V^2}{2g}$$

$$A = \frac{\pi D^2}{4} = \frac{(\pi)(0.5)^2}{4} = 0.196\,m^2$$

$$R = \frac{D}{4} = \frac{0.5}{4} = 0.125\,m \quad \text{(only for \underline{circular} sections!)}$$

$$1.5 + (0.03)(12) = 1.0 + \Big[0.5 + 1.0 + \frac{(2)(9.81)(0.013)^2}{(0.196)^{4/3}} \times (12)\Big] \times \frac{V^2}{(2)(9.81)}$$

$V = 2.81\,m/s$

$Q = VA = (2.81)(0.196) = 0.551\,m^3/s$

Next, assume inlet control (Case 1-4) and use orifice flow formula:

$$Q = C_d A_o \sqrt{2g\left(HW - \frac{D}{2}\right)} \quad \text{where } HW - D/2 = 1.5 + 0.5/2 = 1.25\,m$$

$$Q = (0.62)(0.196)\sqrt{(2)(9.81)(1.25)} = 0.602\,m^3/s$$

The smaller discharge is the design discharge:

$\underline{Q = 0.551\,m^3/s}$ and outlet control.

b) Here the outlet is not submerged. First assume inlet control (Case 1-1) and use orrifice flow f-la:

$Q = C_d A_o \sqrt{2g(HW - D/2)}$

$Q = 0.602\ m^3/s$ (from a)

Next, assume outlet control and case 1-2:

Set $TW = D$ (to be conservative) & use energy eq.:

$$HW + S_0 L = TW + \left(k_L + 1 + \frac{2gn^2}{R^{4/3}} \times L\right)\frac{V^2}{2g}$$

$$1.5 + (0.03)(12) = 0.5 + \left[0.5 + 1 + \frac{(2)(9.81)(0.013)^2}{(0.125)^{4/3}} \times (12)\right]\frac{V^2}{(2)(9.81)}$$

$V = 3.53\ m/s$

$Q = VA = (3.53)(0.196) = 0.693\ m^3/s$

The smaller discharge is the design discharge. Therefore, $Q = 0.602\ m^3/s$ with inlet control.

Lastly, check if the culvert is flowing full by calculating y_n from given nomographs:

Note: $d_o = D = 0.5m$

For $\frac{nQ}{S_0^{1/2} d_0^{8/3}} = \frac{(0.013)(0.602)}{(0.03)^{1/2}(0.5)^{8/3}} = 0.29 \rightarrow \frac{y_n}{D} = 0.76$

$y_n = (0.76)(0.5) = 0.38m < D = 0.5m$, culvert is <u>not</u> flowing full. Case 1-1 is confirmed!