CVG 3109 Soil Mechanics (I) (Fall 2017)

Solutions to Midterm Examination

<u>Question 1 (20 marks)</u>

(i) Draw typical compaction curves for the soils (a) GW and (b) CH. Also, provide approximate value of maximum dry density and optimum moisture content for both these soils along with their SI units. Provide justification or reasons for the values that you are suggesting. **<u>(8 marks)</u>**

<u>Solution</u>:

(a) Compaction curves for GW (Well-graded Gravel) and CH (clay with high plasticity)

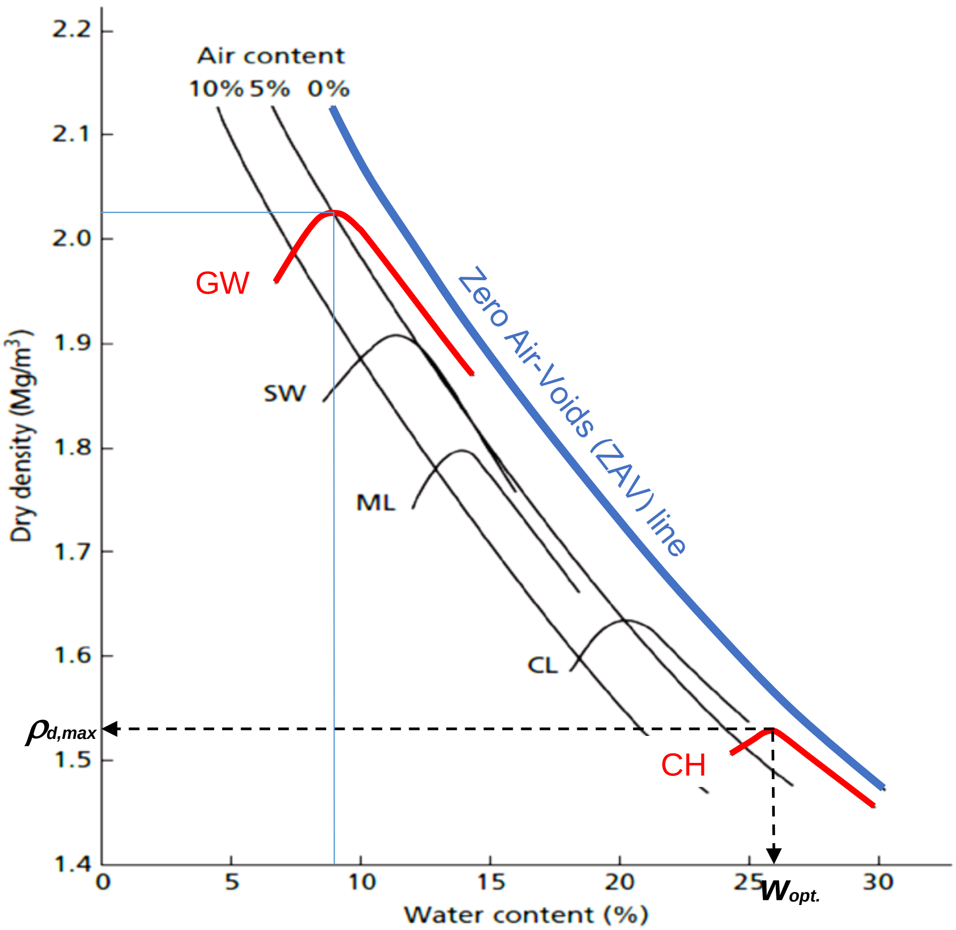


Figure 1. Dry density - water content curves for a range of soil types (modified after Craig's soil mechanics (*Knappett & Craig 2012*)).

(b) Typical values of $\rho_{d,max}$ and $w_{opt.}$ for GW and CH from Standard Compaction Test (i.e., using the same compaction energy effort)

for GW, $\rho_{d,max}$ = (1.8~2.4)×10^3 kg/m^3 , $w_{opt.}$ = (5~15)%;

for CH, $\rho_{d,max}$ = (1.3~1.8)×10^3 kg/m^3 , $w_{opt.}$ = (20~40)%.

(c) Reasons for the suggested values

As the water content is added to the soil particles, larger water films increase around them and lubricate the particles such that they are easier to move and reorient into a denser configuration under the influence of the compaction energy. However, eventually at a particular water content, the soil density does not increase any further. At this point, water starts to replace soil particles in the mold, and since the density of water is less than that of soil particles, the dry density curve starts to fall off.

For coarse-grained soils, the compaction curve is influenced by grain characteristics and grain size distribution. For fine-grained soils, the compaction curve is influenced not only by grain characteristics and grain size distribution, but also by the physical and chemical properties of the soil. In general, CH can adsorb much more water than GW due to their extremely larger specific surface area of clay minerals and can attract more water molecules (e.g., adsorbed water, double-layer water, etc.).

Typically, the void ratio, $e = \frac{Vv}{Vs} = \begin{cases} 0.3 \sim 0.6, & for\ gravel; \\ 0.6 \sim 1.4, & for\ clay. \end{cases}$

And the specific gravity, $Gs = \frac{\rho_s}{\rho_w} = \begin{cases} 2.60 \sim 2.70, & for\ gravel; \\ 2.70 \sim 2.80, & for\ clay. \end{cases}$

According to the phase equation $Se = wG_s$, and **assumed that saturation at optimum moisture content (OMC) is 95%**, thus,

For GW,

$$w = \frac{Se}{G_s} = \frac{0.95 \times 0.4}{2.65} = 14\%$$

$$\rho_{d,\max} = \rho_{d@OMC} = \frac{\rho_s}{1+e} = \frac{G_s \rho_w}{1+e} = \frac{2.65 \times 1000}{1+0.4} = 1893\ kg/m^3$$

For CH,

$$w = \frac{Se}{G_s} = \frac{0.95 \times 1.0}{2.75} = 35\%$$

$$\rho_{d,\max} = \rho_{d@OMC} = \frac{\rho_s}{1+e} = \frac{G_s \rho_w}{1+e} = \frac{2.75 \times 1000}{1+1.0} = 1375\ kg/m^3$$

(ii) What is negative pressure that is typically associated with soil capillarity? How can it be taken into account when calculating the effective stress of soil? Does it contribute towards increasing or decreasing the strength of as soil? **(6 marks)**

Solution:

a) Negative pressure associated with soil capillarity

In soils, especially fine-graded soils, water is capable of rising to a considerable elevation above the ground water table and remain there in a state of equilibrium conditions. In unsaturated zone, part of void space is occupied by water and part by air. The pore water pressure is always less than the pore air pressure due to surface tension. Unless the degree of saturation is close to unity, the pore air will form continuous channels through the soil and the pore water will be concentrated in the regions around the interparticle contacts (Figure 2). The boundaries between pore water and pore air will be in the form of menisci who radii will depend on the size of the pore space within the soil.

b) Calculate the effective stress of soils taking the negative pressure into consideration

For an unsaturated soil, any wavy plane through the soil will pass part through water, and part through air (Figure 3). Then, across a given section of gross area *A*, total force is given by the equation

$$\sigma A = \sigma' A + u_w \chi A + u_a (1-\chi) A$$

where χ is interpreted as the average proportion of any cross-section which passes through water. This leads to the well-known effective stress equation for unsaturated soils proposed by Bishop (1959):

$$\sigma' = \sigma - u_a + \chi (u_a - u_w)$$

where u_w is negative pore water pressure (with negative value in this equaiton), u_a is pore air pressure, ($\sigma - u_a$) is net stress, and ($u_a - u_w$) is soil suction. For a fully saturated soil (S=1), χ=1 (i.e., the above equation thus reduces to **Terzaghi's effective stress equation** for saturated soils); and for a completely dry soil (S=0), χ=0; and for a partially saturated soil (1 > S > 0), 1 > χ > 0. The value of χ is also influenced, to a lesser extent, by the soil structure and the way the particular degree of saturation was brought about.

c) Contribution of negative pressure towards the strength of a soil

Negative pore water pressure will increase the strength of a soil. Because from the pore level point of view, the pore water (with negative pressure relative to pore air pressure) can exert pull forces on the particle surface. Thus, the soil with negative pore water pressure can provide extra bonding forces and therefore greater shear resistance when it is subjected to external loading.

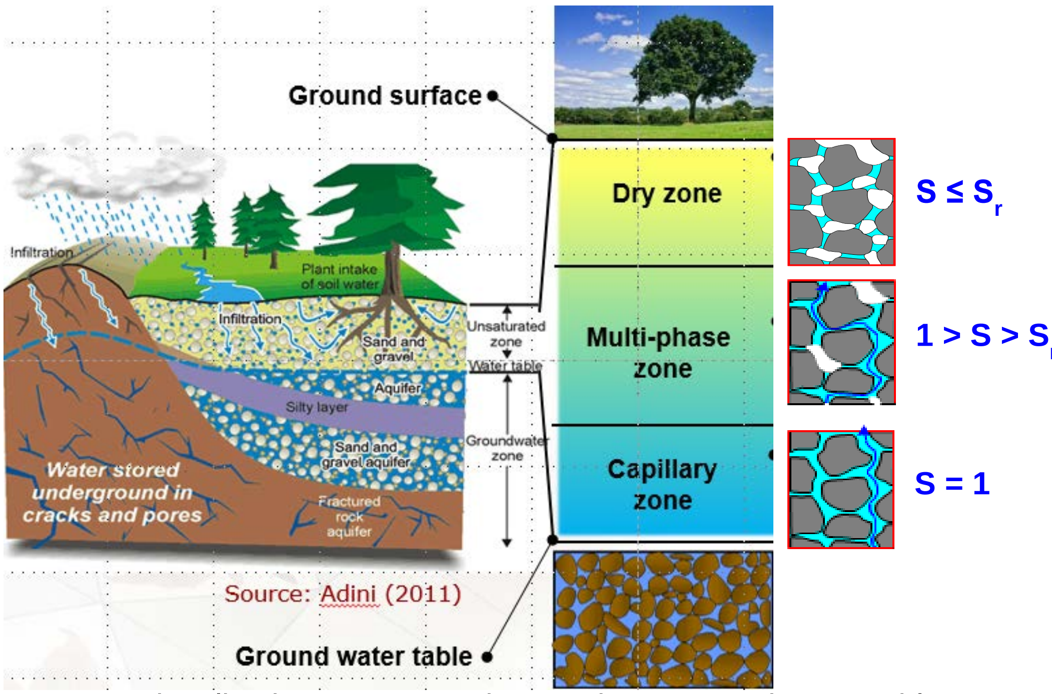


Figure 2. A visualization unsaturated zone above ground water table.

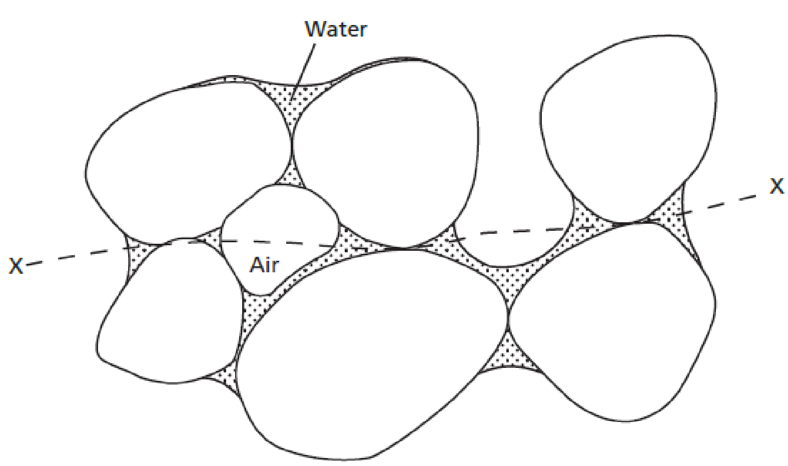


Figure 3. Partially saturated soil (Craig's soil mechanics (*Knappett & Craig 2012*)).

(iii) Which one of the following sandy soils; **Sand A** or **Sand B** will have a higher saturated coefficient of permeability. Give reasons. **<u>(6 marks)</u>**

<u>Solution</u>:

Sand A.

The saturated coefficient of permeability depends primarily on the average size of the pores, which in turn is related to the distribution of particle sizes, particle shape, and soil structure. As grain size distribution (GSD) becomes increasingly well graded, the dry density of the soil will increase. The gap-graded Sand A with could generally have a looser configuration than the well-graded Sand B. Therefore, Sand A will have a greater saturated coefficient of permeability.

Moreover, for sands, Hazen (1892) showed that the approximate value of *k* is given by

$k = 10^{-2} D_{10}^{2}$ (m/s)

where D_{10} is grain size corresponding to 10% by weight passing, also referred to as the effective size (mm). From the grain size distribution cures for Sand A and Sand B (Figure 4), the D_{10} for Sand A is not smaller than the for Sand B. Therefore, Sand A could have a higher saturated coefficient of permeability.

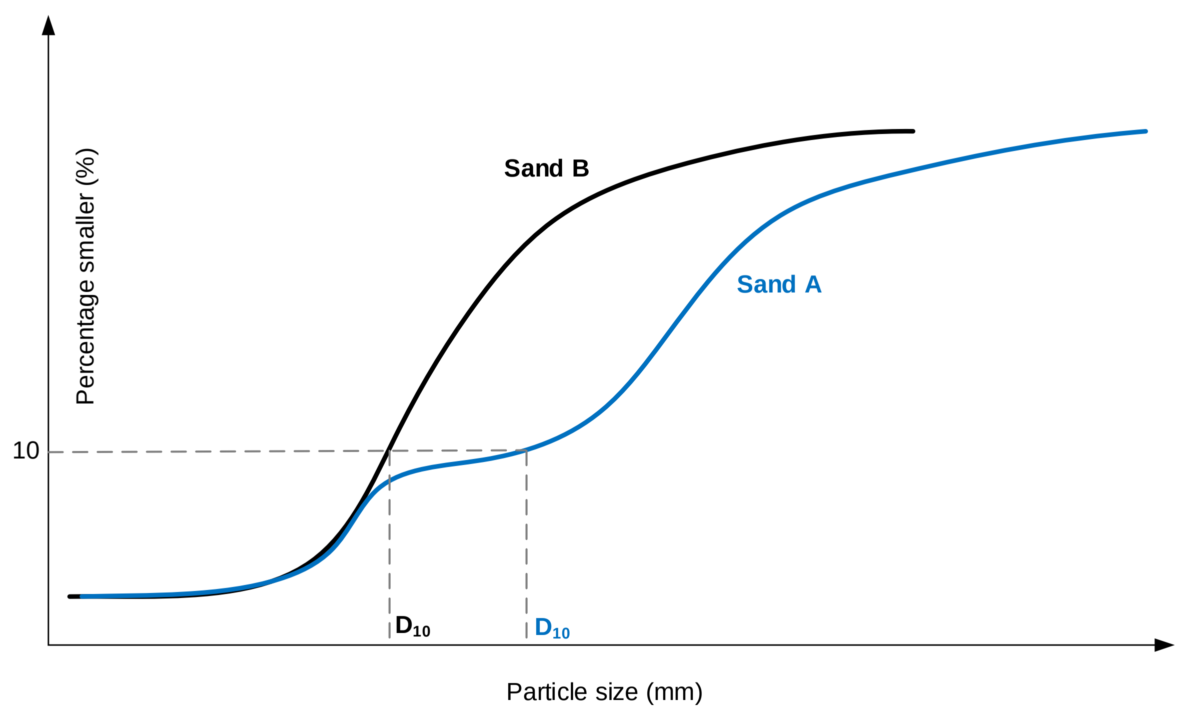


Figure 4. Grain size distribution curves for Sand A and Sand B.

Question 2 (20 Marks)

A proposed embankment fill requires **9000 m^3** of compacted soil. The void ratio of the compacted soil is specified as **0.72**. Four borrow pits are available as described in the following table, which lists the respective void ratios of the soils and the cost per cubic meter for moving the soil to the proposed construction site. Make the necessary calculations to select the pit from which the soil should be bought to minimize the cost. Assume G_s to be equal to **2.67** for Borrow pit A and **2.7** for Borrow pit B.

Borrow Pit	**Void Ratio**	**Unit Cost (\$/m^3)**
A	0.82	9
B	0.93	7

Solution:

① Finding the required weight of soil for the embankment:

$$\gamma_d = \frac{w_s}{V_t} = \frac{G_s}{1+e}\gamma_w = \frac{2.65}{1+0.72} \times 9.81 = 15.11\ kN/m^3$$

$$\gamma_d = \frac{w_s}{V_t} \Rightarrow w_s = \gamma_d V_t$$

$$\boldsymbol{w_s = 15.11 \times 9000 = 135,990\ kN}$$

② Finding the required volume of soil, $V_{treq.}$ to barrow from pit A or B based on the cost.

Borrow Pit	*e*	γ_d (kN/m^3)	$V_{treq.} = \frac{w_s}{\gamma_d}$ (m^3)	Total cost = unit cost * $V_{treq.}$ (\$)
A	0.82	14.39	9,450.3	85,052.70
B	0.93	13.72	9,911.8	**69,382.65**

Borrow pit **B** should be chosen.

Question 3 (35 Marks)

For a line of sheet piling driven 6.00m into a stratum of soil 8.60m thick (**Figure 1**).

a. Establish the flow nets on **Figure 1** by drawing **flow lines and equipotential lines** (Follow all the rules in drawing flow nets). (**15 Marks**)

b. Determine the total volume of water, **q** flowing under the piling per unit time per unit length in m^3/s. Given that coefficient of permeability of the soil, **$k=1.5\times10^{-5}$ m/s**. (**5 Marks**)

c. Calculate effective stress at **Point A**. (**15 Marks**)

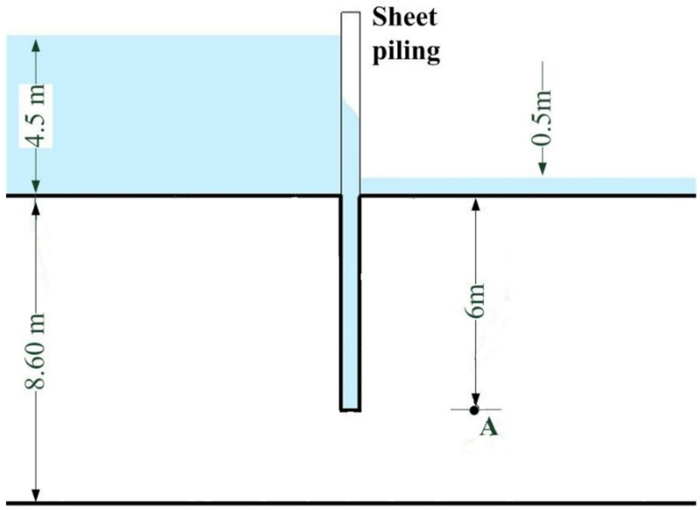
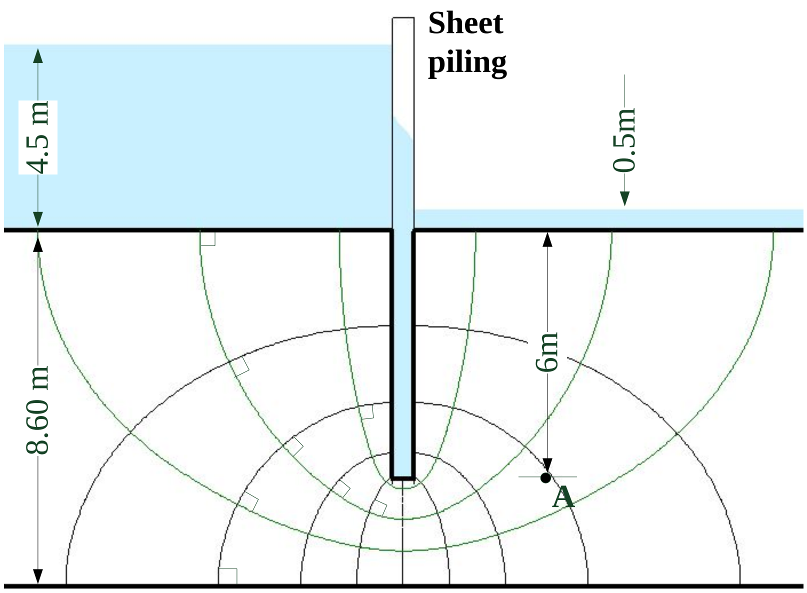


Figure 1

Solution:

(a) Assume **N_f = 4, N_d = 10**



The rules in drawing flow nets:

① The equipotential lines and flow lines are orthogonal to each other;

② The segments are curvilinear squares in which circles can be drawn.

(b) $q = k \times h_w \times \frac{N_f}{N_d} = 1.5\times10^{-5} m^3/s \times (4.5m - 0.5m) \times \frac{4}{10} = 2.4\times10^{-5} m^2/s$

(c) $\sigma' = \sigma - u$

① Calculate the total stress at A

Assume the saturated unit weight of soil γ_{sat} is 20 kN/m^3.

$$\sigma = \gamma_{sat} H_A + \gamma_w (H_{w,A} - H_A) = 20kN/m^3 \times 6m + 9.81kN/m^3 \times (6.5m - 6m) = 124.91kPa$$

② Calculate the pore water pressure at A

$$h_A = h - \frac{\Delta h}{N_d} N_{d,A} = 4.0m - \frac{4.5m - 0.5m}{10} \times 8 = 0.8m$$

$$u = \gamma_w (h_A - z_A) = 9.81 kN/m^3 \times (0.8m + 6m) = 66.71 kPa$$

③ Calculate the effective stress at A

$$\sigma' = \sigma - u = 124.91 kPa - 66.71 kPa = 58.2 kPa$$

Question 4 (25 Marks)

The plan of a flexible rectangular loaded area is shown in **Figure 2**. The uniformly distributed load on flexible area, **q is 100 kN/m^2**.

1. Determine the vertical stress increase, **$\Delta\sigma_z$, at depth of z = 2m below Point C using any ONE of the suitable methods**. Comment on the relative stress values for point A and B at the same depth of z=2m, without any mathematical calculations.

2. Also, will the vertical stress values at A, B and C would increase or decrease at depth of z=5m in comparison to vertical stress, values at z = 2m. AGAIN, NO MATHEMATICAL CALCULATIONS ARE REQUIRED FOR ANSWERING THIS QUESTION. Give reasons for your comments.

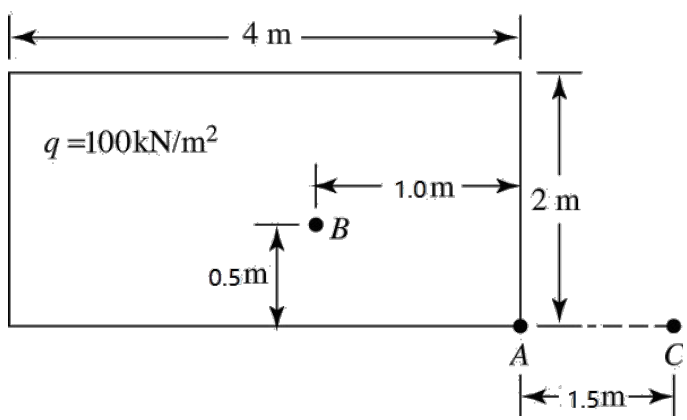
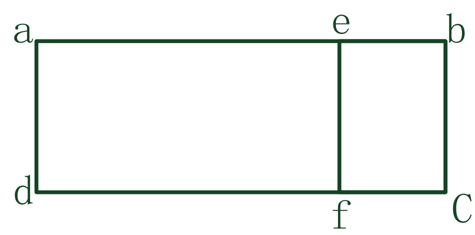


Figure 2

Solution:

(1) Stress increase at depth of z=2m

(a) Stress increase at point C



The stress in the C should be the stress caused by rectangle 5.5m × 2m (abCd) minus the stress caused rectangle 2m × 1.5m (ebCf).

① For rectangle 5.5m ×2m

$$m=\frac{L}{z}=\frac{5.5}{2}=2.75 \quad n=\frac{B}{z}=\frac{2}{2}=1, \quad I_{r1}=0.2030$$

② For rectangle 2m × 1.5m

$$m=\frac{L}{z}=\frac{2}{2}=1 \quad n=\frac{B}{z}=\frac{1.5}{2}=0.75, \quad I_{r2}=0.1547$$

③ $\sigma_z=q\left(I_{r1}-I_{r2}\right)=100\times\left(0.2030-0.1547\right)=4.83kPa$

(b) **Comment on relative stress values at A, B and C**

$\Delta\sigma_{z,B} > \Delta\sigma_{z,A} > \Delta\sigma_{z,C}$

Generally, at the same depth, the stress increase due to a rectangular uniform loads is highest at the point below the center of the loaded area. The stress decreases with the distance away from the center. The father the point goes away from the center, the less the stress increase is.

(2) Comment on vertical stress values at depth of z=5m

The vertical stress increase at depth of z=5m is less than that at depth of z=2m.

m and n coefficients, m=L/z and n=B/z. With increasing z, the values of m and n will decrease and consequently the value of I_{qr} will decrease. Therefore, the values of vertical stress increase due to loading will decrease with increasing depth.

In terms of the stress dispersion, stress increase due to loading will spread on a wider area at a greater depth. Both the trapezoidal rule and isobar diagram are good examples of this theory. According to the trapezoidal rule, $\Delta\sigma_z = \frac{qBL}{(B+z)(L+z)}$. This means with increasing depth, the vertical stress increase will decrease. According to the isobar diagram, it can be found that the value vertical stress increase due to loading decrease along depth and the area of soil mass affected by the loading on ground surface increase along depth.

For a system of forces in equilibrium applied to some segment of a solid body produces stresses in that body that rapidly diminish with increasing distance from the segment. Thus, in this question, the point of interest is farther away from the loaded area, the stress increase due to the loading is smaller.