Assignment #5 - Shear Strength of Soils - Solutions

Q1: Solution

(a)

For dry sand, c' = 0, and the failure equation is $\tau_f = \sigma' \cdot \tan\phi'$,

$$\tau_f = \frac{250/1000}{(0.065)^2} = 59.2\, kN/m^2$$

Thus, $\tan\phi' = \frac{\tau_f}{\sigma'} = \frac{59.2}{95} = 0.62$, and $\phi' = \tan^{-1}(0.62) = 31.9°$.

(b)

For σ' = 150 kN/m^2,

$$\tau_f = \sigma' \bullet \tan\phi' = 150 \times \tan(31.9°) = 93\, kN/m^2$$

Therefore, the shear force needed is

$$S = (93)(0.065)^2(1000) = 392.92\, N.$$

(c)

Draw the Mohr circle tangent to the known shear strength envelope at point T, and the center of the circle C is on the horizontal axis (see Figure 1),

$$\overline{ED} = \frac{\overline{TD}}{\tan(45° + \phi'/2)} = \frac{\tau_f}{\tan(45° + 31.9°/2)} = \frac{59.2}{\tan(60.95°)} = 32.9\, kN/m^2,$$

$$\overline{DC} = \frac{\overline{TD}}{\tan(90° - \phi')} = \frac{\tau_f}{\tan(90° - 31.9°)} = \frac{59.2}{\tan(58.1°)} = 36.8\, kN/m^2,$$

$$\sigma_3' = \sigma' - \overline{ED} = 95 - 32.9 = 62.1\, kN/m^2,$$

$$\sigma_1' = \sigma' + \overline{ED} + 2\overline{DC} = 95 + 32.9 + 2 \times 36.8 = 201.5\, kN/m^2, \text{ \underline{or}}$$

$$\begin{aligned}\sigma_1' &= \sigma_3' \tan^2(45° + \phi'/2) + 2c' \tan(45° + \phi'/2) \\ &= 62.1 \times \tan^2(45° + 31.9°/2) + 0 \times \tan(45° + 31.9°/2) \\ &= 201.3\, kN/m^2\end{aligned}$$

(d)

Draw horizontal line TP drawn from the tangent point T, and intersects the Mohr circle at the pole P. Therefore, find the orientation of the major principal plane with the horizontal plane (horizontal direction), which is given by the angle (see Figure 1):

$$\alpha = 180° - [(270° - \phi')/2] = 45° + \phi'/2 = 61°$$

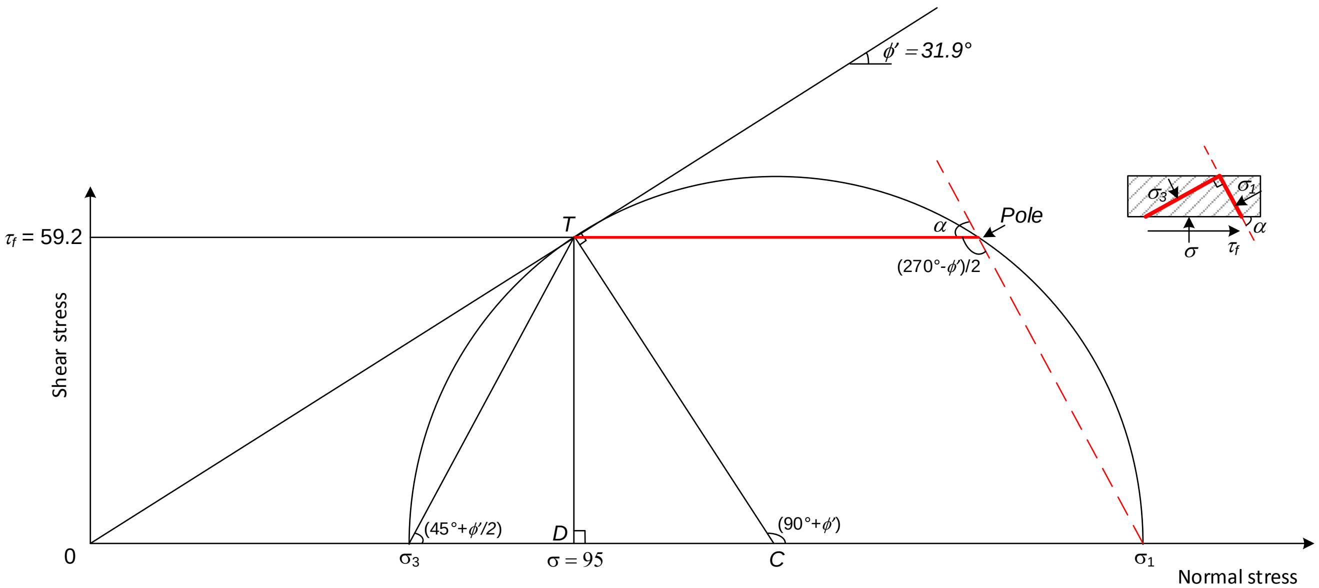

Figure 1

Q2: Solution

The failure envelope can be determined as follows:

Test no.	**(1)** Normal force (N)	**(2)** Normal contact area (mm^2)	**(3)** Normal stress, σ' (kN/m^2)	**(4)** Failure shear force (N)	**(5)** Horizontal displacement (mm)	**(6)** Corrected Area for shearing (mm^2)	**(7)** τ_f (kN/m^2)
1	145	2500	58	157.5	5	2250	70.0
2	230	2500	92	199.9	7	2150	93.0
3	330	2500	132	257.6	9	2050	125.7
4	540	2500	216	363.4	11	1950	186.4

Note:

$$Col(2) = B \times L$$

$$Col(3) = \frac{N}{A} = \frac{N}{B \times L} = \frac{Col(1)}{Col(2)}$$

$$Col(6) = B \times (L - \Delta L) = B \times (L - Col(5))$$

$$Col(7) = \frac{S}{A_c} = \frac{Col(4)}{Col(6)}$$

In order to obtain the Mohr-Coulomb shear strength envelope, $\tau_f = c' + \sigma_f' \cdot \tan\phi'$,

Approach 1

The procedure of Least Squares Best Fit (LSBF) should be used to fitting the straight line with the four known points on the line:

The error at each point, $e_i = \tau_{f,i} - (c' + \sigma'_i \bullet \tan\phi'), \quad i = 1,2,3,4$

To minimize the sum of error square, $\sum_{i=1}^{4}(e_i)^2 = \sum_{i=1}^{4}\left[\tau_{f,i} - (c' + \sigma'_i \bullet \tan\phi')\right]^2$,

Results in $\begin{cases} \dfrac{\partial}{\partial(c')}\left[\sum_{i=1}^{4}(e_i)^2\right] = 0 \\ \dfrac{\partial}{\partial(\tan\phi')}\left[\sum_{i=1}^{4}(e_i)^2\right] = 0 \end{cases}$, thus, $\begin{cases} \tan\phi' = \dfrac{4\sum_{i=1}^{4}(\sigma'_i \bullet \tau_{f,i}) - \left[\sum_{i=1}^{4}(\sigma'_i)\right] \bullet \left[\sum_{i=1}^{4}(\tau_{f,i})\right]}{4\sum_{i=1}^{4}(\sigma'_i)^2 - \left[\sum_{i=1}^{4}(\sigma'_i)\right]^2} = 0.742 \\ c' = \dfrac{\sum_{i=1}^{4}(\tau_{f,i}) - (\tan\phi')\sum_{i=1}^{4}(\sigma'_i)}{4} = 26.4 \end{cases}$,

Therefore, the effective shear strength parameters, c' = 26.40 kPa, and ϕ' = 36.6°.

Approach 2

The average procedure is shown as follows:

Test no.	Normal stress, σ' (kN/m^2)	Shear stress, τ_f (kN/m^2)	Combination of tests	Shear strength parameters	Average values
1	58	70.0			
2	92	93.0	1 & 2	ϕ' = 34.1° c' = 30.7 kPa	ϕ' = 36.4° c' = 26.2 kPa
3	132	125.7	2 & 3	ϕ' = 39.3° c' = 17.7 kPa	
4	216	186.4	3 & 4	ϕ' = 35.9° c' = 30.1 kPa	

Q3: Solution

Using the failure envelope equation $q' = m + p' \bullet \tan\alpha$ for the modified angle of friction

$$\begin{cases} p' = \dfrac{\sigma_1' + \sigma_3'}{2} \\ q' = \dfrac{\sigma_1' - \sigma_3'}{2} \end{cases}$$

Test no.	*p*' (kN/m^2)	*q*' (kN/m^2)
1	199	106
2	345	160

Then using the failure envelope equation, we have

$$\begin{cases} 106 = m + 199 * \tan\alpha \\ 160 = m + 345 * \tan\alpha \end{cases}, \text{ thus } \begin{cases} \alpha = 20.3° \\ m = 32.37\ kN/m^2 \end{cases}.$$

Therefore, the effective shear strength parameters are

$$\phi' = \sin^{-1}(\tan\alpha) = \sin^{-1}(\tan 20.3°) = 21.7°;$$

$$c' = \frac{m}{\cos\alpha} = \frac{32.37}{\cos 20.3°} = 34.51\, kPa.$$

Q4: Solution

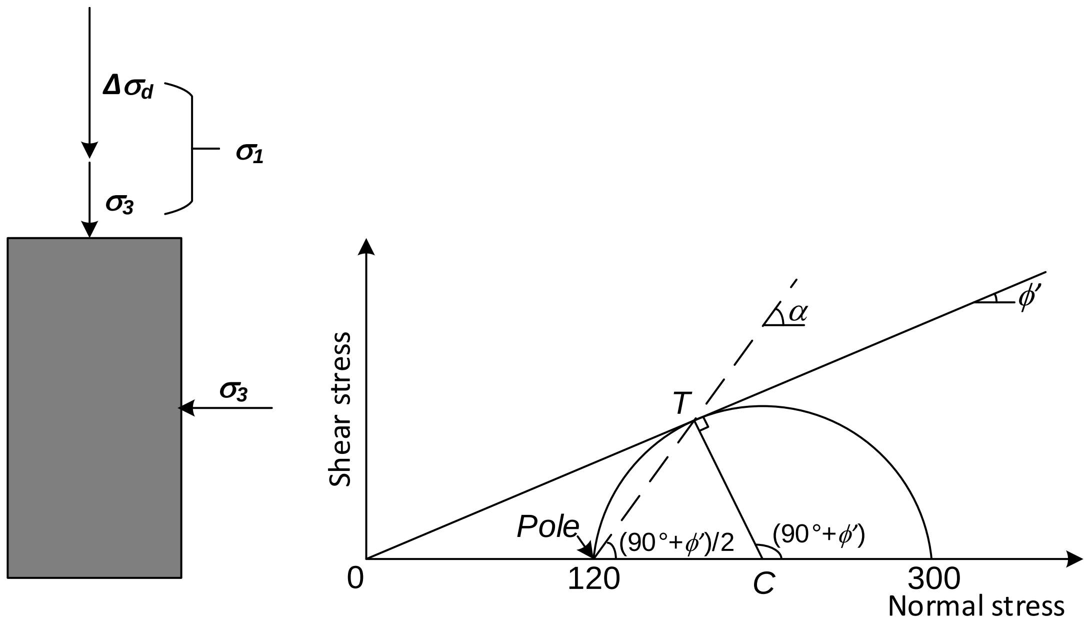

Figure 2

(a)

For a normally consolidated clay, c' = 0.

For the CD test at failure (Figure 2), $\sigma_3' = \sigma_3$ = 120 kPa, $\sigma_1' = \sigma_1 = \sigma_3 + \Delta\sigma_d$ = 120 + 180 = 300 kPa.

$\sin\phi' = \dfrac{CT}{CO} = \dfrac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2} = \dfrac{(300-120)/2}{(300+120)/2} = 0.429$, thus ϕ' = 25.4°.

(b) & (c)

From Figure 2, using the pole method, the failure angle that the failure plane makes with the major principal plane (horizontal plane) is

α = (90° + ϕ') / 2 = 57.7°;

Or the angle that the failure plane makes with the major principal stress (vertical direction) is

90° - α = 90° - 57.7° = 32.3°.

Q5: Solution

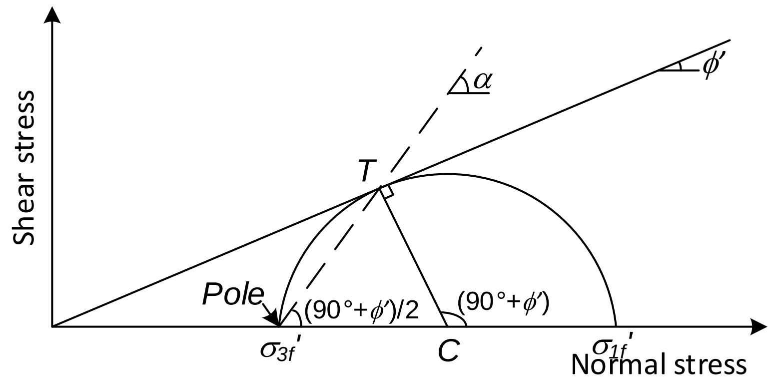

Figure 3

(a) For *c'* = 0, and ϕ' = 25°, the Mohr-Coulomb failure envelope is shown as Figure 3.

Thus, $\sigma_{1f}' = \sigma_{3f}' \tan^2\left(45° + \frac{\phi'}{2}\right) = 100 \times \tan^2\left(45° + \frac{25°}{2}\right) = 246.4\ kPa$,

Therefore, $\Delta\sigma_d = \sigma_{1f}' - \sigma_{3f}' = 246.4 - 100 = 146.4\ kPa$.

(b) From Figure 3, using the pole method, the failure angle that the failure plane makes with the major principal plane (horizontal plane) is
α = 45° + ϕ' / 2 = 45° + 25° / 2 = 57.5°.