CVG3140 – Theory of Structures 1

Structural Analysis Course Notes

Theory notes

Fall 2017

References

1. Hibbeler, R.C., 2015. Structural analysis. 9th édition. Pearson/Prentice-Hall.
2. McCormac, J.C., 2006. Structural analysis: Using classical and matrix methods. 4th Edition. Wiley.
3. Hibbeler, R.C., 2014. Mechanics of materials. 9th Edition. Pearson/Prentice-Hall
4. Philpot, T.A., 2012. Mechanics of materials: An integrated learning system. 3rd Edition. Wiley.
5. Gere, J.M. et Goodno, B.J., 2009. Mechanics of materials. 7th Edition. Cengage Learning.

Lecture 1.1: Introduction to Structural Engineering

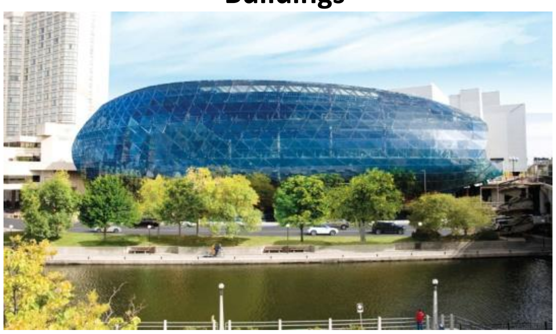
Lecture outline:

1. Background - structures and structural engineering
2. Structural engineering definition
3. Structural engineering design process
4. Building materials
5. Material and sectional properties for this course
6. Structural components and systems

1. Background - structures and structural engineering

Structure: A general term referring to a system of connected parts that support a load. In civil engineering the primary types of structures of concern are:

Buildings



Ottawa Convention Center

Burj Khlaifa

Warehouse

Bridges

Confederation Bridge

Golden Gate Bridge

Overpass

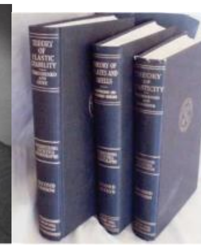
Early civil engineering structures were designed **empirically** and by using rules of thumb based on past experience.

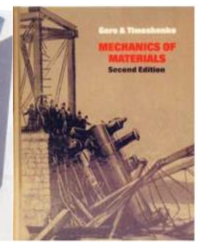
Ancient Egyptian structures

Ancient Roman structures

Ancient Greek structures

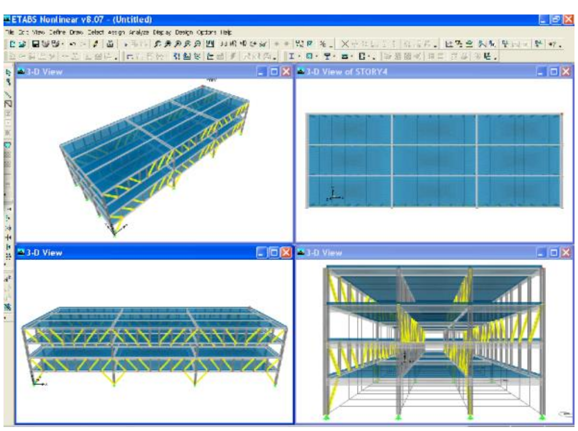
At the 17th century, engineers began applying their knowledge of **mathematics and mechanics** when designing structures → <u>structural engineering as a **science**</u>





These analytical accomplishments were accompanied by

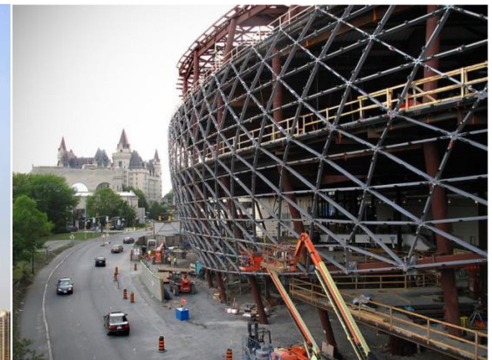
- **Innovations** in **building materials**
- **Innovations** in methods of **analysis**
- **Innovations** in methods of **construction**



These innovations led to modern structural engineering and design.

Burj Khlaifa, UAE, 800 m

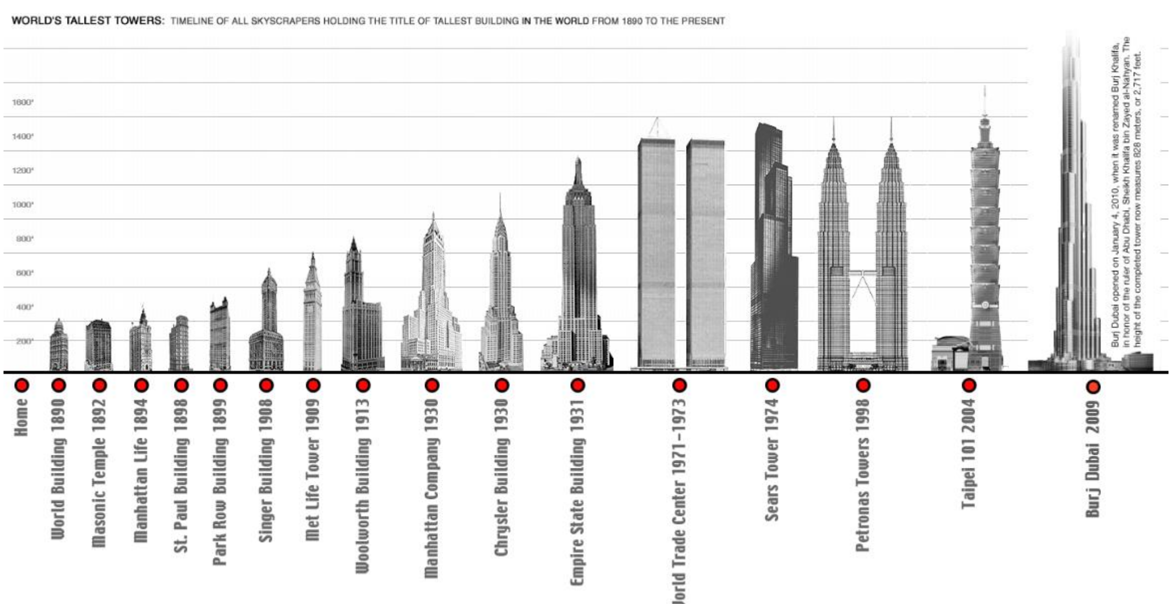
During construction ...



Ottawa convention center - Canada

Donghai Bridge, China, 32.5 km

Confederation Bridge - Canada



(source: http://www.skyscraper.org/TALLEST_TOWERS/tallest.htm)

2. Structural Engineering - Definition

Definition:

*The art of using **materials** …* **[that have properties that can only be approximated]**

*….to build **real structures** ...* **[that can only be analyzed approximately]**

*… to resist **forces** …* **[that are not precisely known]**

*… such that we satisfy our responsibilities regarding the **safety** of the public.*

Safety then (Code of Hammurabi)

A. If a builder build a house for a man and do not make its construction firm and the house which he has built collapse and cause the death of the owner of the house – that builder shall be put to death

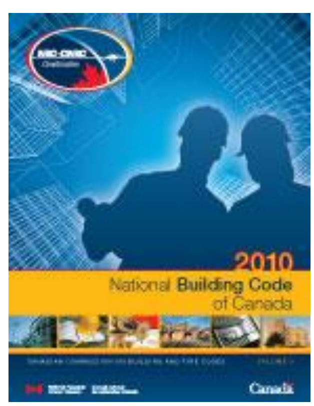
B If it cause the death of the son of the owner of the house – they shall put to death a son of that builder.

D. If it destroy property, he shall restore whatever it destroyed, and because he did not make the house which he built firm and it collapsed, he shall rebuild the house which collapsed at his own expense.

Translated by R.F. Harper

"Code of Hammurabi" p. 83 – seq.

Safety now .... In Canada the **National Building Code of Canada (NBC)** gives guidance on how to design safe buildings.



3. Structural engineering design process

The design process for a structure can be generalized into a number of key steps. Depending on the scope of the project, some of these steps may be more time consuming than others. This process is outlined below.

Phase	Goal	Description
Planning	Specification of various design criteria	• Quantifying the intended use of the building • Determining the occupancy of the building • Development of architectural plans and layout • Defining design criteria requirements: Safety, serviceability, durability etc.

Preliminary Design	Selection of structural solution (Feasibility Study)	• Selection of building material or materials
		• Selection of structural system
		  ○ Gravity load resistant system
		  ○ Lateral load resistant system
		• Determination of general structural layout
		• Approximate structural member sizing
		  ○ 'Back of napkin" calculations
		  ○ Experience from previous projects
		  ○ Simplified computer models

Idealization and Structural Analysis	Determine accurate member forces for design	• Build structural model
		• Determine loads on structure
		  ○ Gravity loads
		  ○ Lateral loads
		• Apply loads
		  ○ Pattern loading
		  ○ Load combinations
		• Analyze the structure
		  ○ Individual member analysis
		  ○ Global or system level analysis

Structural Design	Design of all structural members and connections	• Design members to withstand all loads to required design criteria
		  ○ Determine member size
		  ○ Detailing various structural elements
		  ○ Detailing connections specific to each member
		• Produce structural drawings and specifications

Note that this design process is ***iterative***.

4. Building materials

Structures are typically built with common materials including:

- Reinforced concrete
- Structural steel
- Timber
- Masonry



The selection of the building material needed for each structure is based on a number of factors including:

- Cost
- Effectiveness
- Aesthetics

Each material has its own stress-strain constitutive properties.

The design rules are covered in material-specific CSA standards:

<table>
<tr>
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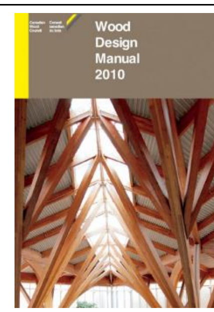
Concrete - CSA A23.3</td>
<td>

Steel - CSA S16</td>
<td>

Timber - CSA SO86</td>
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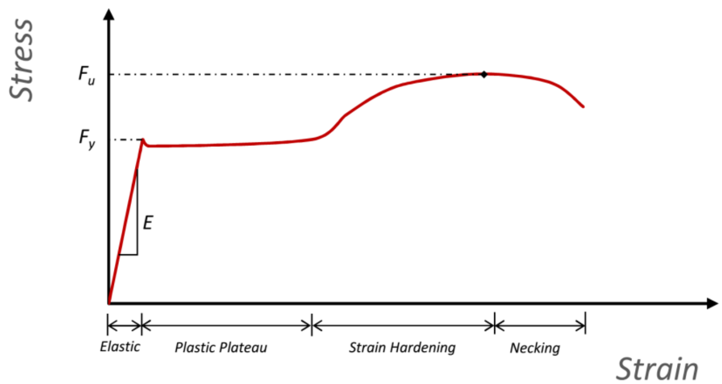
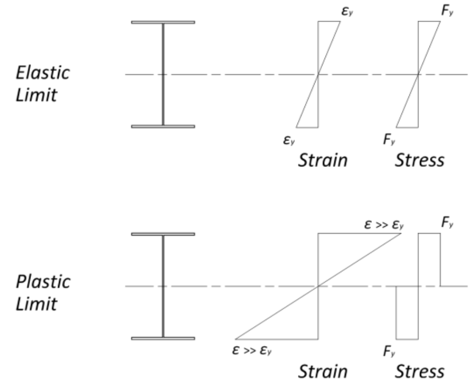
Material specific structural design courses at the University of Ottawa include:

- CVG3147 – Structural Steel Design I
- CVG3148 – Reinforced Concrete Design I
- CVG4143 – Structural Steel Design II
- CVG4145 – Reinforced Concrete Design II
- CVG4146 – Structural Design in Timber

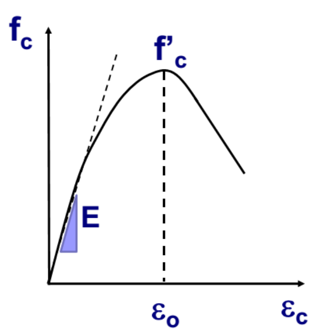
A sample of the actual material constitutive properties used in design for steel and concrete are given below:



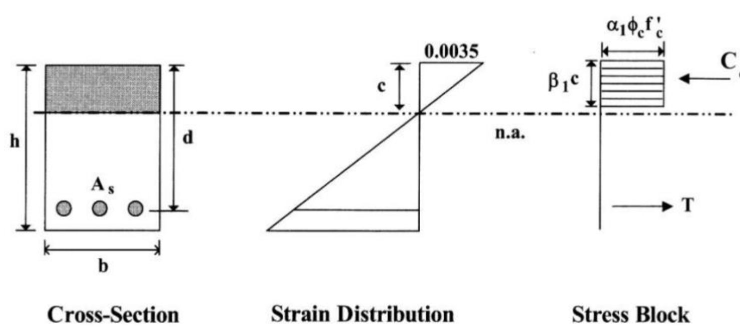
Structural Steel Material Properties



Steel Sectional Response



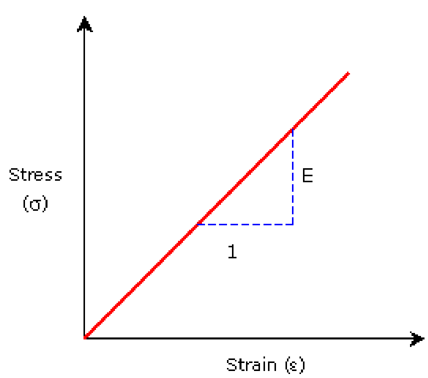
Concrete Material Properties

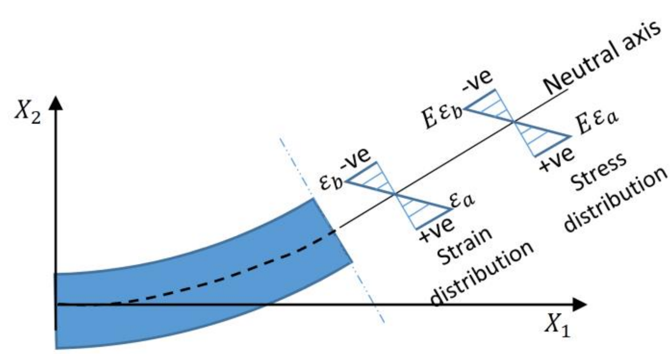


Reinforced Concrete Sectional Response

For this course, the following assumption will be used:

Assumption: All structures will be built with members that are ***linear-elastic.***





Assumed material and sectional response used in this course

4. Material and Sectional Responses for this Course

All materials for this course will have what is termed “elastic sectional properties”. These properties are not affected by stress the stress within the section. They are defined by the following:

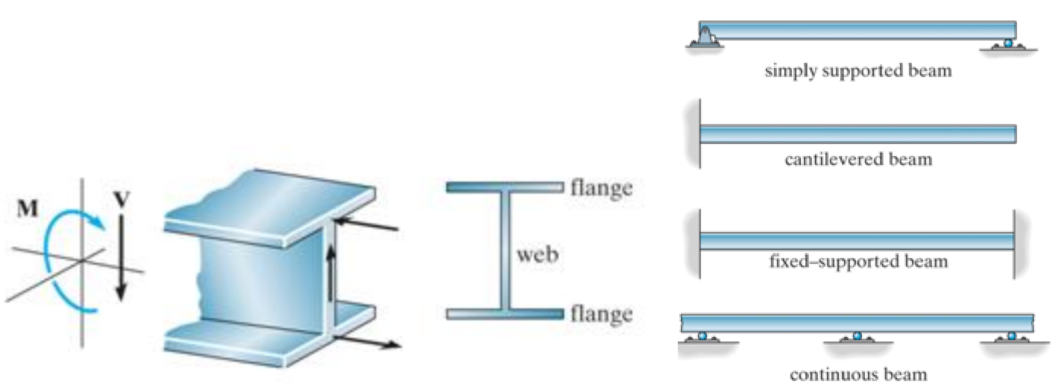
Name	**Symbol**	**Typical Units**	**Description**
Area	*A*	mm^2 (in^2)	The total (gross) cross-sectional area of a member. For example: the area of a chord in a truss
Elastic Modulus	*E*	MPa (ksi)	The ratio of stress and strain for specified material. For example: Steel usually has a modulus of elasticity of 200,000 MPa
Moment of Inertia	*I*	mm^4 (in^4)	A geometrical property of the section related to flexural resistance. Note: 1) By default, members will be considered bent about principle axis for this course 2) Members may be subjected to moments about both strong and weak axis (I_{xx}, *and* I_{yy})
Rigidity	*EI*	N.mm^2 (k.in^2)	The product of the moment of inertial and modulus of elasticity for a section.

5. Structural components and systems

(1) Beams:

Beams are typically straight, horizontal members used to carry vertical loads between supports. There is negligible axial load (neither tension nor compression) on beams. These members respond in flexure (*bending moment, M*) and shear (*V*).

Beams are often classified in the way they are supported.



(Structural Analysis, Hibbeler, 2009)

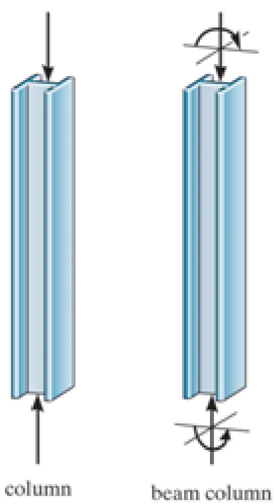
(2) Compression members

A compression member is a structural component that only carries axial load in compression. The most common type of compression members in civil structures are:

- Compression chords of trusses
- Pure columns

(3) Beam-columns

In some cases columns are subjected to bending moments as well as axial loads. In these cases, the members are referred to as ***beam-columns***. The distinction between columns, beams, and beam columns is important for a designer to recognize as the stress state within each member type is considerably different.



(Structural Analysis, Hibbeler, 2009)

The different stress states result in different design considerations including:

- Buckling
- Lateral-torsional buckling
- Local failure of section
- Tension fibre governed failure
- Compression fibre governed failure
- etc.

These different design possible responses to different stress states are handled differently for each material (eg. concrete, steel, timber etc).

(3) Tension members:

Tension members are components loaded under pure tension forces. These members are often pin ended or flexible enough to not carry any moment.

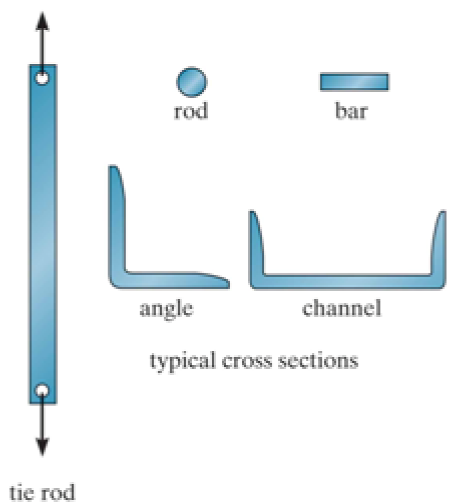
Typical tension members include:

Truss Chords

Tie Rods and Hangers

Lateral Load Braces

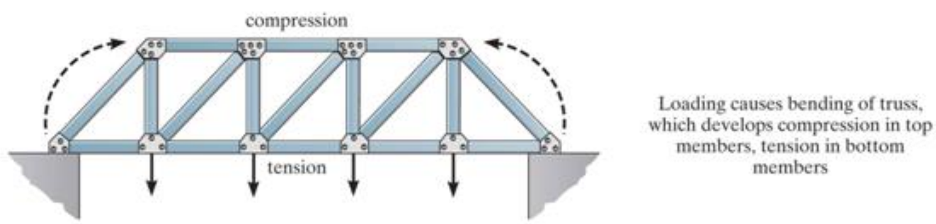
Typical section used in tension members:



(Structural Analysis, Hibbeler, 2009)

(4) Trusses:

Triangulated structure composed of ties (tension) and struts (compression), where all joints are pins. Trusses are only loaded at the joints.



(Structural Analysis, Hibbeler, 2009)

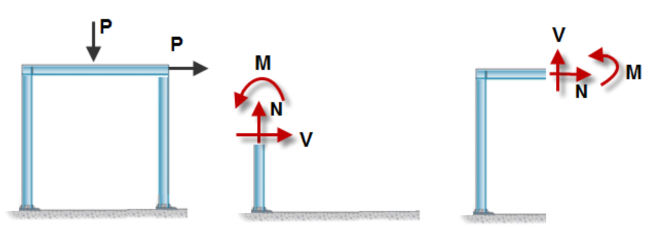
Trusses are commonly used in large, open structures where long spans are desired:


Trusses in arena


Over-Truss steel bridge

(5) Frames:

The combination of beams and columns results in a frame. The straight members are connected either by rigid or pinned joints.

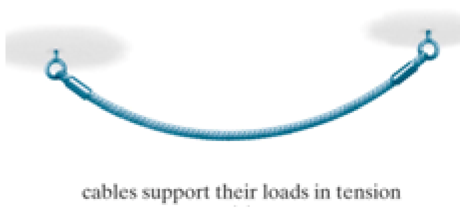


(6) Cables:

Cables are <u>flexible</u> structures that carry load in <u>tension</u>.

Load is not applied along axis (as in the case of tension ties), therefore takes a form.

Commonly used to support long bridge spans (usually spans > 46 m)



Cable component (*Structural Analysis, Hibbeler 2009*)

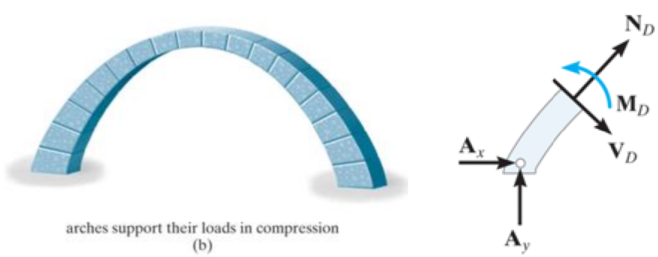
Cable supporting bridge

(7) Arches:

Arches are <u>rigid</u> structures that achieve their strength in <u>compression</u>

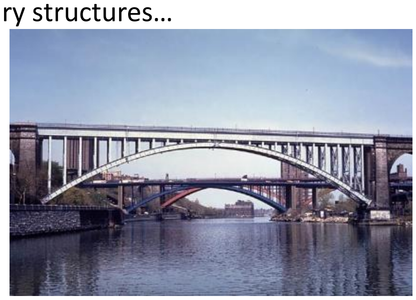
- Secondary shear and moments forces must be considered in design

Often used in bridge structures, dome roofs, openings for masonry structures…



Arch and internal component forces

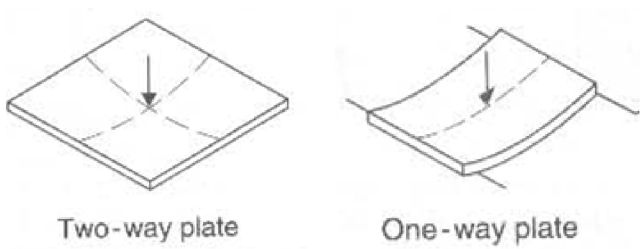
(*Structural Analysis, Hibbeler 2009*)



Bridge arch

(8) Surface structures: Plates and walls

A plate is a thin member (relative to its span) that caries load by one-way or two-way action in bending. In civil structures, the most common plate example is a reinforced concrete slab.



One and two-way plates (*Structures, Schodek, 1998*)

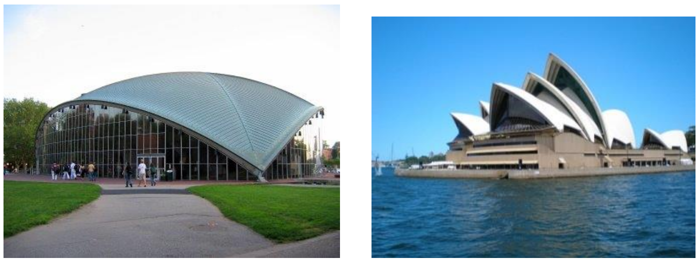
Reinforced concrete slabs

(9) Surface structures: Shells and domes

Shells are structures that have a curved surface and the number of shapes is boundless.

The most common shape is the spherical shell (double curvature) and cylindrical shell (single curvature)

Shells and domes are very efficient structures capable of spanning large spans



Shell buildings

(10) Surface structures: membranes and nets

A membrane is a thin flexible sheet often used in tent structures that support load in mutli-axial tension

Both simple and complex forms can be made

Nets are 3D surface structures made of a series of crossed cables

Membranes and Nets on Buildings

Factors influencing choice of structural system?

Factors in selection of structural form	
Cost	cost of design
	cost of materials
	cost of transportation
	cost of installation
	cost of protection and maintenance
Schedule	shop drawings and approvals
	materials acquisition
	construction productivity
Function	clear span
	depth of members and clear height
	location of bearing walls and other supports
	incorporation of mechanical and electrical systems
	thermal insulation
Code requirements	fire protection
	sound transmission
	structural capacity
Appearance	appearance of materials
	appearance of the finished systems
	architectural blend of form and function
Environmental issues	environmental impact of harvesting, mining, quarrying, and manufacture
	energy required to manufacture, transport, and install materials
	thermal efficiency of building systems
	disposal, recycle, or reuse at end of life cycle

Lecture 1.2: Introduction to Loads on buildings

Lecture outline:

1. Introduction to building design loads
2. National Building Code of Canada
3. Loads in the NBCC
4. Load path in buildings
5. Load transfer concepts
6. Structural idealization

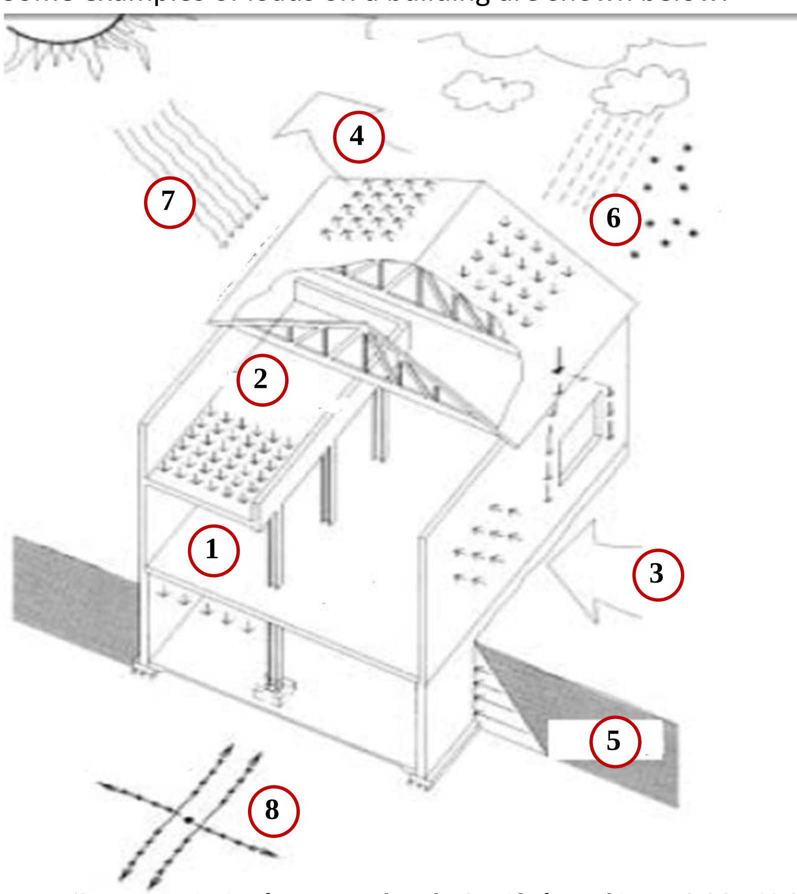
1. Introduction to building design loads

All structures are subjected to forces that are imposed by:

1. Gravity loads
2. Lateral loads

The combination of these loads will determine the maximum design force a member must sustain. A structural engineer will select a member that meets the strength and serviceability criteria for the specific project under the governing loads.

Some examples of loads on a building are shown below:



(Source: Designing for structural steel - A guide for architects, AISC, 2002)

Types of Loads on Structures

1) Dead loads

2) Live loads

3) Wind loads

4) Wind (uplift) loads

5) Soil pressure loads

6) Snow and rain loads

7) Thermal loads

8) Earthquake loads

Other:

9) Impacts

10) Explosions (Blast loads)

2. National Building Code of Canada

In Canada, the National Building Code of Canada (NBCC) is used to determine the nature and magnitude of loads that will act on building structures.

- Published by the Canadian Commission on Building and Fire Codes (CCBFC)
- Written for the purpose of protecting the health and safety of the public
- Represents the consensus opinion of experienced engineers/architects/others
- Current edition: **NBCC 2010 & NBCC 2015**

The NBCC is divided into **9 part**s that specify minimum requirements for buildings within Canada. These parts detail requirements for fire safety, structural design, heating, ventilation, plumbing, construction site safety etc.

- **Part 4** of the NBCC is used to determine the design loads on structures within Canada.

- The **Structural Commentaries** addition to the NBCC contains more guidance on calculation of Snow Loads, Wind Loads, Earthquake Loads, etc.

NBCC 2010 and Structural Commentaries 2010

Note: Part 4 of the NBCC and some excerpts from the structural commentaries are provided on the course website. Students are not required to purchase these documents.

Note: The online seminars give an introduction to loads in Part 4 of the NBCC :

- Seminar 2 - Intro to the NBCC and Limit states design
- Seminar 3A - Dead loads
- Seminar 3B - Live loads
- Seminar 4 - Snow loads
- Seminar 5 - Wind loads (intro)
- Seminar 6 - Earthquake loads (intro)

3. Loads in the National Building Code of Canada

Loads in the National Building Code of Canada (NBCC 2010):

Dead load (D)

Permanent load representing the self-weight of members + superimposed dead loads.

- Covered in Cl. 4.1.4

Live load (L)

Variable use &occupancy loads

- Covered in Cl. 4.1.5

Snow load (S)

Variable environmental load acting mainly on the roof areas of buildings.

- Covered in Cl. 4.1.6 + Commentary G

Wind load (W)

Variable environmental load acting on the wall and roofs of a building.

- Covered in Cl. 4.1.5 + Commentary I

Earthquake load (E)

Variable environmental load that occurs rather rarely (also known as seismic load).

- Covered in Cl. 4.1.5 + Commentary J

How are loads combined ?

The **factored loads** are obtained by considering various "load combinations" defined in the National Building Code of Canada (NBCC , 2010).

Some examples of load combinations we would need to consider…

Permanent load	+	Principal load	+	Companion load
1.4D				
1.25D	+	1.5L	+	(0.5S or 0.4W)
1.25D	+	1.5S	+	(0.5L or 0.4W)
1.25D	+	1.4W	+	(0.5L or 0.5S)
1.0D	+	1.0E	+	(0.5L or 0.5S)

(1.0 in NBCC 2015) [annotation pointing to the 0.5 factors in the 1.5L and 1.5S rows]

4. Load paths in buildings

"Load path" = the way loads flow through the building.

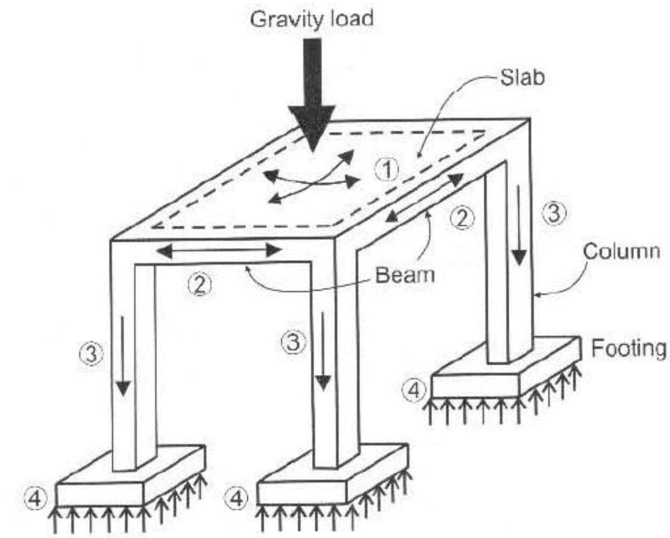
The load path can be identified by considering the elements in the building that contribute to resisting the load and by observing how they transmit load to the next element

2 basic load paths (depending on the type of load to be transferred):

- Gravity load path
- Lateral load path

(1) Gravity load path (dead, live, snow loads)

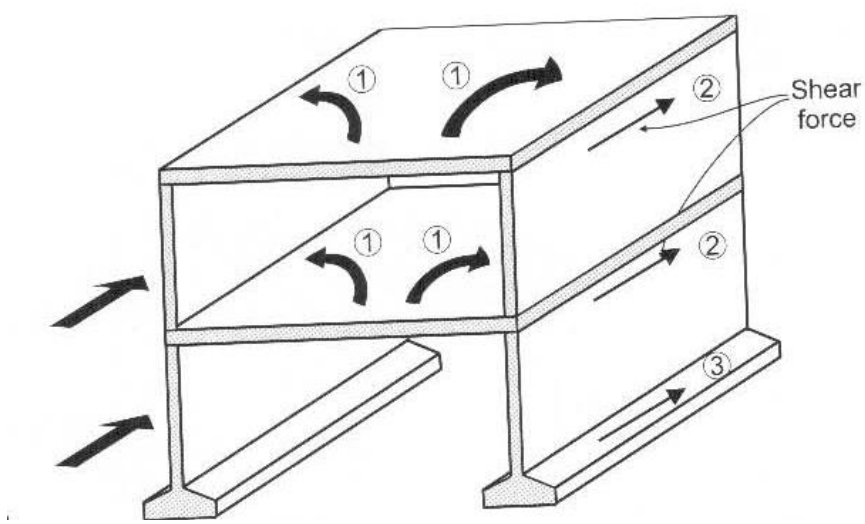
- The way in which gravity loads (dead, live, snow loads) are transferred through the building.



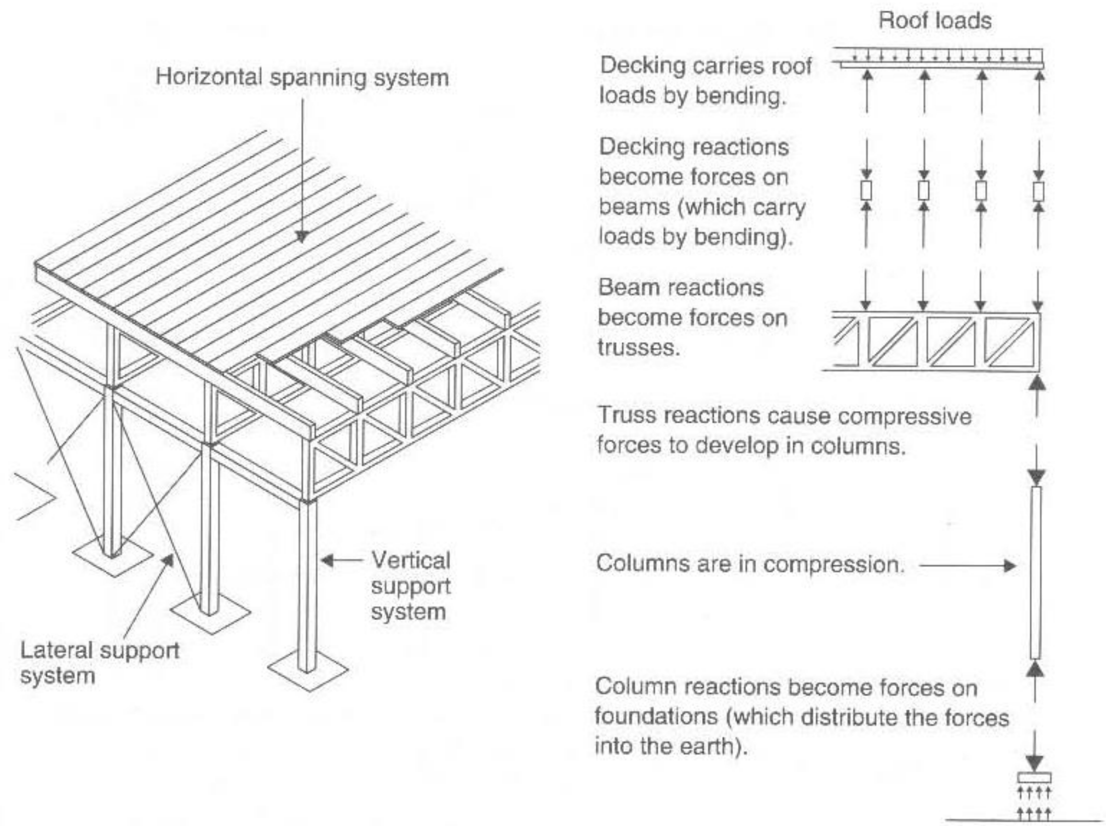
(*Source: Brzev and Pao, Reinforced concrete design, 2006)*

(2) Lateral load path (wind, earthquake)

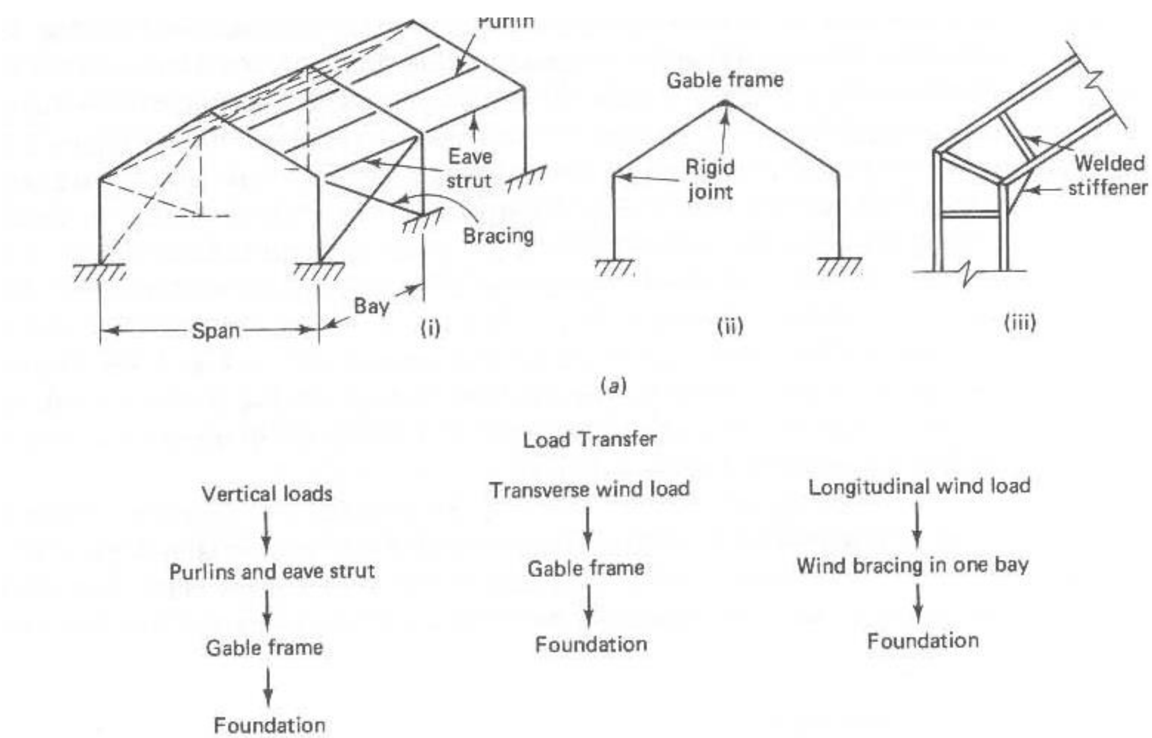
- The way in which lateral loads (wind, earthquake loads) are transferred through the building.



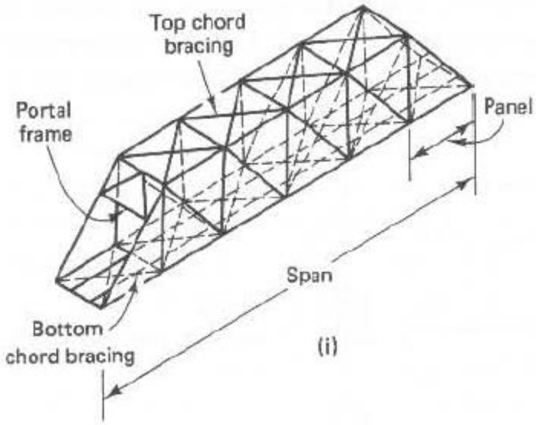
(*Source: Brzev and Pao, Reinforced concrete design, 2006)*

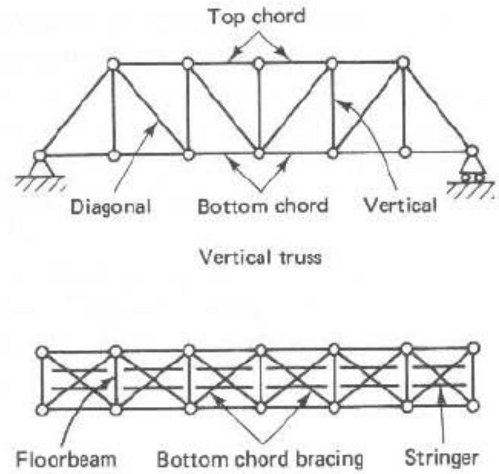


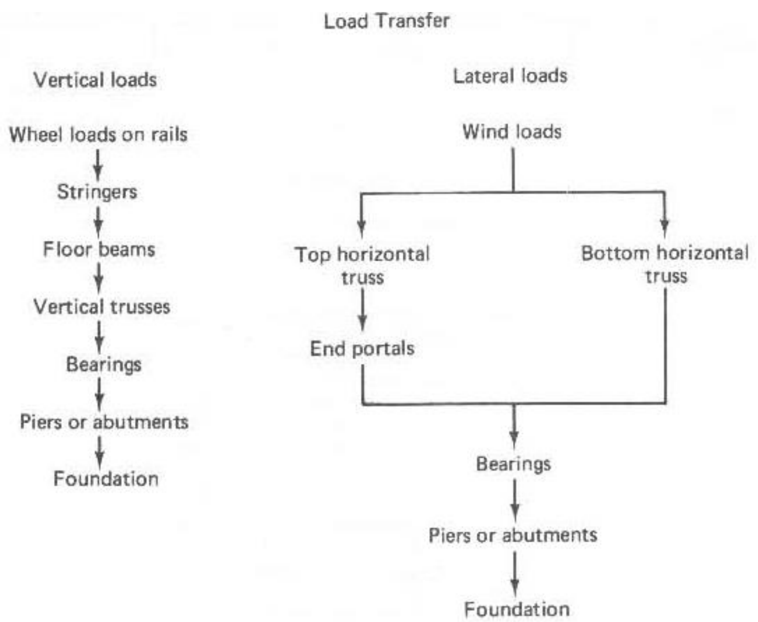
(Source: Schodek, 2015, Structures)

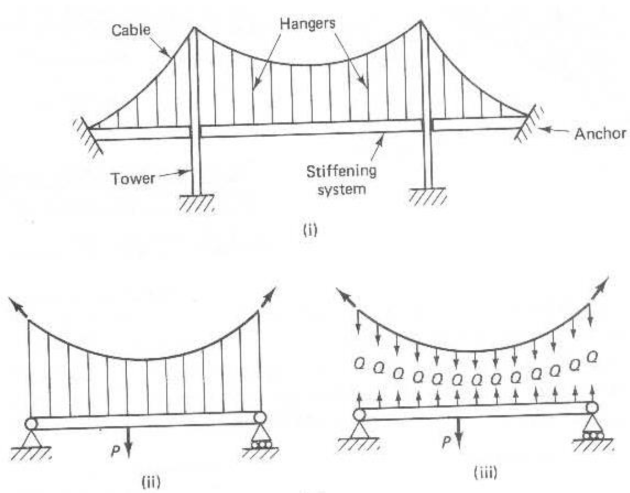


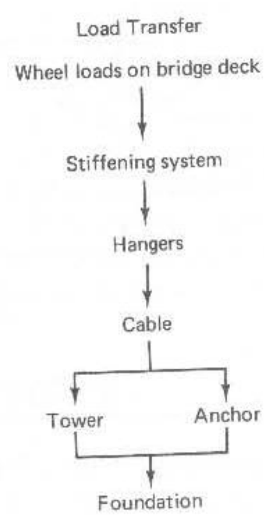
(Source: Kennedy, 1990, Elastic Analysis of Structures: Classical and Matrix Methods)





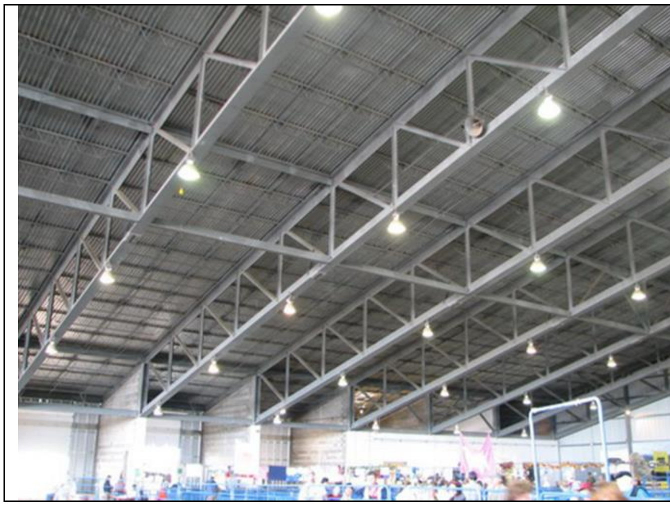






(Source: Kennedy, 1990, Elastic Analysis of Structures: Classical and Matrix Methods)

Some additional examples of load transfer:



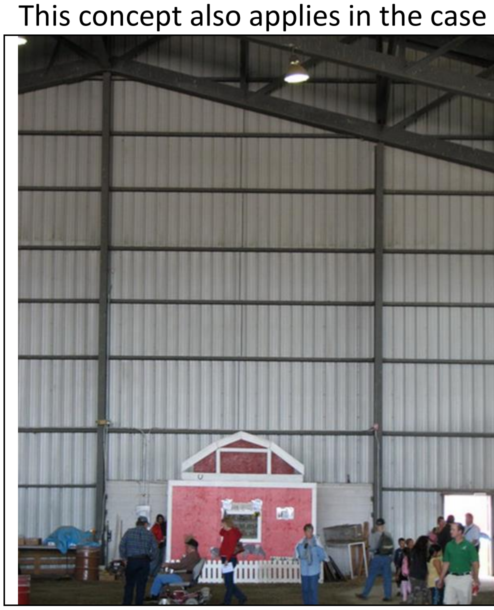
Load transfer:

1) decking to joists
2) joists to trusses
3) trusses to piers/columns
4) Columns to foundation

Load transfer:

1) decking to beams
2) beams to girders
3) girder to columns
4) Columns to foundation

This concept also applies in the case of lateral loads on walls/siding …



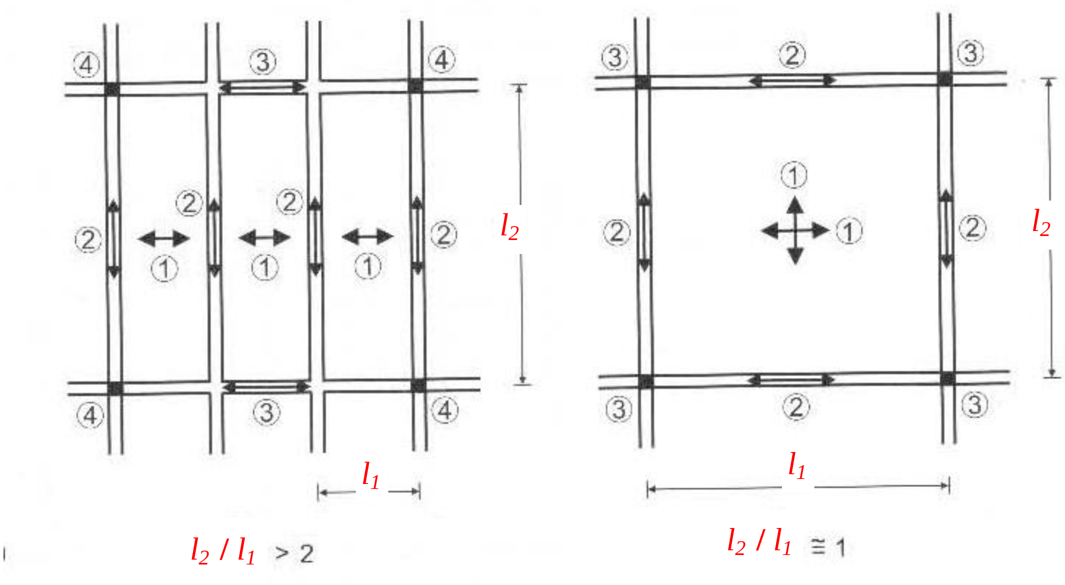
Load transfer:

1) Wall siding to girts
2) Girts to beam-columns
3) Columns to foundation

5. Load transfer concepts

One way versus two-way load transfer:

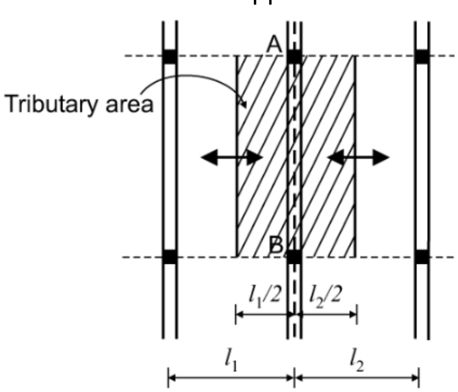
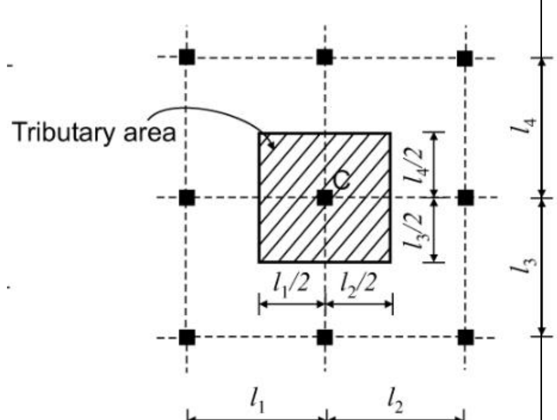
- **One-way**: when the ratio between the longer span (L2) and shorter span (L1) of a floor bay is greater than 2, load transfer will occur in one direction (short direction)
- **Two-way:** If this ratio is smaller than 2 then there will be some load transfer in both directions.



(Source: Brzev and Pao, Reinforced concrete design, 2006)

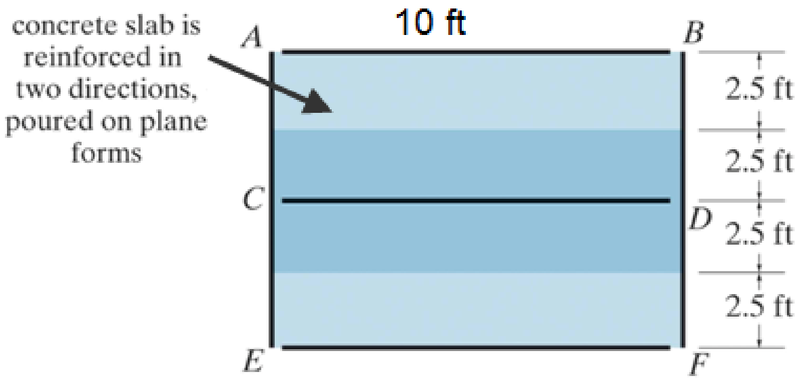
Tributary area:

- The concept of **tributary area** is used to determine the loads that beams, columns and walls carry.

The tributary area of a **beam** or **girder** supporting a portion of a floor is equal to the area enclosing the member and bounded by the lines of zero shear, which can be approximated as halfway between the lines of support : 	The tributary area of a **column** is the area bounded by the lines located halfway between the lines of support in both directions 

(Source: Brzev and Pao, Reinforced concrete design, 2006)

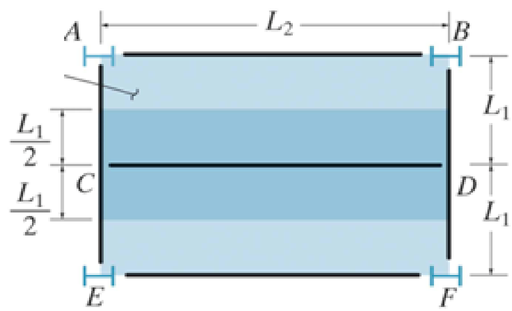
Example: The flooring system below is loaded with q = 100 lb/ft^2 , determine the loading on Beam CD and girder BF:



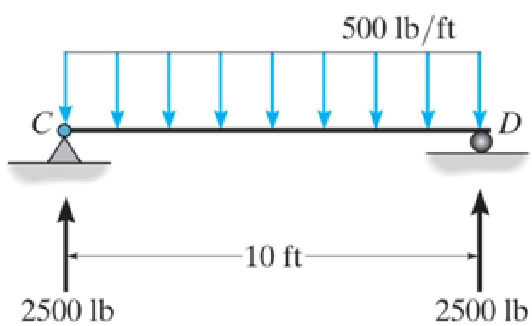
idealized framing plan

Since L2/L1 ≥ 2 this is a one-way system, Load transfer will occur in the short direction:

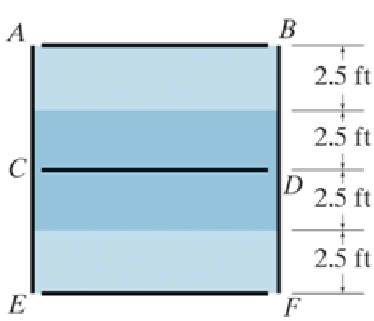
- Trib width for CD = 5 ft
- Load on Cd = 100 lb/ft^2 * 5 ft = 500



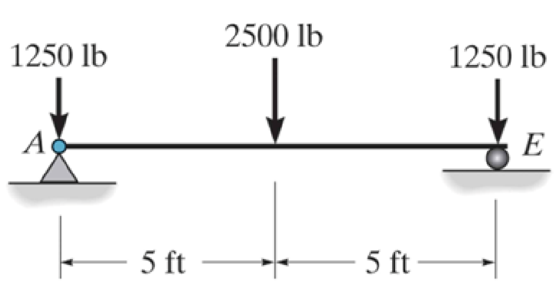
idealized framing plan for one-way slab action requires $L_2/L_1 \geq 2$



Since the beams rest on the girder, the loading on the girder results from the end reactions of the 3 beams.



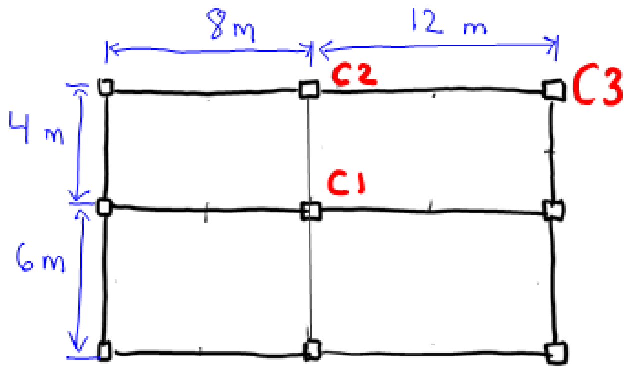
idealized framing plan



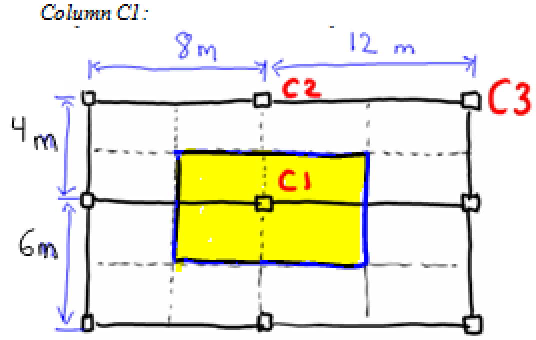
idealized girder

(Source: Hibbeler, 2005, Structural Analysis)

Example: What is the tributary area of columns C1, C2, C3 below ?



Column C1 :

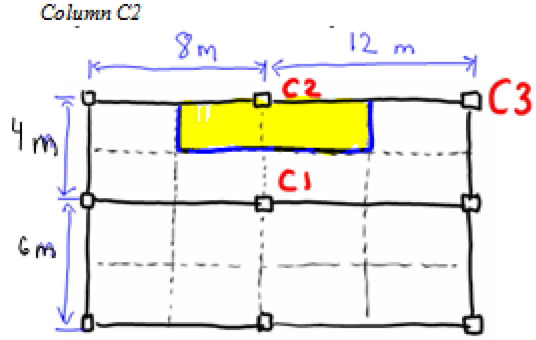


TA_{C1}:

$$\left(\frac{8}{2} + \frac{12}{2}\right) \times \left(\frac{4}{2} + \frac{6}{2}\right)$$

$$= 10 \times 5 = 50\ m^2$$

Column C2

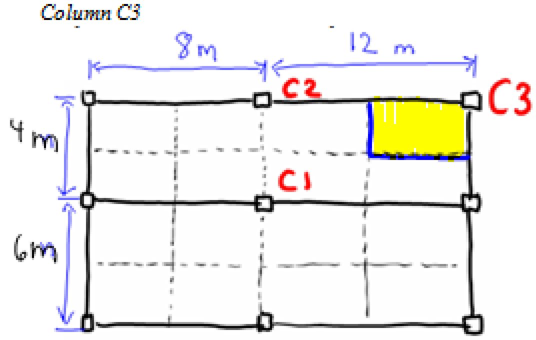


TA_{C2}:

$$\left(\frac{8}{2} + \frac{12}{2}\right) \times \frac{4}{2}$$

$$= 10 \times 2 = 20\ m^2$$

Column C3



TA_{C3}:

$$(12/2) \times (4/2)$$

$$= 6 \times 2 = 12\ m^2$$

6. Structural Idealization

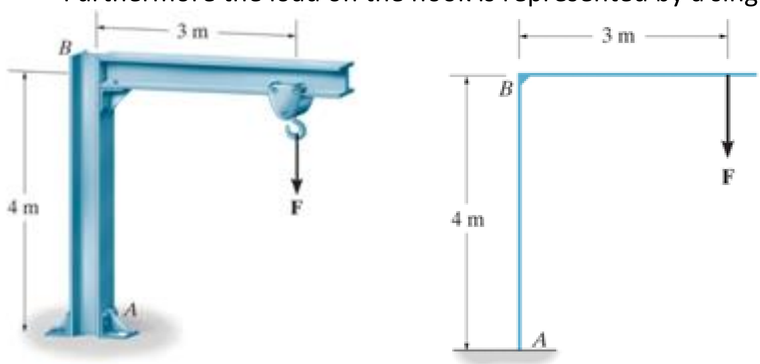
Question: Can an exact analysis of a structure be carried out?

- No, because ***APPROXIMATIONS*** need to be made regarding: dimensions, boundary conditions (supports & connections), material properties, loads, etc ...

Thus it is important to understand it is not possible to analyse the exact structure we will build, we only analyse an idealized model of that structure.

Example: For the jib crane below:

- we can neglect the thickness of the two main structural members and the two members are represented by two connected lines.
- We will assume the connection at joint B to be rigid, and the connection at A can be assumed to be fixed.
- Furthermore the load on the hook is represented by a single point load F.

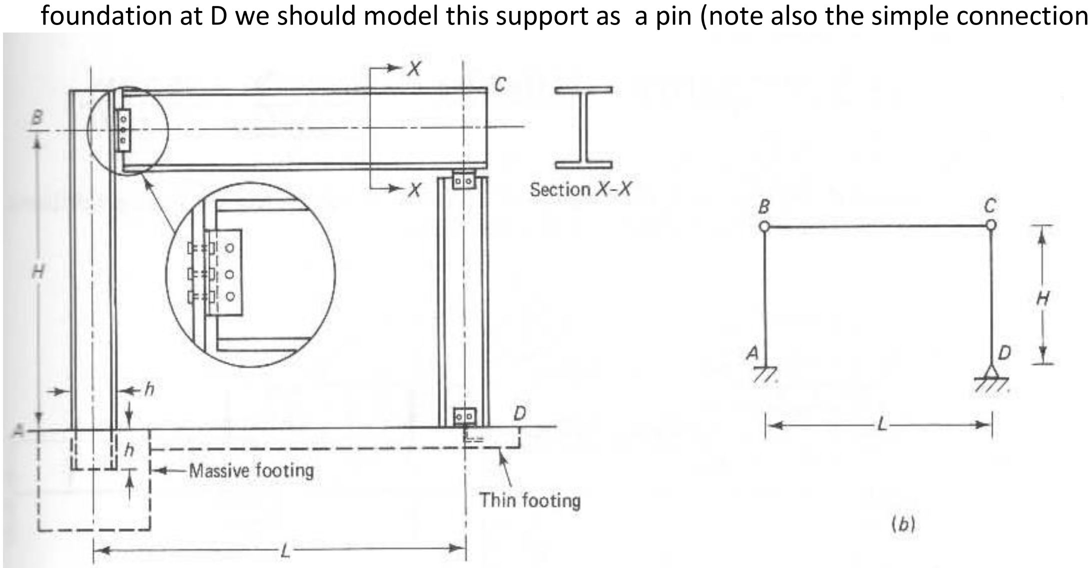


actual structure idealized structure

(Source: Hibbeler, 2005, Structural Analysis)

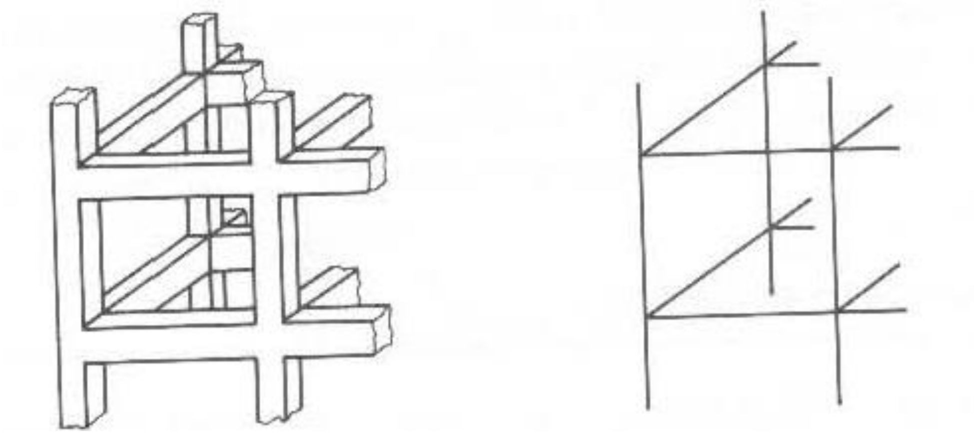
Example: idealizing the connections and supports in the simple frame structure:

- If the connection at B allows some rotation we can idealize it as a pin
- The massive footing at A will have some restraint to rotations, so we can model this support as fixed (note also that the column is anchored into the support to ensure the transfer of moments we are assuming in analysis). On the other hand because of the thin foundation at D we should model this support as a pin (note also the simple connection).



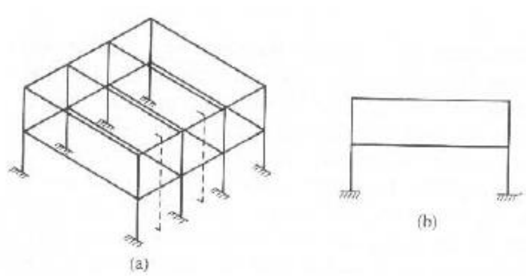
(Source: Kennedy, 1990, Elastic Analysis of Structures: Classical and Matrix Methods)

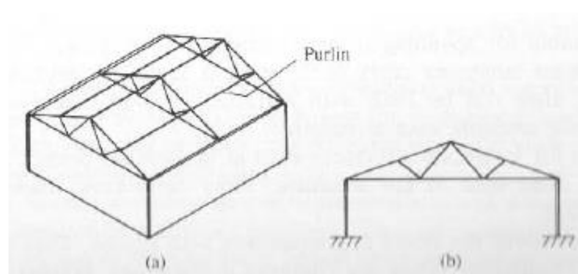
Example: A 3D frame structure is idealized by line diagrams where the lines coincide with the actual centroidal axes of the members (members become linear with the material concentrated on their centroidal axes → called "skeletal structure" or "line drawing" which we can use in analysis)

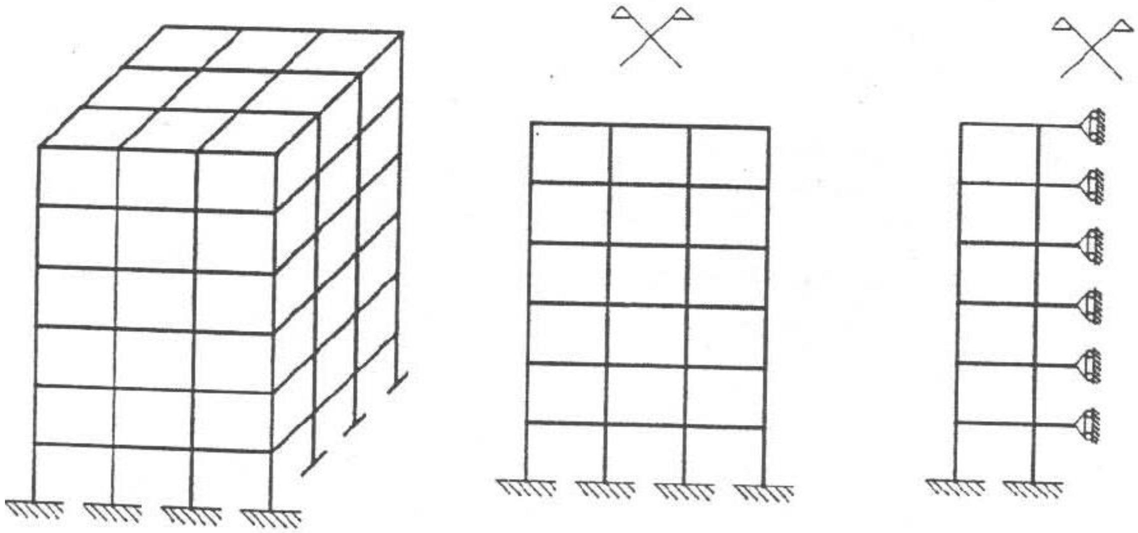


(Source: Kennedy, 1990, Elastic Analysis of Structures: Classical and Matrix Methods)

Example: different levels of structural idealization (3D vs. 2D models)



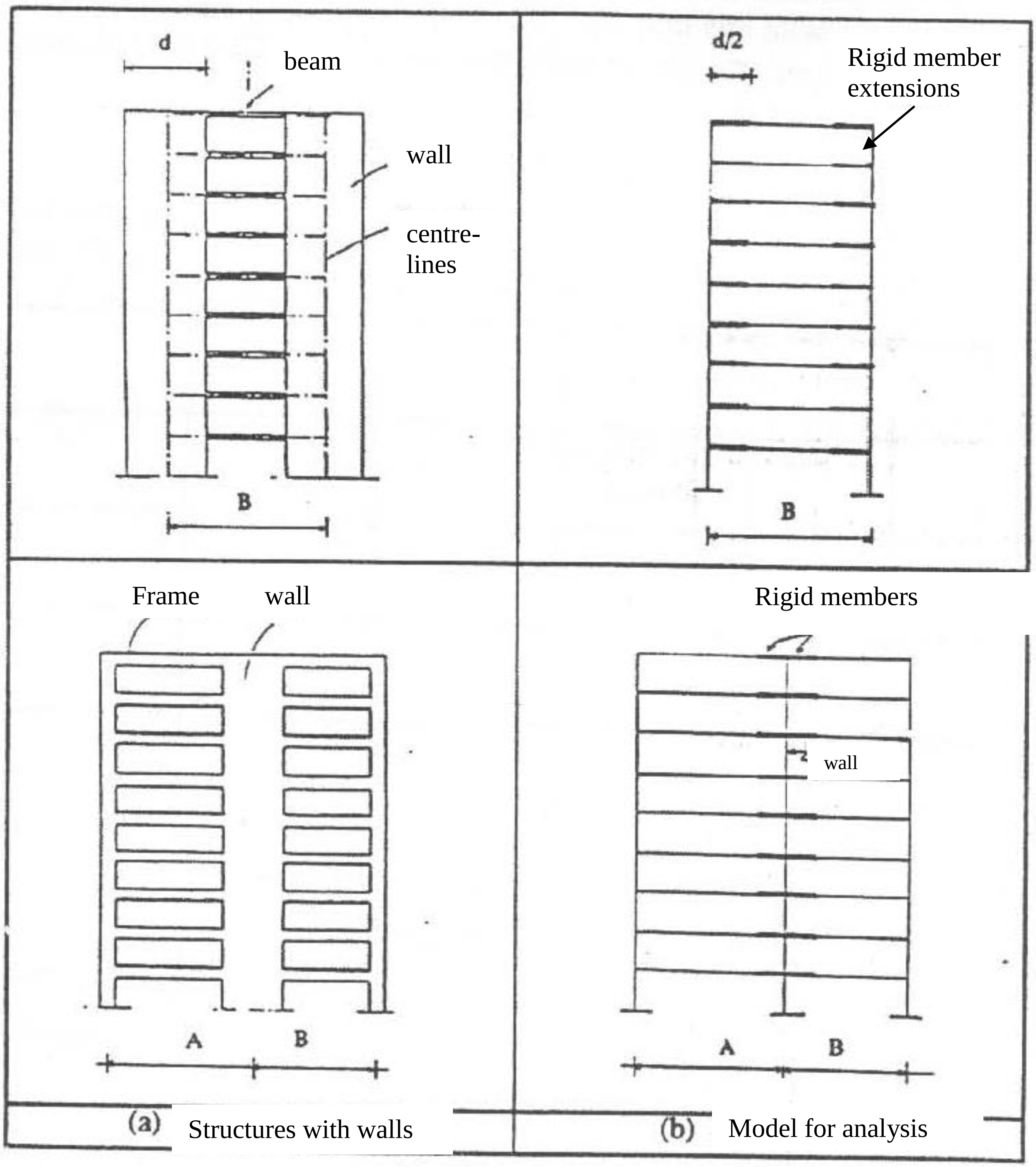




(Source: Kennedy, 1990, Elastic Analysis of Structures: Classical and Matrix Methods)

Another example of idealization: incorporating walls in computer models ...

- Beams and columns are flexible in comparison to walls.
- If a structure contains a mixture of these elements one can choose to model the walls by idealizing them as “large columns”.
- One keeps the location of the centerlines between the beams and the wall by providing a member which is infinitely rigid

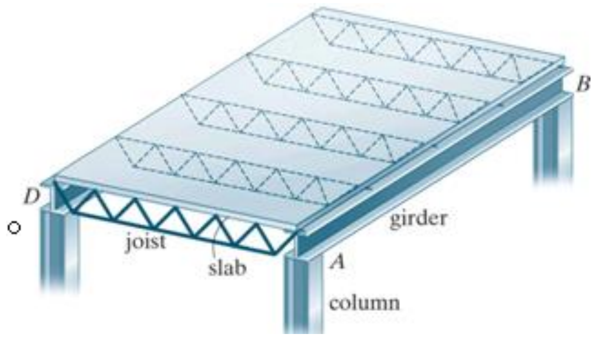


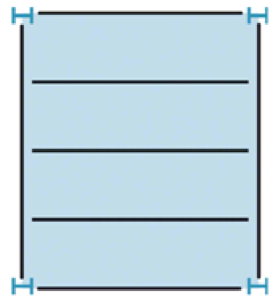
(Chaallal et al., Analyse des structures, 2007)

Representation on drawings

(1) Framing where members are not rigidly connected

- Joists/beams are pin or roller connected to the girders
- The girders are pin or roller connected to the columns
- Notice that the lines representing the beams and girders do not touch

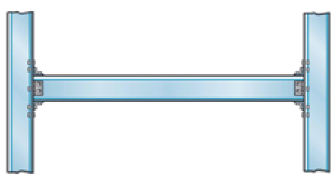




idealized framing plan

(2) Framing where members are rigidly connected (welded versus simple bolting)

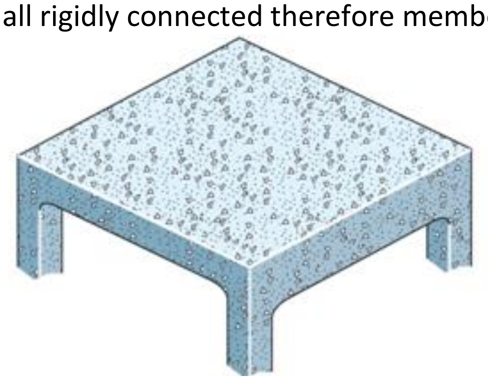
- Notice that the lines representing the beams and girders do touch

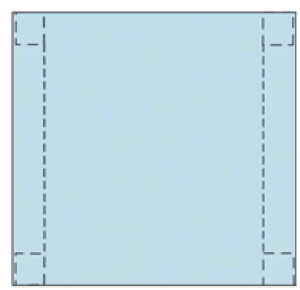


fixed-connected beam

idealized beam

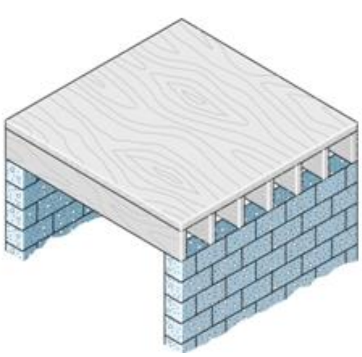
In reinforced concrete the beams and girders are represented by double lines (members are all rigidly connected therefore members are drawn to the supports)

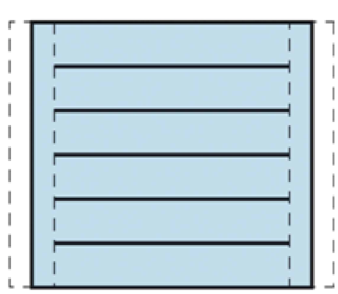




idealized framing plan

Timber system (beams are simply supported on masonry walls: again lines do not touch)





idealized framing plan