Lecture 2.1: Determinacy & Stability of structures

1. Determinacy - review
2. Determinacy of trusses
3. Stability of trusses
4. Determinacy of Beams & Frames
5. Stability of Beams & Frames

1. Determinacy - review

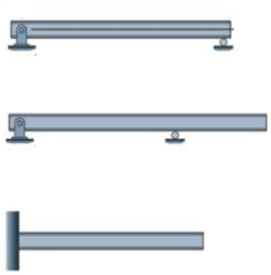
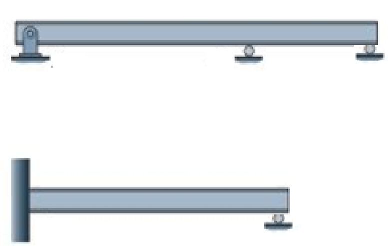
Understanding if a structure is determinate is essential to selecting appropriate design and analysis methods. Ensuring a truss is stable is necessary for sound design.

A structure is considered ***determinate*** when the equations of equilibrium are sufficient to determine:

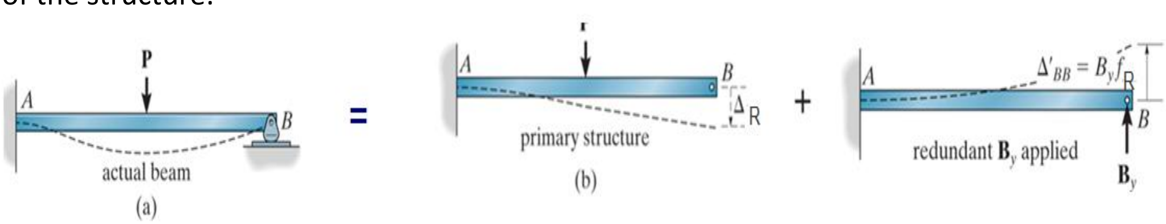
- all the support reactions
- and internal member forces in the system

Statically indeterminate structures have:

- more unknowns forces than the available equilibrium equations

Examples of DETERMINATE BEAMS:	***Examples of INDETERMINATE BEAMS:***
	 (Philpot,2009)

Note: If the structure is indeterminate, we need additional equations = compatibility equations. These compatibility equations must be equal to the ***degree of indeterminacy*** of the structure.



For the above example we will have the following "compatibility equation": $\Delta_{total} = \Delta_R + \Delta'_{BB} = 0$ (where $\Delta'_{BB} = B_y \times f_R$)

2. Determinacy of trusses

Statically determinate truss:

- When the equations of equilibrium are sufficient to determine all the **support reactions** and **internal member forces** in the system, the structure is **statically determinate**

Statically indeterminate truss:

- Trusses having more unknown forces than available equilibrium equations are called **statically indeterminate**

To see if a truss is determinant or indeterminate, we need to compare the **number of unknowns** and the **number of equations** that we have based on equilibrium.

To simplify the process for determinacy, the following procedure can be applied. The procedure is based on the number of equilibrium equations for the entire truss (2 at each joint) and the number of internal and external unknowns based on the number of members and number of reactions.

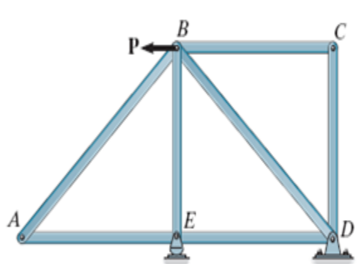
1. Count the number of joints, number of reactions, and number of members in the truss.

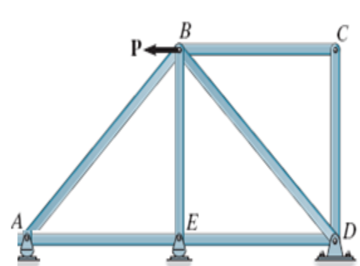
Number of Joints	j
Number of Members	b
Number of Reactions	r

2. Using the following, determine if the truss is determinant or indeterminate.

Truss is **Determinate**	$b + r = 2j$
Truss is **Indeterminate**	$b + r > 2j$
Degree of truss indeterminacy	$SI = b + r - 2j$

Example:





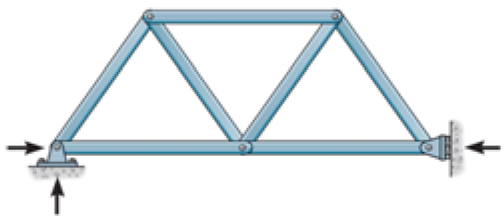
3. Stability of trusses

To ensure equilibrium of a structure it is not only necessary to satisfy the equations of equilibrium, but the members must also be properly held or constrained by their supports

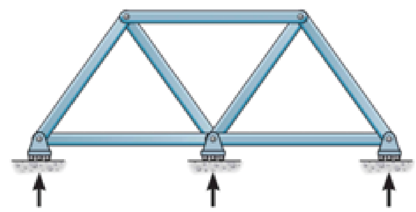
Instability #1: partial constraints
In some cases a structure may have fewer reactive forces than equations of equilibrium (i.e. $b+r<2j$), in this case the truss is unstable.

Instability #2a: External instability
A truss is externally unstable if:

- … all its reactions are concurrent
- … all its reactions are parallel



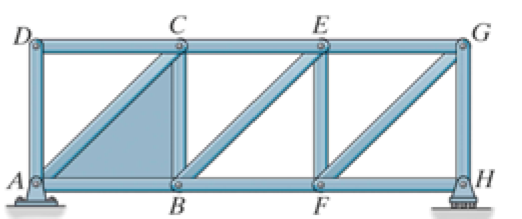
unstable concurrent reactions

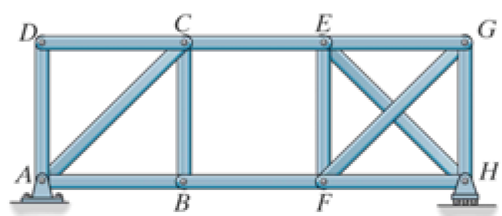


unstable–parallel reactions

Instability #2b: Internal instability
The internal stability of a truss can be checked by examining the arrangement of members in the truss and examining if a “collapse mechanism” exists (i.e. joint can move in a “rigid body” sense with respect to other joints)

A simple truss will always be internally stable since by the nature of its construction we start from a basic triangular arrangement and add successive “rigid elements”:





The truss to the right is internally unstable because of the non triangular arrangement in CEFB since it will not be able to hold its joints in a fixed position = “collapse mechanism”
(note: it may be more difficult to identify the stability of complex trusses)

Summary:

$SI<0$	Truss is unstable
$SI\geq 0$	Truss can be unstable if: (1) truss support reactions are parallel or concurrent (2) or if some of the components of the truss form a collapse mechanism

- For obvious reasons the use of an unstable truss is to be avoided in practice

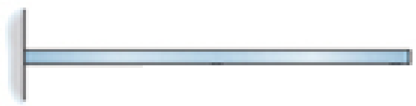
4. Determinacy -Beams & Frames

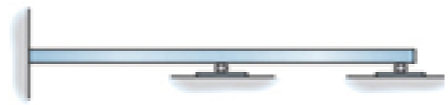
Statically determinate beams or frame:

- Equations of equilibrium **are** sufficient to determine all the **reactions** and **internal forces**

Statically Indeterminate beam or frame:

- Equations of equilibrium **are not** sufficient to determine all the **reactions** & **internal forces**





Method 1:

In beams and frames (2D or co-planar frames) we have a total of three equations of equilibrium for each part. Using the same logic as with trusses, we can add up all the possible equations of equilibrium for a frame or beam and compare that with the total number of unknowns to see if the structure is determinate.

We define:

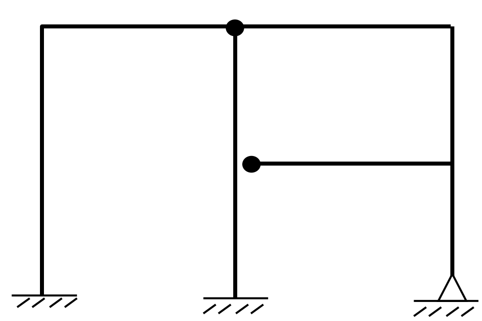
- m = # of members
- j = # of joints
- r = # of unknown support reactions
- e_c = number of equations of conditions (example internal hinge)
  - e_c = # of members meeting at release joint - 1

Unknowns - for each member, *m*, there are 3 force components 3m
- reactions at supports r

Knowns - for each joint, *j*, there are 3 equilibrium equations 3j
- equations of euilibrium at internal releases (hinges) e_c

Comparing the knowns and unknowns we get:

Beam/frame is statically **Determinate**	$(3m+r)=(3j+e_c)$
Beam/frame statically **Indeterminate**	$(3m+r)>(3j+e_c)$
Degree of beam/frame indeterminacy	$SI=(3m+r)-(3j+e_c)$



<u>Method 2</u>: In this method we sum the number of reactions in the beam or frame and compare this with the total number of equilibrium equations:

$r < 3n$	externally unstable
$r = 3n$	statically determinate
$r > 3n$	statically indeterminate
$SI = r-3n$	Degree of indeterminacy

where *n* is the number of parts and *r* is the number of force and moment reactions.

- For beams and frames with pin connections, we must separate them into parts at the location of the pin.
- For frames with closed loops we break them into parts at the loops

Beams

(b)

$r = 5$, $n = 1$, $5 > 3(1)$

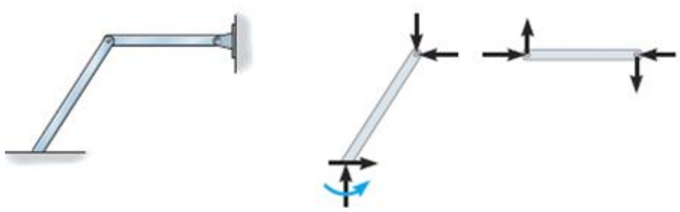
SI =2

(d)

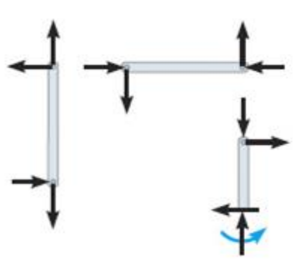
$r = 10$, $n = 3$, $10 > 3(3)$

SI =1

Pin -Frames



$r = 7$, $n = 2$, $7 > 6$
Statically indeterminate to the first degree



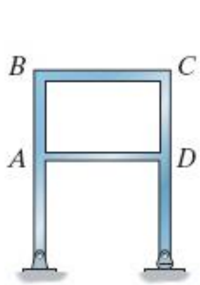
$r = 9$, $n = 3$, $9 = 9$,
Statically determinate

Frames with "closed loops"

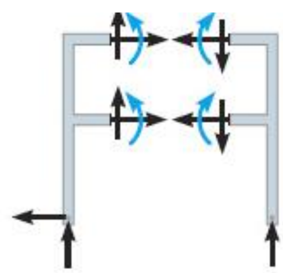
- When frames have a closed loop we need to cut the frame into parts by cutting through the loop. Read the below on your own to understand this method of determining the degree of indeterminacy for these structures.

Solution

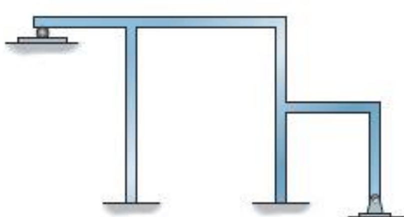
Unlike the beams and pin-connected structures of the previous examples, frame structures consist of members that are connected together by rigid joints. Sometimes the members form internal loops as in Fig. 2–20*a*. Here *ABCD* forms a closed loop. In order to classify these structures, it is necessary to use the method of sections and "cut" the loop apart. The free-body diagrams of the sectioned parts are drawn and the frame can then be classified. Notice that only *one section* through the loop is required, since once the unknowns at the section are determined, the internal forces at any point in the members can then be found using the method of sections and the equations of equilibrium. The frame in Fig. 2–20*b* has no closed loops and so it does not need to be sectioned between the supports when finding its determinancy. The resulting classifications are indicated in Fig. 2–20*a* and 2–20*b*.



(a)

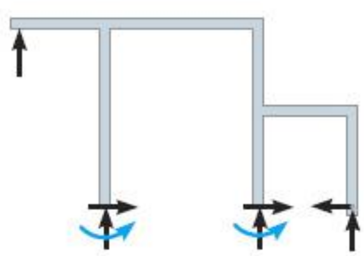


$r = 9$, $n = 2$, $9 > 6$,
Statically indeterminate to the third degree **Ans.**



(b)

(This frame has no closed loops.)



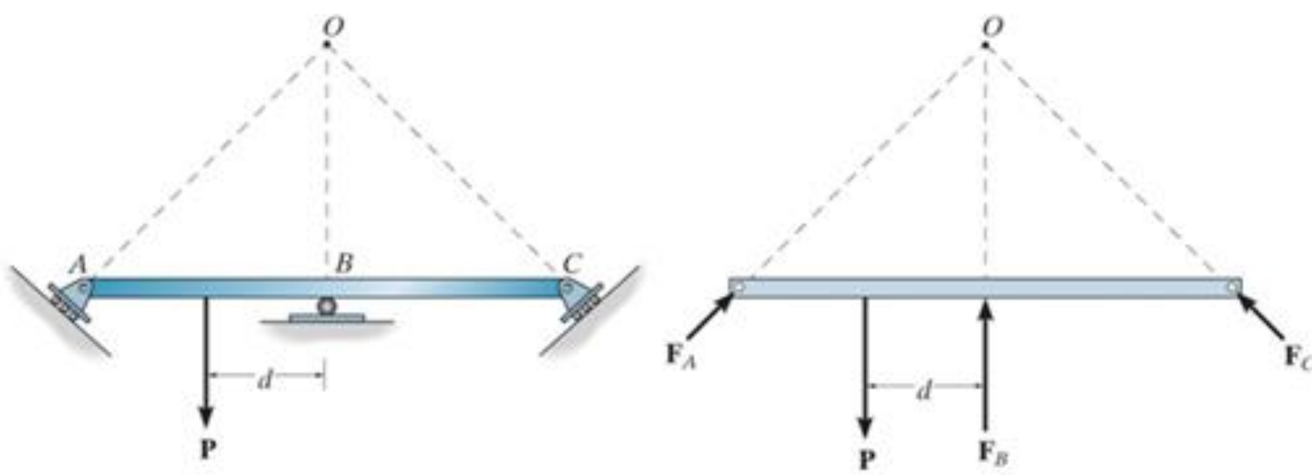
$r = 9$, $n = 1$, $9 > 3$,
Statically indeterminate to the sixth degree **Ans.**

5. Stability - Beams & Frames

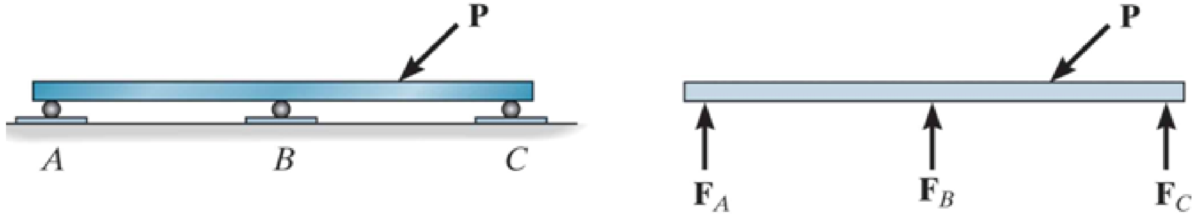
The rules for stability of beams and frames are the same as those for trusses. In general we must satisfy both external and internal stability requirements.

Therefore:

- **External Instability**
  - concurrent reactions
  - parallel reactions
  - SI<0
- **Internal Instability**
  - Internal collapse mechanism



concurrent reactions



parallel reactions

Lecture 2.2: V & M diagrams - Beams

1. Beam characteristics
2. Internal member releases
3. Principles for drawing shear and moment diagrams
4. Procedure - V & M diagrams

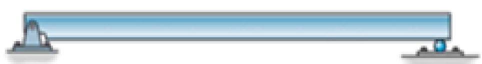
1. Beam characteristics

Definition:

A beam is a structural element that is subject to transverse loading (i.e. ⊥ to its longitudonal axis)***. A beam carries transverse laod primarily in bending*** (flexure)

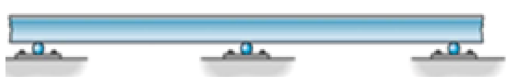
Three main physical characteristics to classify beams:

(1) Support configuration:



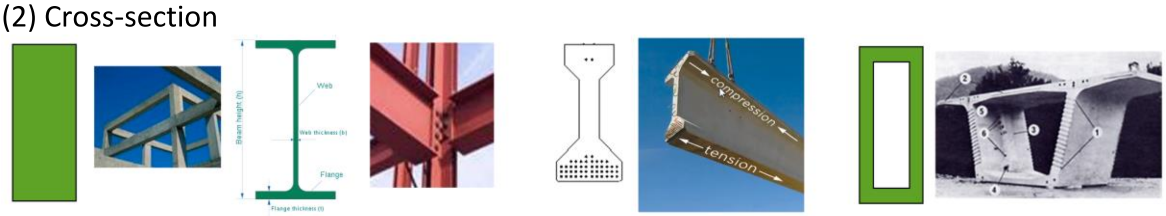
simply supported beam

cantilevered beam



continuous beam

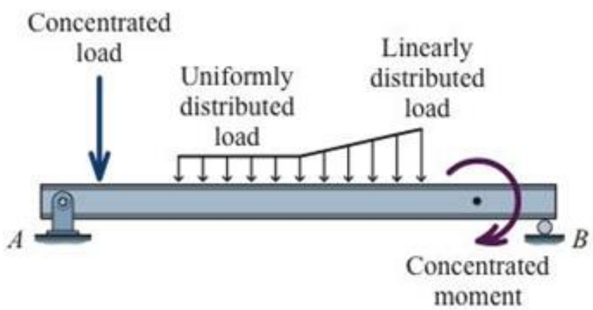
(2) Cross-section



(3) Profile

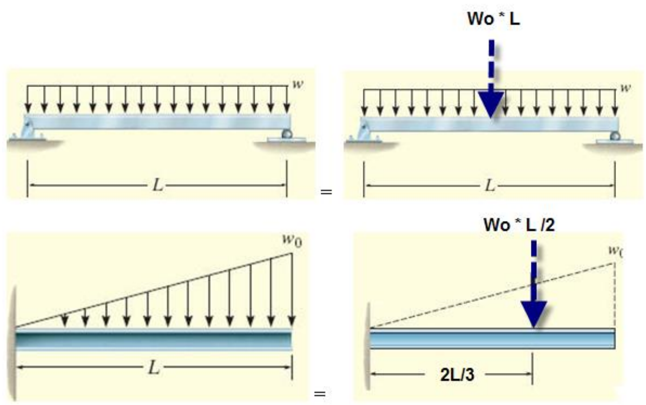
- Simple
- Tapered

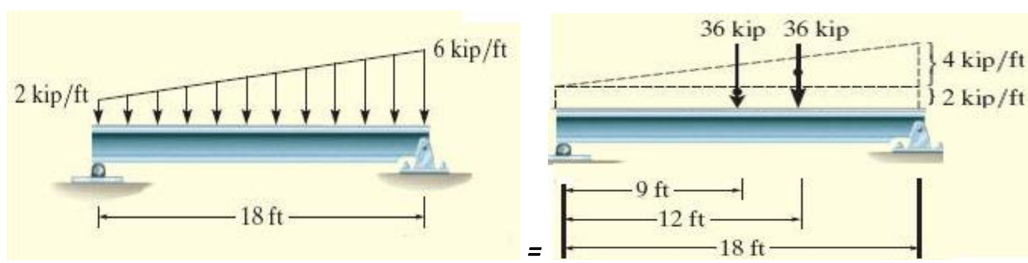
Beams can be subjected to a variety of external forces:



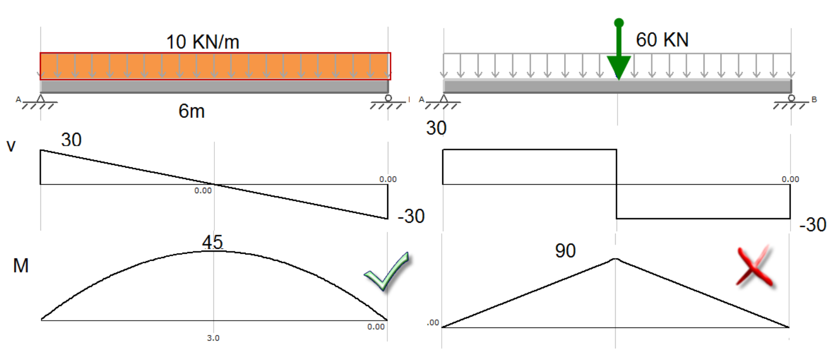
(Philpot, 2009)

For **determining** **reactions** a distributed force can be replaced by a concentrated force representing the RESULTANT of the distributed load





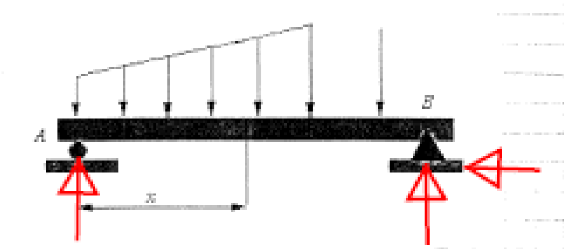
However, when we draw shear and moment diagrams we need to make sure we use the actual distributed load, not the resultant !



we can use the resultant to help find the reactions, but we cannot use it to replace the distributed load when drawing the M diagram

To determine external reactions → draw Free-Body-Diagram (**FBD**) of the **entire structure**

- **FBD** of the entire structure **must be in equilibrium**

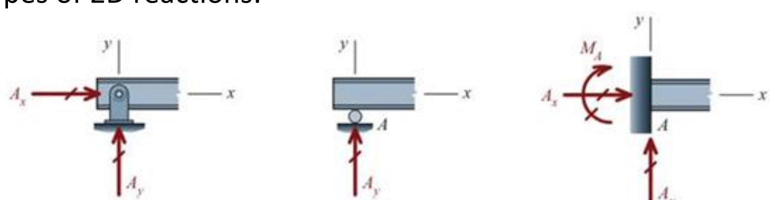


(Léger, 2004)

Note that we can indicate the direction of the forces in an arbitrary manner:

- if reaction = **positive** ... we chose the correct direction
- if reaction = **negative** ... direction is opposite to what we chose

Types of 2D reactions:



(*a*) Pin support (*b*) Roller support (*c*) Fixed support

Beams can have a variety of reactions (2D or 3D)

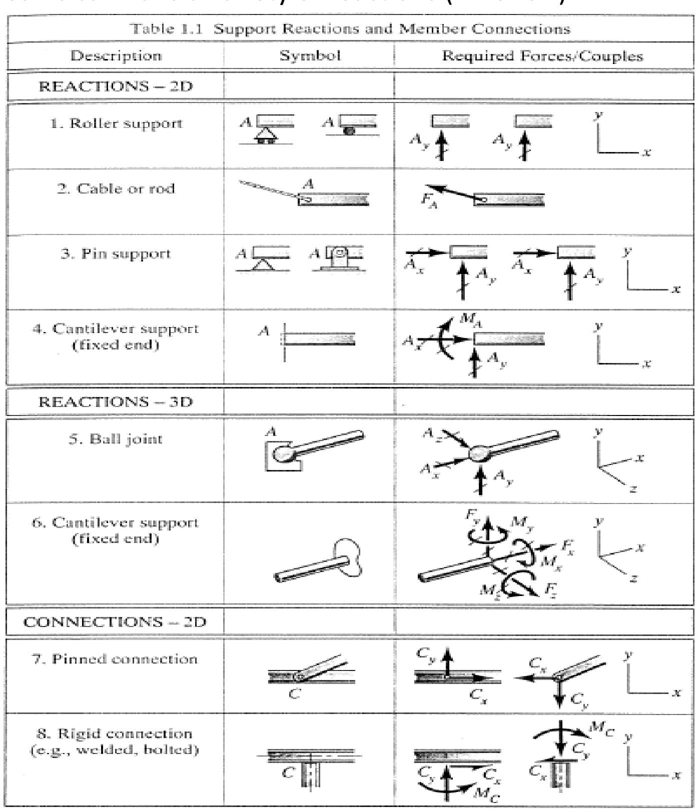
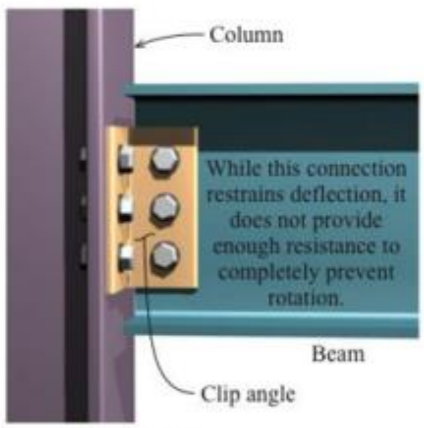


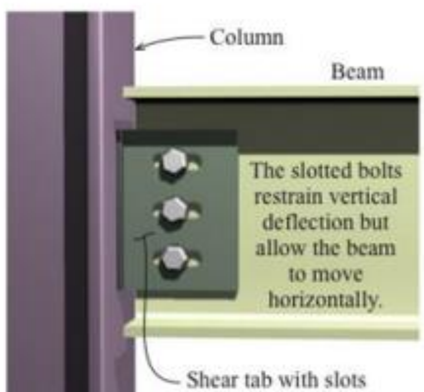
Table 1.1 Support Reactions and Member Connections

Description	Symbol	Required Forces/Couples
REACTIONS – 2D		
1. Roller support		A_y, A_y
2. Cable or rod		F_A
3. Pin support		A_x, A_y, A_x, A_y
4. Cantilever support (fixed end)		A_x, M_A, A_y
REACTIONS – 3D		
5. Ball joint		A_z, A_x, A_y
6. Cantilever support (fixed end)		F_y, M_y, F_x, M_x, M_z, F_z
CONNECTIONS – 2D		
7. Pinned connection		C_y, C_x, C_x, C_y
8. Rigid connection (e.g., welded, bolted)		C_y, C_x, M_C, M_C, C_y, C_x

(Craig, 2000)

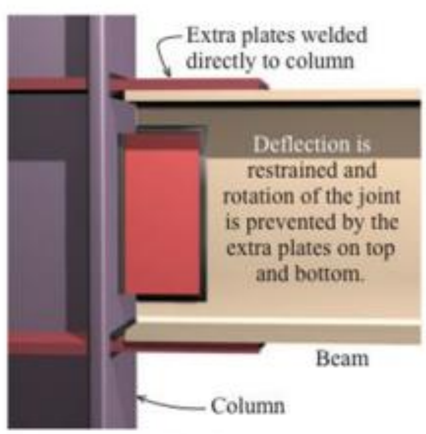


(a) Pin support

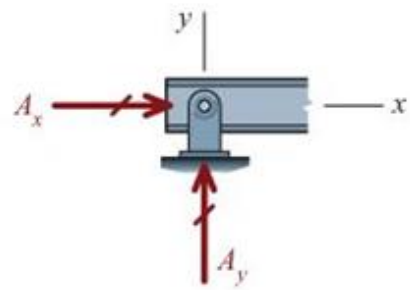


(b) Roller support

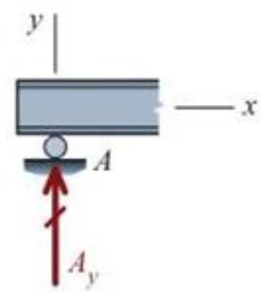
rts.



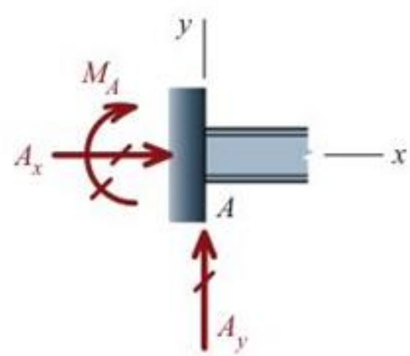
(c) Fixed support



(a) Pin support



(b) Roller support



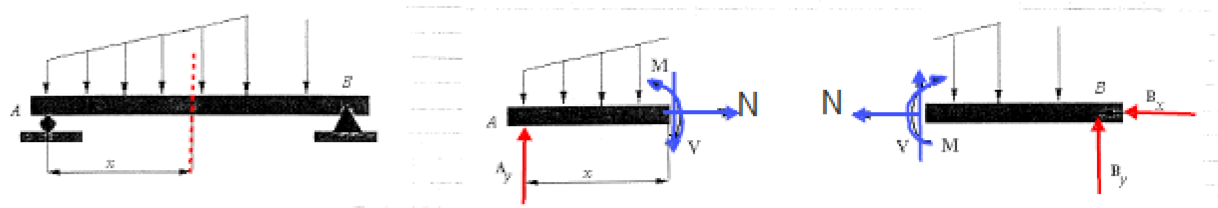
(c) Fixed support

(Philpot,2009)

If we cut a section through a beam:

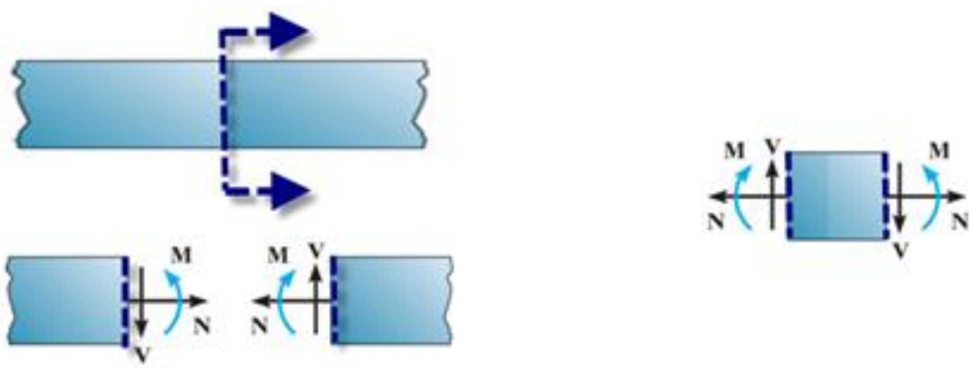
- Three possible **internal forces (N,V,M)**:
  - **N** = axial force
  - **V** = shear force
  - **M** = moment

- **Each portion**'s **FBD** needs to be in **equilibrium**
- The "**action-reaction**" principle → internal forces are **equal & opposite** on each side of section



(Léger, 2004)

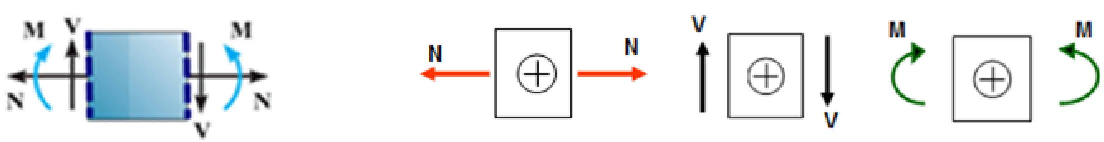
The **"positive sign convention"** for **internal forces** (N, V, M)**:**



After applying equilibrium equations:

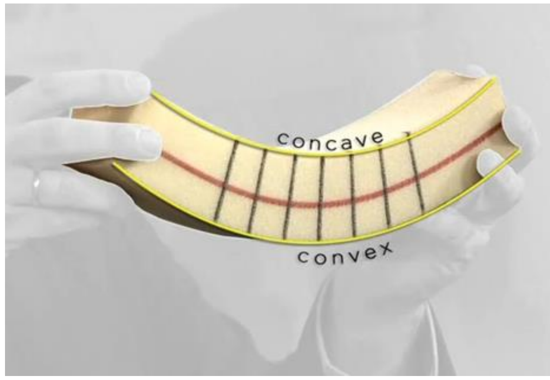
- if V = **positive** ... internal shear force at this section is **positive**
- if M = **positive** ... internal moment at this section is **positive**

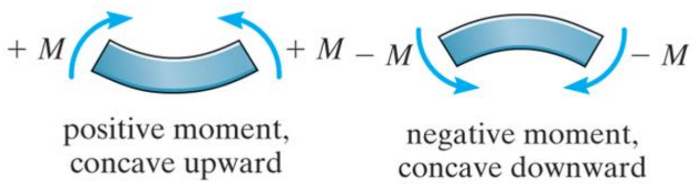
Easy way to remember the convention : " Pull - Clockwise - Smile "

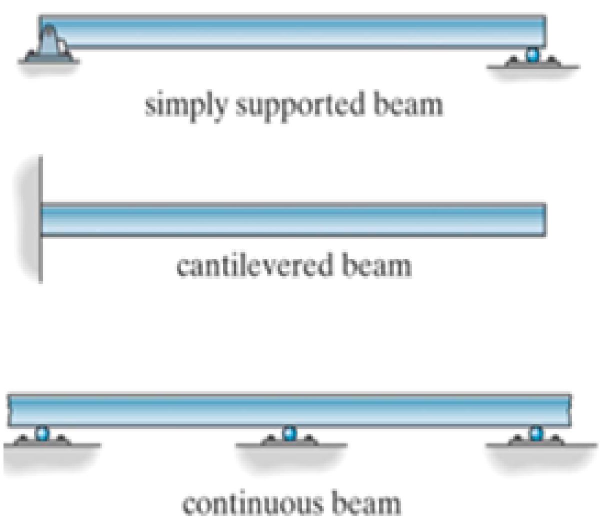


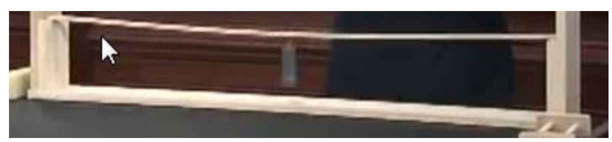
When the beam bends, the beam deforms and curvature results

- positive bending = curvature is **concave upward** (i.e. compression on top, tension on bottom)
- negative bending = curvature is **concave downward** (i.e. compression on bottom, tension on top)



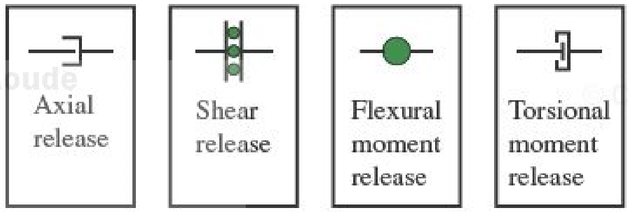




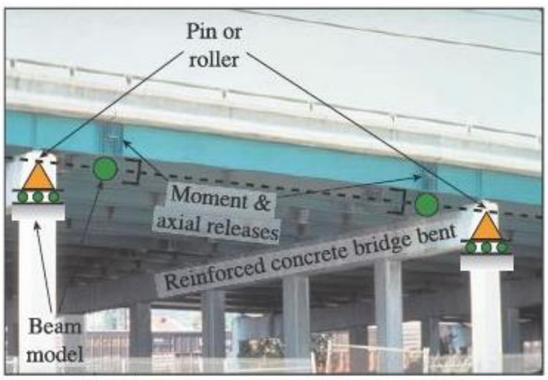


2. Internal member releases

In certain beams we can have "internal releases" in our idealized model.



For example in the bridge beam below structural details have been inserted to ensure that moment and axial force are zero at these locations.



To represent this detail in the idealized structure we can:

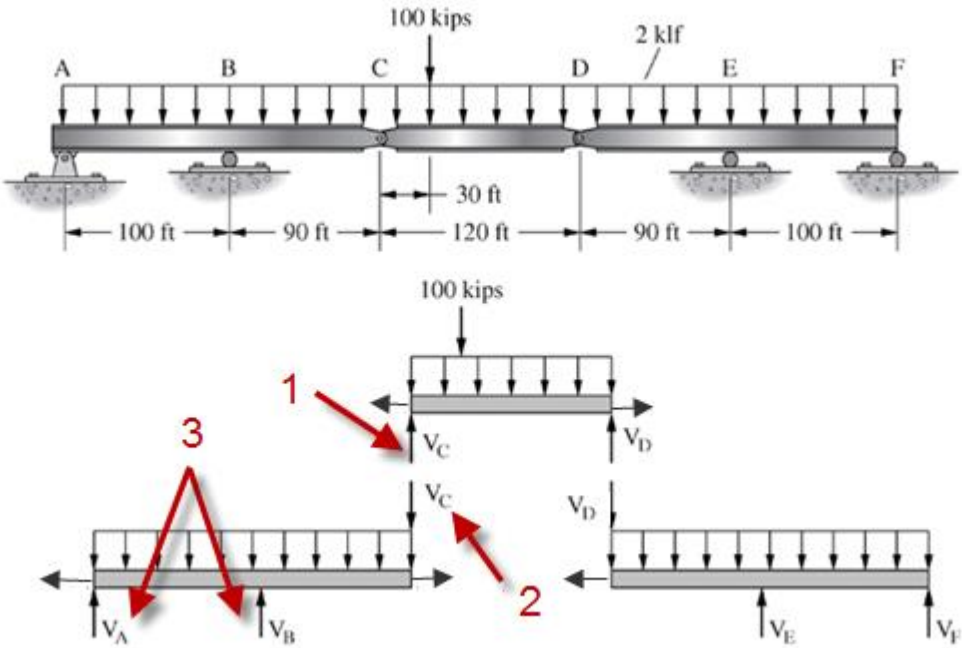
- add an internal hinge / moment release (M=0)
- add an axial release (N=0)...

Steps in the analysis of a beam with an "internal hinge" (internal moment release)

- we know that M=0 at the internal hinge (but V ≠ 0)
- we section the beam at this location to determine V & N
- we analyse the FBD of one portion of the beam →determine V & N
- we use the "action-reaction principle" to find the other unknowns

Example:

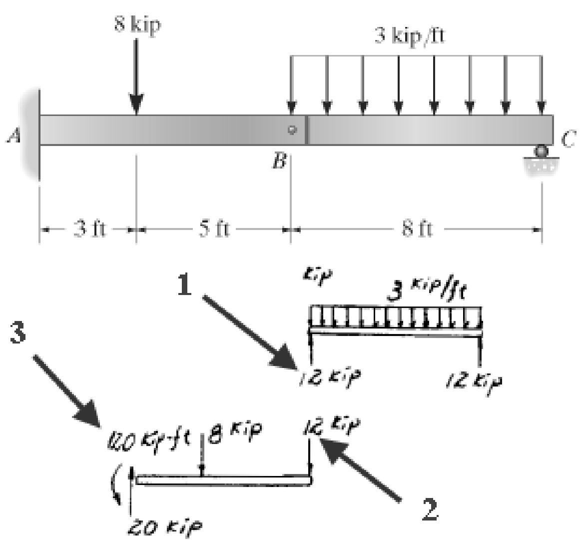
- start with drop-in beam CD, determine reactions (@C and D)
- use action-reaction principle to transfer forces onto cantilever spans
- go to cantilever spans and determine remaining reactions (@ A & B)
- reassemble beam to draw shear and moment diagrams

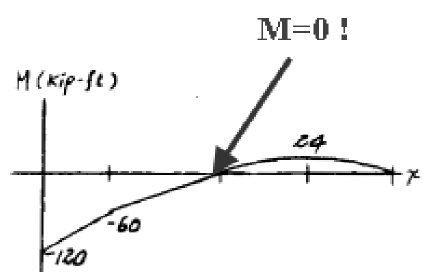


(McCormac, 2006)

Example:

- start with span BC, determine reactions (@B & C)
-use action-reaction principle to transfer forces onto span AB
- analyse span AB and determine reactions (@ A)
- reassemble beam to draw shear and moment diagrams





(Hibbeler, 2005)

3. Principles for drawing V and M diagrams

$$\frac{dV}{dx} = w(x)$$ Slope of Shear curve = Intensity of loading	$$\frac{dM}{dx} = V$$ Slope of Moment curve = Intensity of the shear

Practical point #1:

- if load w(x) has a curve of degree **n** ...
  - ... **V** diagram has a curve of degree **n+1**
- if **V** has a curve of degree **n** ...
  - ... **M** diagram has a curve of degree **n+1**

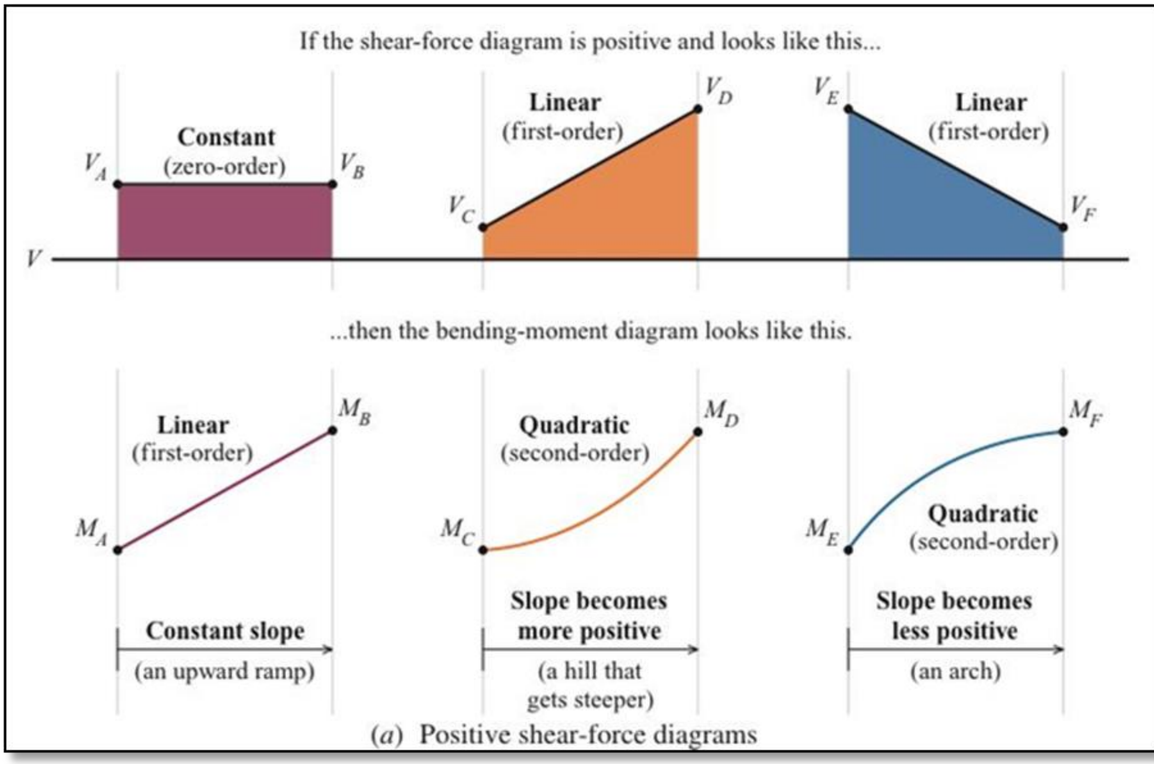
Practical point #2:

- if $|V|_{a \to b}$ **decreases** between 2 points:
  - .... the slope of M between these 2 points will **decrease** (more and more **flat)**
- if $|V|_{a \to b}$ **increases** between 2 points:
  - .... the slope of M between these 2 points will **increase** (more and more **steep)**

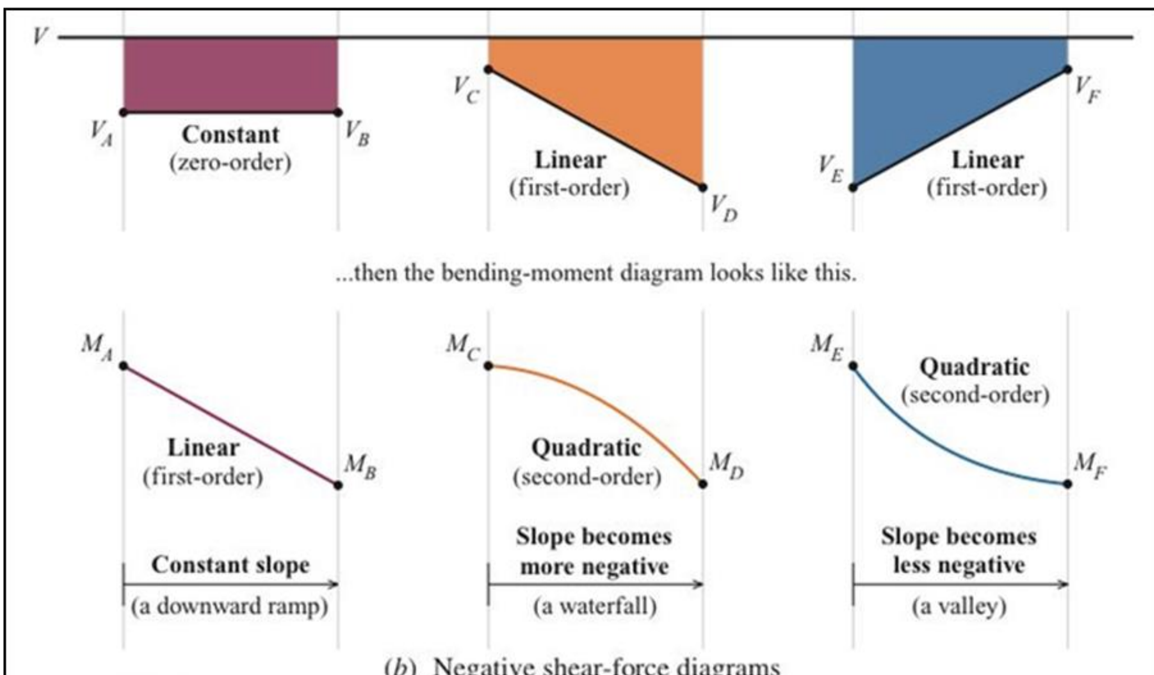
$$\Delta V = V_2 - V_1 = \int_{x_1}^{x_2} w(x)\,dx$$ Change in Shear = Area under loading curve	$$\Delta M = M_2 - M_1 = \int_{x_1}^{x_2} V\,dx$$ Change in Moment = Area under V diagram

Practical point #3

- After we obtain the shear diagram "V" we only have to find the different "areas" under this diagram to determine the "M" diagram!



(*a*) Positive shear-force diagrams

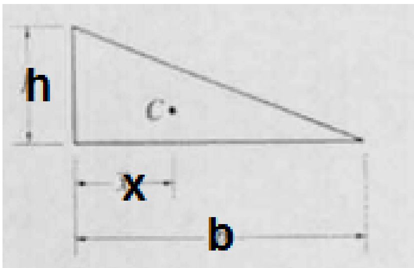


(*b*) Negative shear-force diagrams

(Philpot, 2009)

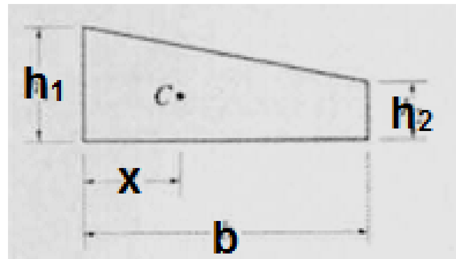
Triangle:

$$A = \frac{1}{2}bh \qquad \bar{x} = \frac{1}{3}b$$



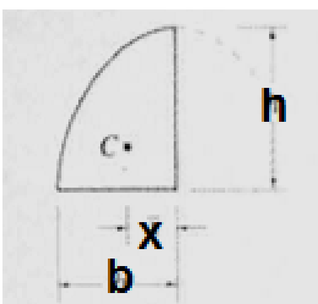
Trapezoid:

$$A = \frac{1}{2}b(h_1 + h_2) \qquad \bar{x} = \frac{b}{3}\left(\frac{2h_2 + h_1}{h_1 + h_2}\right)$$



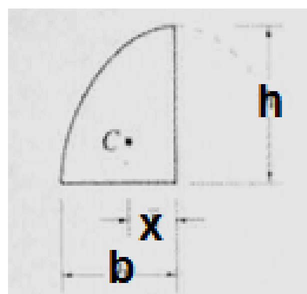
Semi-Parabola (n = 2 only)

$$A = \frac{2}{3}bh \qquad \bar{x} = \frac{3}{8}b$$



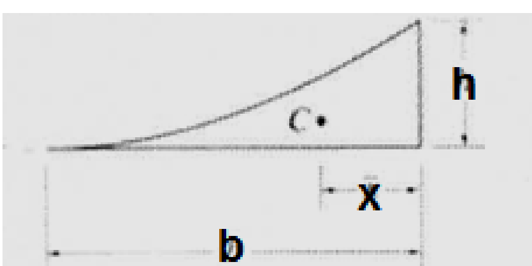
Semi-Parabola (n ≥2)

$$A = bh\left(\frac{n}{n+1}\right) \qquad \bar{x} = \frac{b}{2}\left(\frac{n+1}{n+2}\right)$$



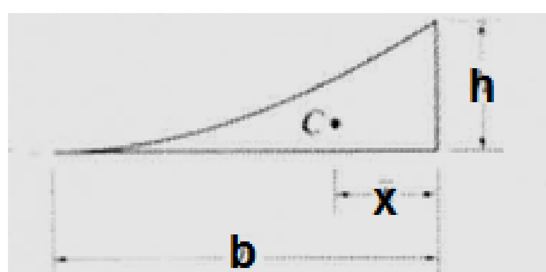
Spandrel (n = 2 only)

$$A = \frac{1}{3}bh \qquad \bar{x} = \frac{1}{4}b$$

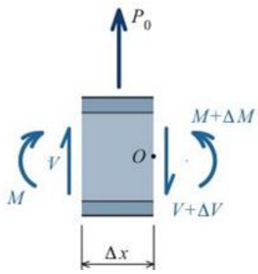


Spandrel (n ≥2)

$$A = bh\left(\frac{1}{n+1}\right) \qquad \bar{x} = b\left(\frac{1}{n+2}\right)$$



Concentrated point loads…



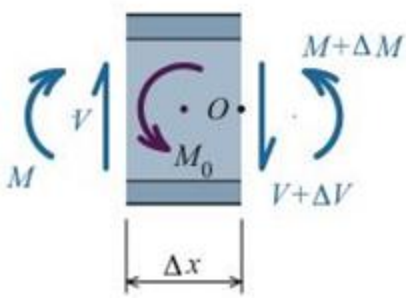
$$\Sigma F_y = V + P_0 - (V + \Delta V) = 0 \qquad \therefore \Delta V = P_0$$

$$\Delta V = P_o$$

Practical point #4:

- if "Po" acts **upwards**
  - **...** ΔV = positive
  - ... the “jump” in the V diagram is **upwards (+)**
- Si “Po” acts **downwards**
  - **...** ΔV = negative
  - ... the “jump” in the V diagram is **downwards (-)**

Concentrated moments…



$$\Sigma M_O = -M - V\Delta x + M_0 + (M + \Delta M) = 0$$

$$\Delta M = -M_o$$

Practical point #4:

- If “Mo” acts **clockwise**
  - **...** ΔM = positive
  - ... the “jump” in the M diagram is **upwards (+)**
- Si “Mo” est **counter-clockwise**
  - ... ΔM = negative
  - ... the “jump” in the M diagram is **downwards (-)**

4. Procedure: V/M diagrams (beams)

Procedure (beams)

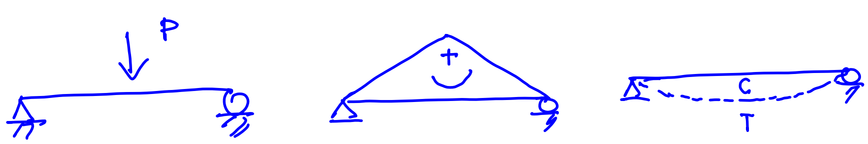
1. Determine if beam determinate (if yes, proceed …)

2. Draw FBD of entire beam and determine support reactions by applying ($\sum F_x = 0$, $\sum F_y = 0$, $\sum M = 0$)

3. Construct Shear diagram
   - Establish the "V" and "x" axes
   - Plot the value of shear at left end of the beam
   - For curvature recall … $\frac{dV}{dx} = w(x)$
   - To determine value of shear at any point recall … $\Delta V = \int w(x)dx$
   - If distributed loading is a curve of degree "n"; shear = degree "n +1"
   - Note: concentrated load ↑ … causes **+ve** "jump" in shear

4. Construct Moment diagram
   - Establish the "M" and "x" axes
   - Plot the value of moment at left end of the beam
   - For curvature recall … $\frac{dM}{dx} = V$
   - To determine value of shear at any point recall … $\Delta M = \int V(x)dx$
   - If shear is a curve of degree "n"; moment = degree "n +1"
   - Note: concentrated load ↻ … causes **+ve** "jump" in moment

* Note in our convention: we will draw moments on the **compression side** of the member (this is the convention typically used in the *Canada/US*).



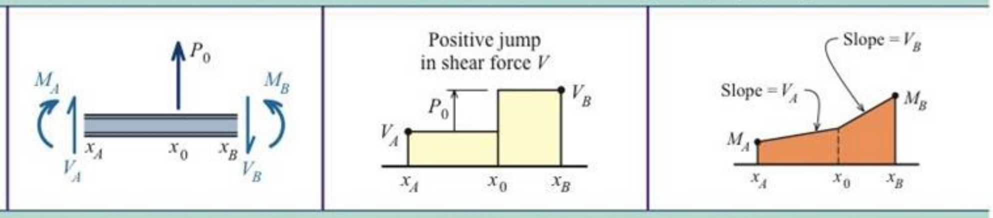
6 rules for constructing shear and moment diagrams (Philpot, 2009)

Table 7.1 Construction Rules for Shear-Force and Bending-Moment Diagrams

Equation	Load Diagram w	Shear-Force Diagram V	Bending-Moment Diagram M

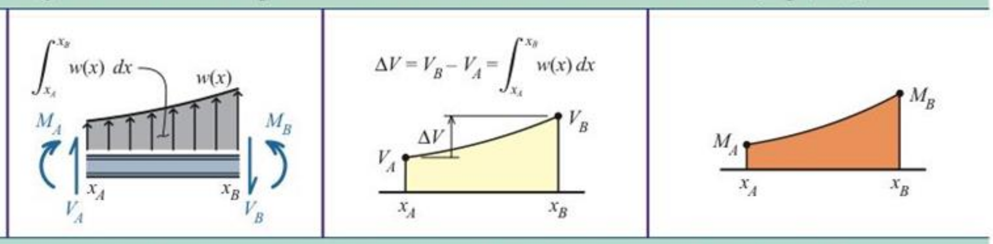
Rule 1: Concentrated loads create discontinuities in the shear-force diagram. [Eq. (7.5)]

$$\Delta V = P_0$$



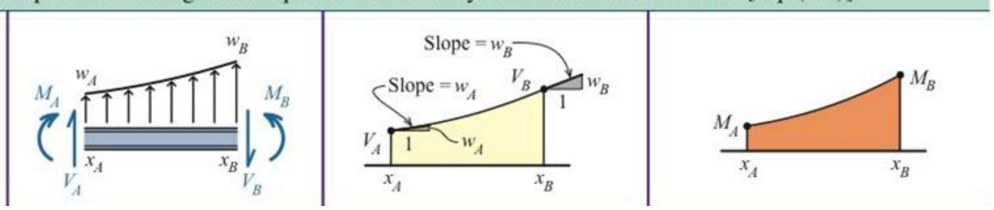
Rule 2: The change in shear force is equal to the area under the distributed-load curve. [Eq. (7.3)]

$$V_B - V_A = \int_{x_A}^{x_B} w(x)\,dx$$



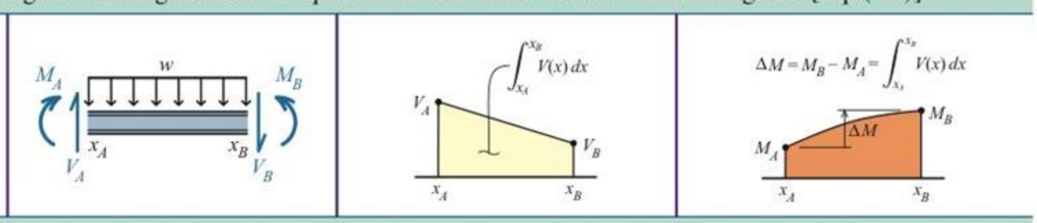
Rule 3: The slope of the V diagram is equal to the intensity of the distributed load w. [Eq. (7.1)]

$$\frac{dV}{dx} = w(x)$$



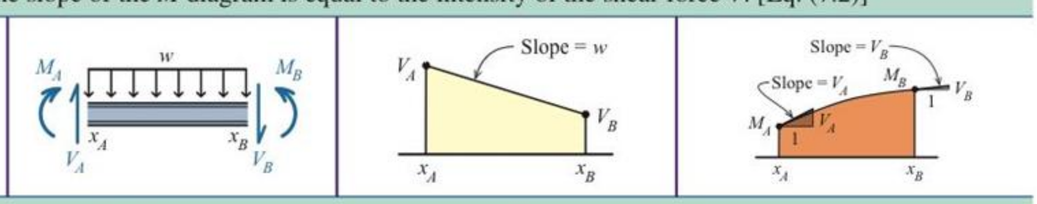
Rule 4: The change in bending moment is equal to the area under the shear-force diagram. [Eq. (7.4)]

$$M_B - M_A = \int_{x_A}^{x_B} V\,dx$$



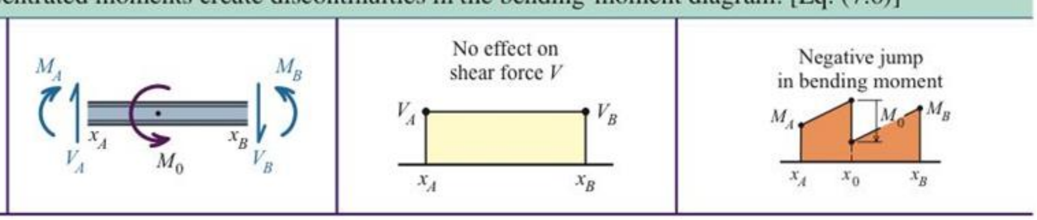
Rule 5: The slope of the M diagram is equal to the intensity of the shear force V. [Eq. (7.2)]

$$\frac{dM}{dx} = V$$



Rule 6: Concentrated moments create discontinuities in the bending-moment diagram. [Eq. (7.6)]

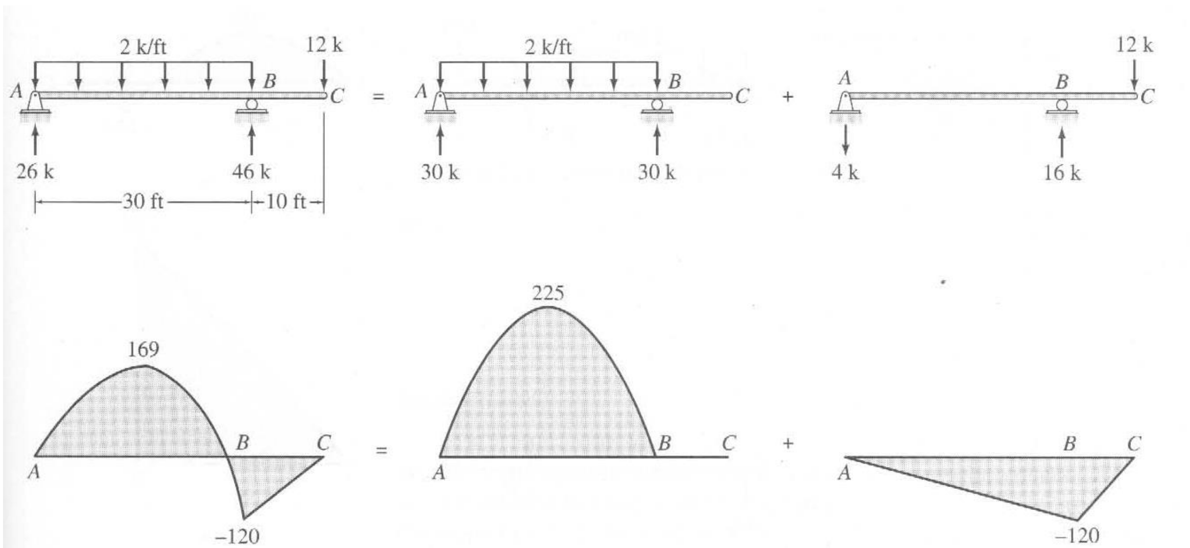
$$\Delta M = -M_0$$

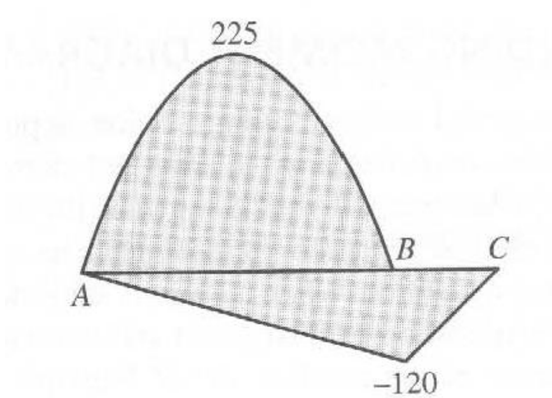


Drawing M diagrams by parts:

Using superposition:
This procedure simply involves applying each of the loads separately on the beam and constructing the corresponding bending moment diagrams.

For example, consider the beam below which has two different loadings. Rather than drawing the M diagram in one step, we will draw the M diagrams by parts by applying the two loads separately on the beam.

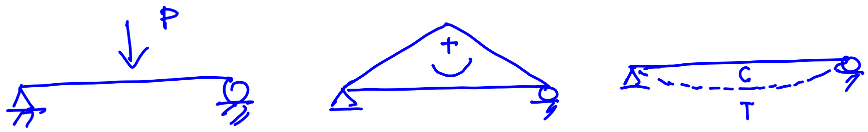




A note on sign conventions

Representing stress resultants graphically on line models is one of the most widely used features of communicating information in structural analysis. In general though, graphical representations without the use of a consistent sign convention can lead to misleading understanding.

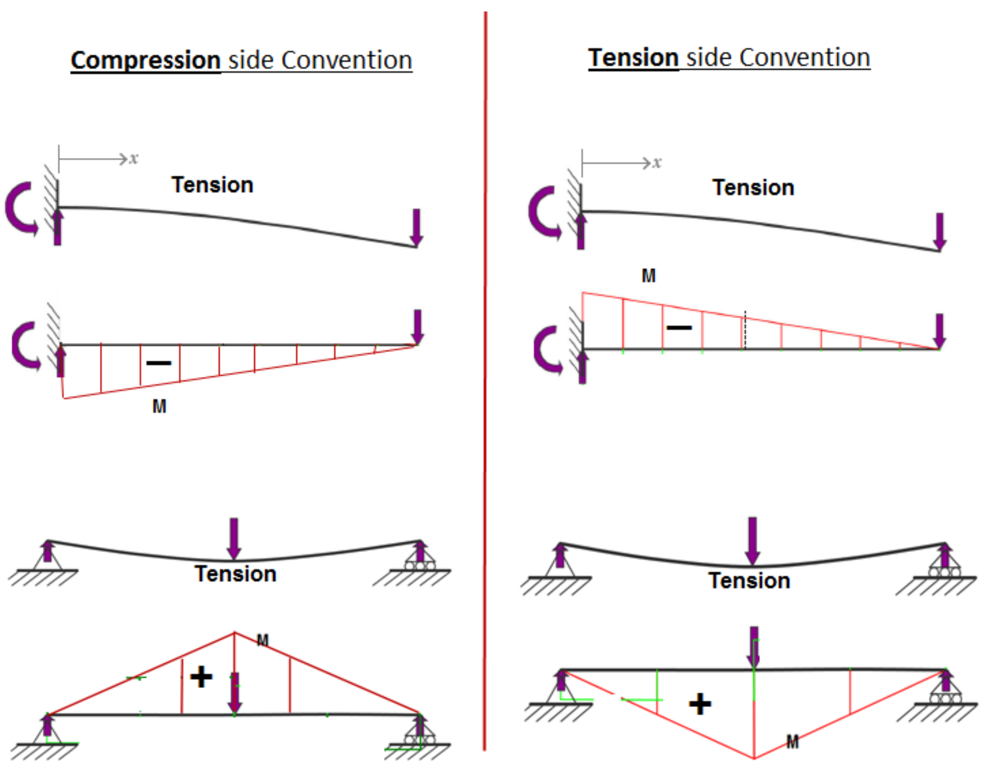
In this course we will (typically) draw moments on the **compression side** of the member (this is the convention typically used in North America).



Other engineers like to use the opposite convention - moments drawn on the **tension side** (this is the convention typically used in Europe).

It is important to understand in both conventions, positive moment remains positive and negative moment remains negative.

Therefore if I am designing a cantilever reinforced concrete beam in Canada or England, the moment diagram will be negative and I will place the reinforcement at the top of the beam.



Lecture 2.3: V & M diagrams- Frames

1. Introduction - frames
2. V and M diagrams (frames)

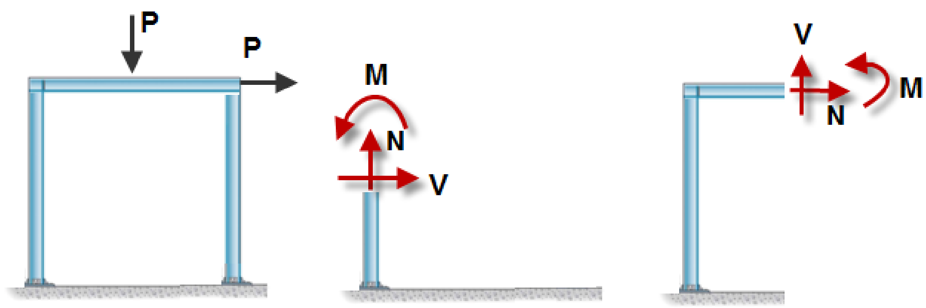
1. Introduction - Frames

The combination of beams and columns results in a frame.

A frame is an assembly of elements in which at least one member is carrying load in flexure

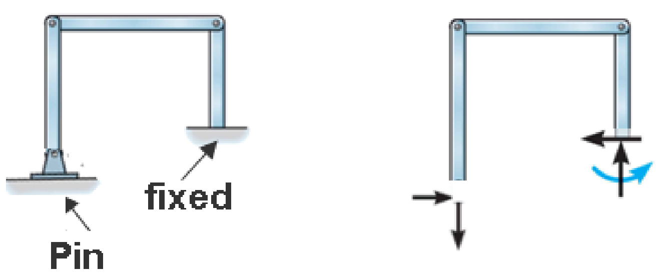
Members of a frame can develop three types of internal forces (just like beams):

- internal axial force (N) ... along the member axis
- internal shear force (V) ... perpendicular to the member
- internal bending moment (M)



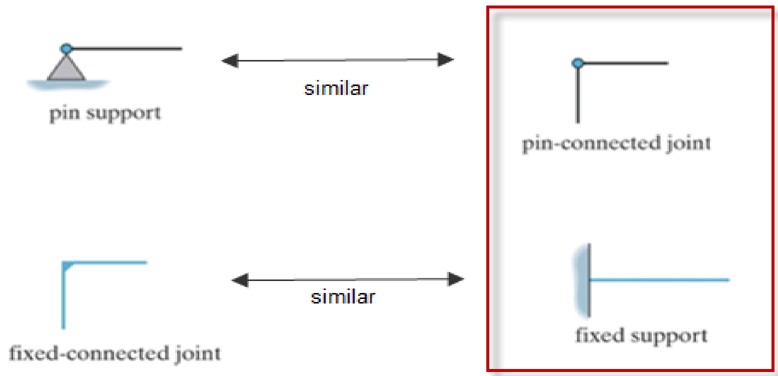
Just like beams frames can have three types of supports:

- "fixed" support
- "pin" support
- "roller" support



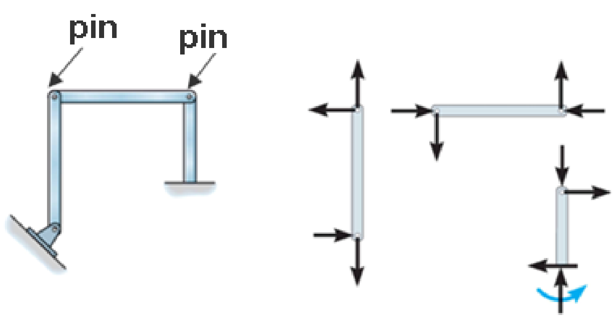
The members of a frame are attached by either:

- **pinned joints** ... typically used in a braced frame
- **rigid joints** ... typically used in a rigid frame
- we sometimes can have a mix of pin and rigid joints in the same frame



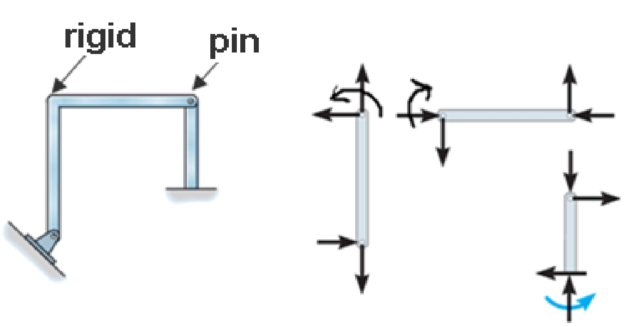
If we have a pin joint we have transfer of 2 types of forces between members:

- axial force (N)
- shear force (V)
- but M = 0
- The joint cannot resist rotation



If we have a rigid joint we have transfer of 3 types of forces between members:

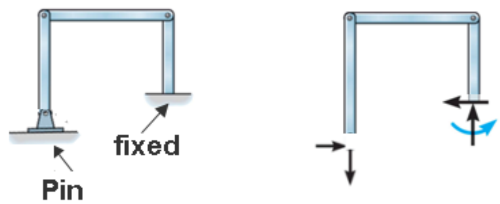
- axial force (N)
- shear force (V)
- Bending moment ($M \neq 0$)
- The joint can resist rotation (angle between adjoining members will stay at 90º)



3. V/M diagrames (Frames)

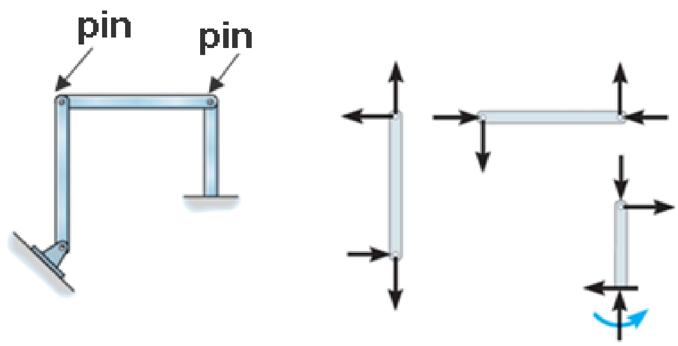
External reactions

- "fixed" support
- "pin" support
- "roller" support

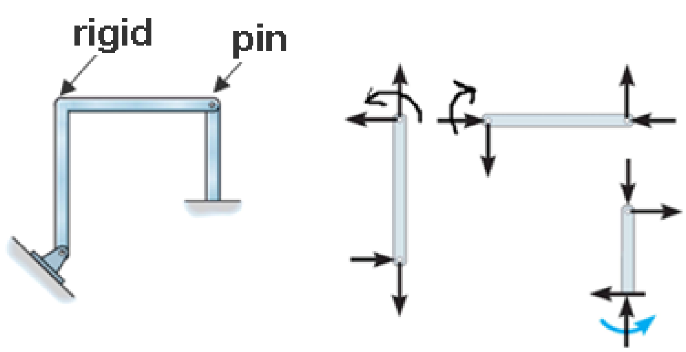


Connections

- **Pin** connection → transfer of 2 types of forces between members (N and V ; but M =0)



- **Rigid** connection → transfer of 3 types of forces between members (N and V ; and M ≠ 0)



Action-reaction principle

We note that the forces at connections follow the "action-reaction" principle:

- forces are equal and opposite
- **N** in vertical member ... becomes ....**V** in horizontal member
- **V** in vertical member ... becomes ....**N** in horizontal member
- **M** in vertical member ... will be the same value in horizontal member

Essentially the same procedure used for beams is used in the analysis of frames.

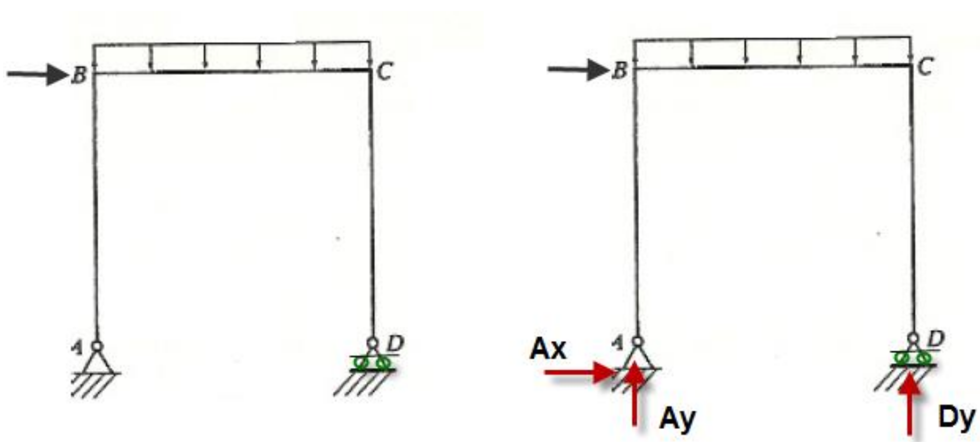
To help us, let us remember the following principles:

- When the equilibrium conditions are satisfied:
    - The external reactions of the entire structure as a whole are in equilibrium
    - Each member is in equilibrium
    - Each joint is in equilibrium

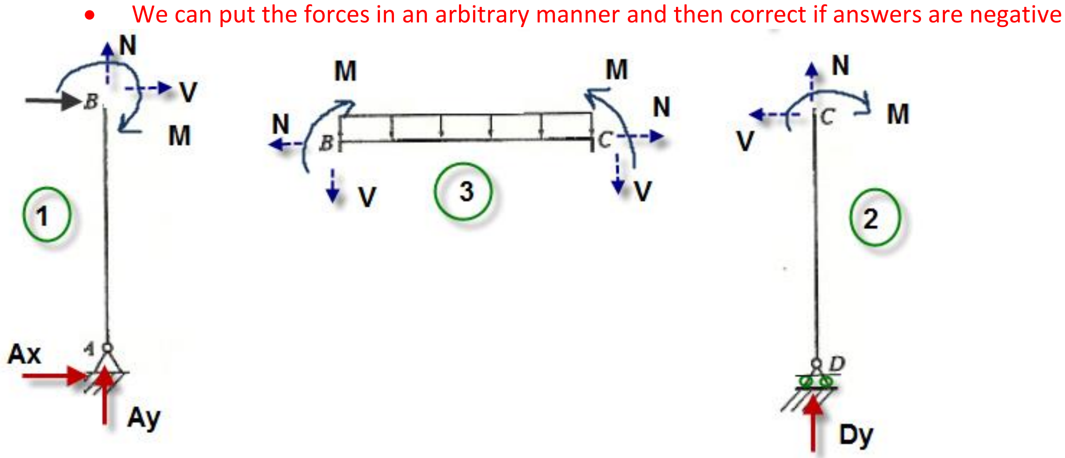
Procedure (frames)

1. Determine if the frame is **determinate** (if yes proceed…)

2. Find the **external reactions** by taking the **FBD** of the entire frame and applying the equations of equilibrium ($\sum F_x = 0$, $\sum F_y = 0$, $\sum M = 0$)
    - To start, we indicate the direction of the reactions in an arbitrary manner:
        - if positive = we chose the correct direction
        - if negative = the reaction is in the opposite direction to what we assumed

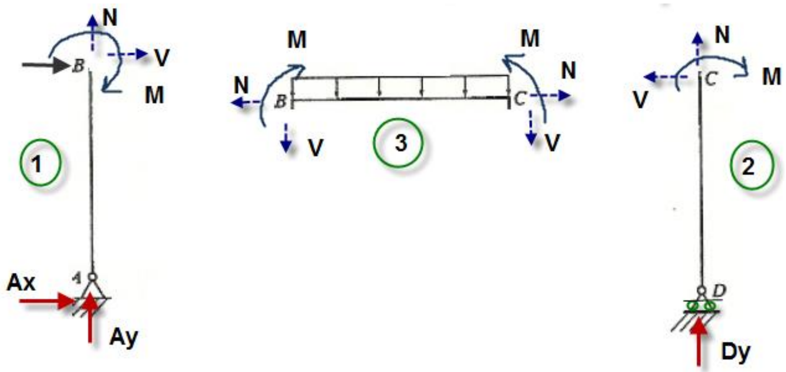


3. Cut the frame and draw the **FBD** of **each member**
    - Draw unknown internal forces (N, V, M)
    - We can put the forces in an arbitrary manner and then correct if answers are negative

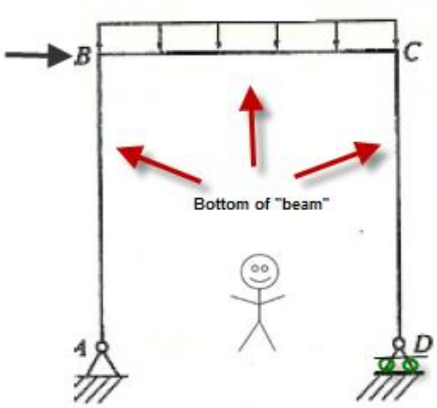


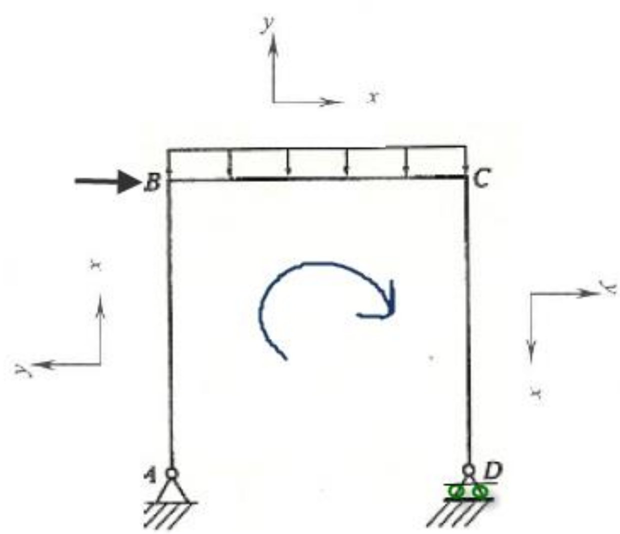
4. Apply **equations of equilibrium** on **each FBD**:
   - $\overset{+}{\rightarrow}\sum F = 0$
   - $+\uparrow \sum F = 0$
   - $+\circlearrowleft \sum M = 0$
     - We start with one of the members and try to find N,V,M:
     - If internal force = positive (+) = we chose the correct direction
     - If internal force = negative (-) = the internal force is in the opposite direction

5. To <u>transfer</u> internal loads from one member to another we use the **action-reaction** principle
   - Forces will be equal and opposite
   - **N** in a vertical member ... becomes ....**V** in a horizontal member
   - **V** in a vertical member ... becomes ....**N** in a horizontal member

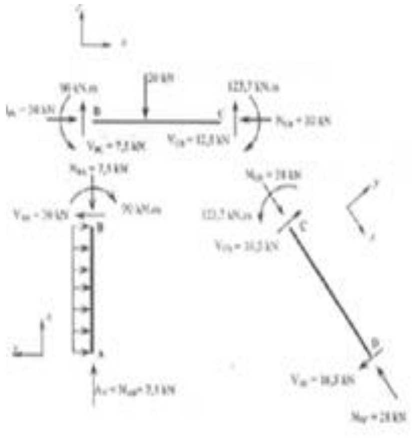


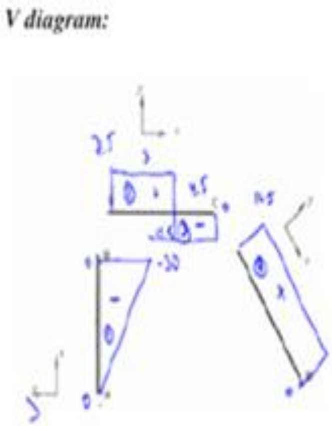
6. Draw a local axis (x'-y') for each member
   - X' = along the length of the member
   - Y'= perpendicular to X' using the right hand rule

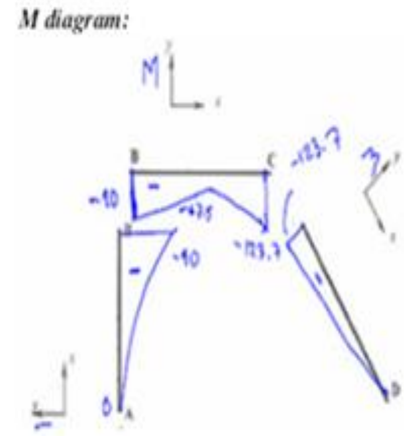




7. Construct the shear and moment diagrams for each segment <u>just as we saw for beams</u>

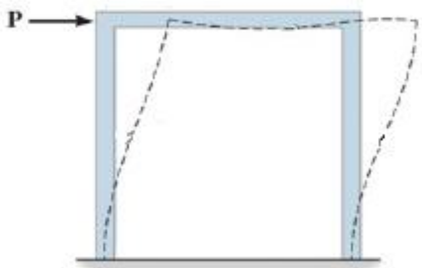


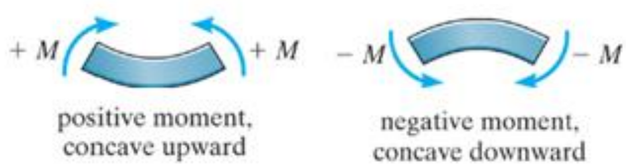


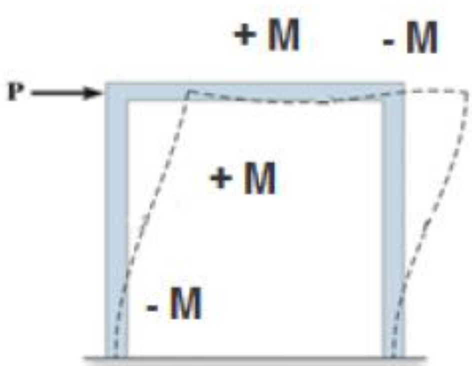


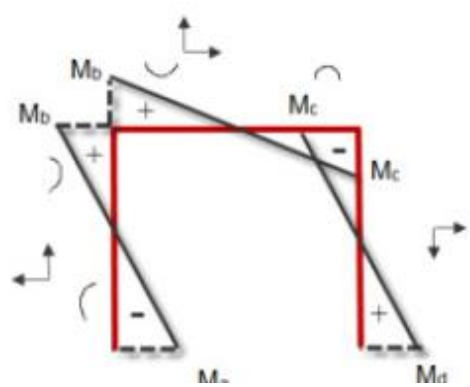
<u>Note #1</u> : At rigid joints moments must be equal.

<u>Note #2:</u> In this course we will use a convention where moments are drawn on the **<u>compression</u> face of the members** (like we do for beams). Some engineers prefer to use the opposite convention (i.e. moments drawn on the tension face of the members)



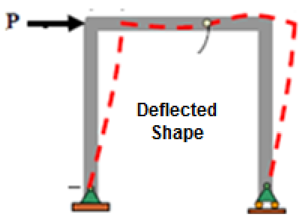
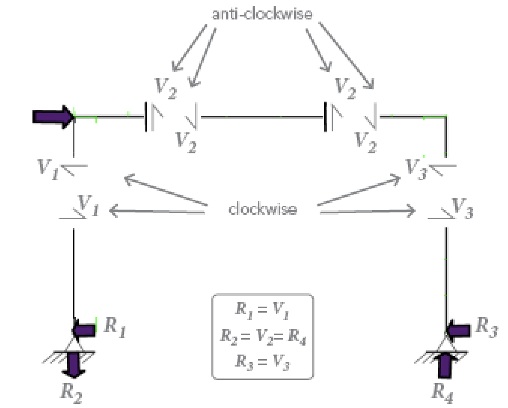
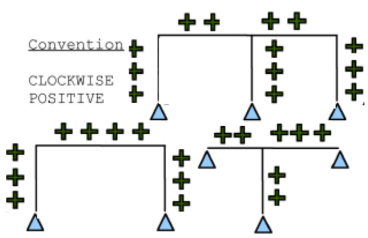
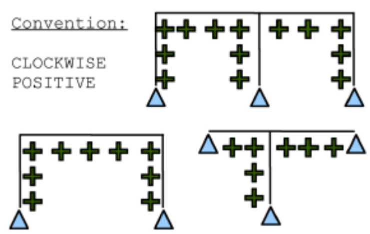
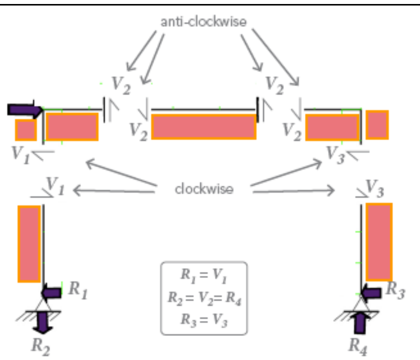
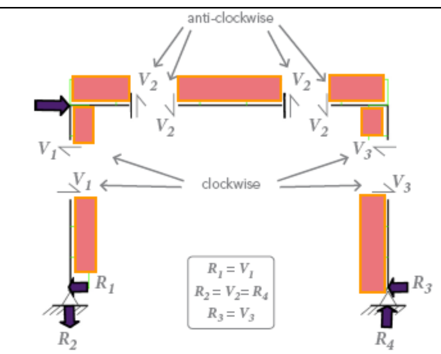
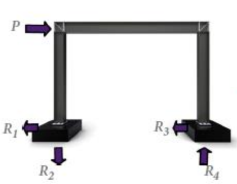
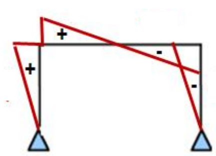
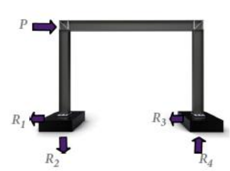
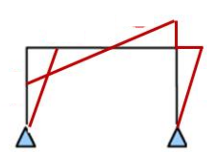






FRAMES: A note on sign conventions - Shear and Moment diagrams

<table>
<tr><th></th><th>Compression-side convention</th><th>Tension-side convention</th></tr>
<tr><td rowspan="3">Convention Shear</td><td colspan="2">A clockwise shear is taken as positive
An anti-clockwise shear is taken as negative


</td></tr>
<tr><td>A positive shear force is plotted on the Outside of the Frame.
</td><td>A positive shear force is plotted on the Inside of the Frame.
</td></tr>
<tr><td></td><td></td></tr>
<tr><td rowspan="2">Convention Moment</td><td>Moments are drawn on the Compression side</td><td>Moments are drawn on the Tension side</td></tr>
<tr><td> </td><td> </td></tr>
</table>

Lecture 2.4: Review - Trusses

1. Truss characteristics
2. Zero force members
3. Method of joints
4. Method of sections
5. Visual technique (based on method of joints)
6. Practical considerations
7. Trusses in older bridges - a primer

1. Truss characteristics

Truss is a assembly of members connected at joints and arranged into a network of triangles arranged such that all the elements in the structure carry load either in axial tension or compression

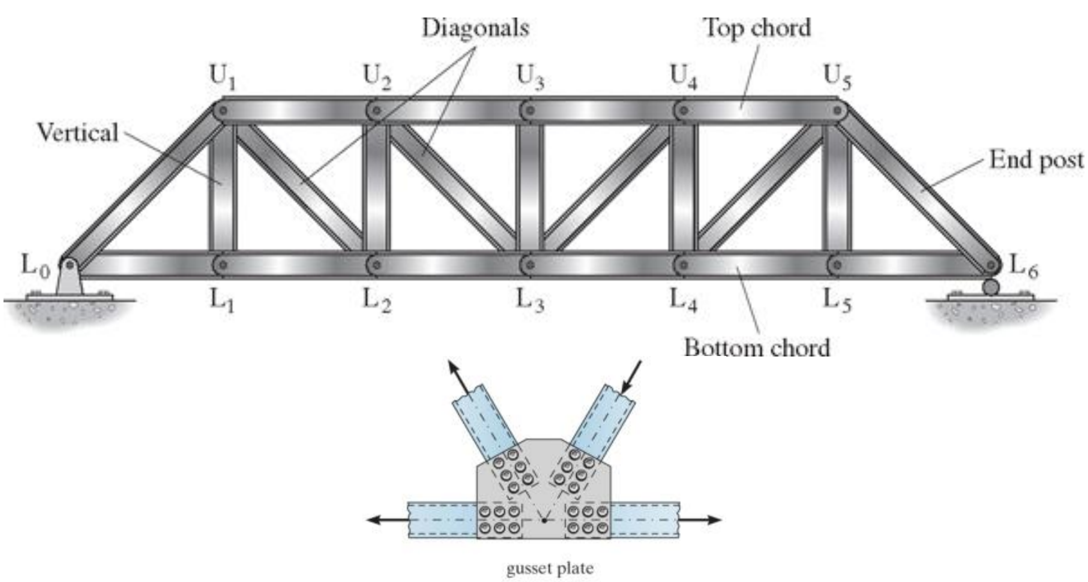


Figure: 03-01

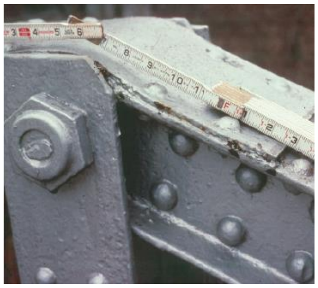
Assumptions for trusses:

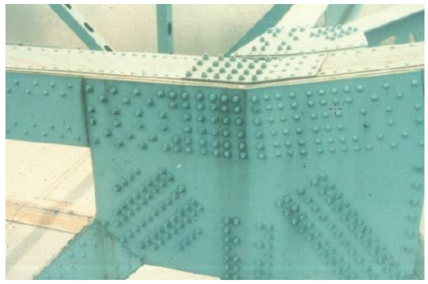
- ... (1) triangular arrangement of members
- ... (2) elements are connected at their ends by **pins** (frictionless hinges)
- ... (3) centroidal axes of members at a joint are concurrent
- ... (4) external loads are applied at joints
  - no loads in between members
  - self-weight is small compared to external loads because members are slender (so we typically ignore self-weight in analysis)
  - If self-weight is significant →separate self-weight into point load and place @ ends.

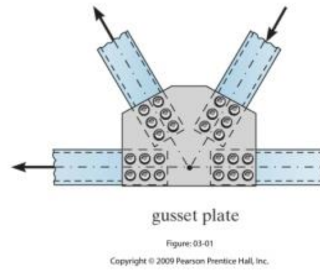
- 1+2+3+4 = all members carry pure **axial loads**
  - two-force members (C or T)
  - no moments

Members in trusses are pin-connected:

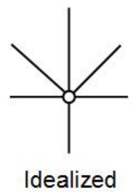
- frictionless hinges
- gusset plates with rivets/bolts

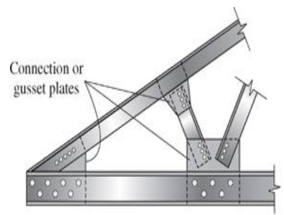




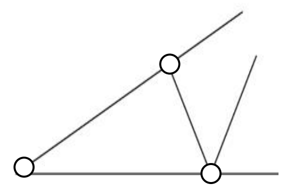


Idealization...

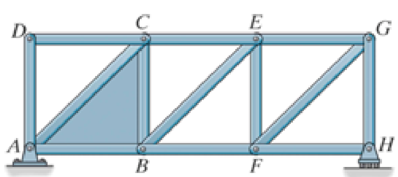


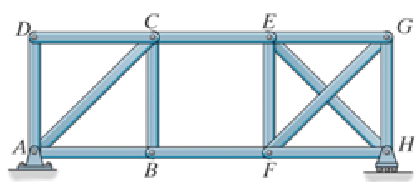


(a)

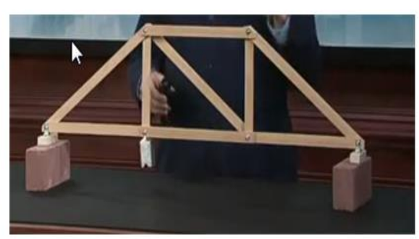

(b)

Note: trusses that have a non-triangular arrangement are **unstable.**





Why triangles ... Answer: Stability



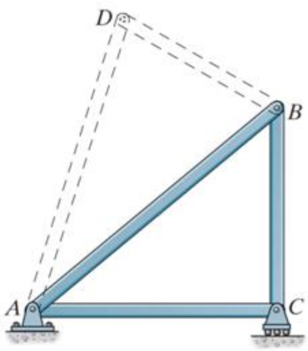
(source: Understanding Great Structures - Ressler)

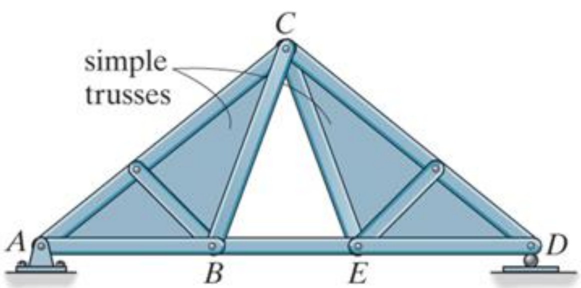
Classification of planar trusses:

1. **Simple truss**: Formed by starting with an initial triangular element and connecting to it 2 other members and a joint to form a second triangle, etc.

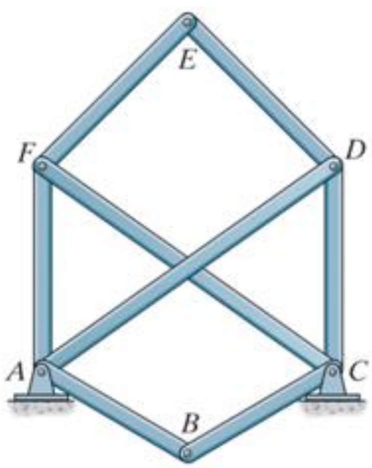
2. **Compound truss:** Formed by connecting together 2 or more simple trusses using a common joint and/or additional member

3. **Complex truss:** Those that cannot be qualified as either simple or compound





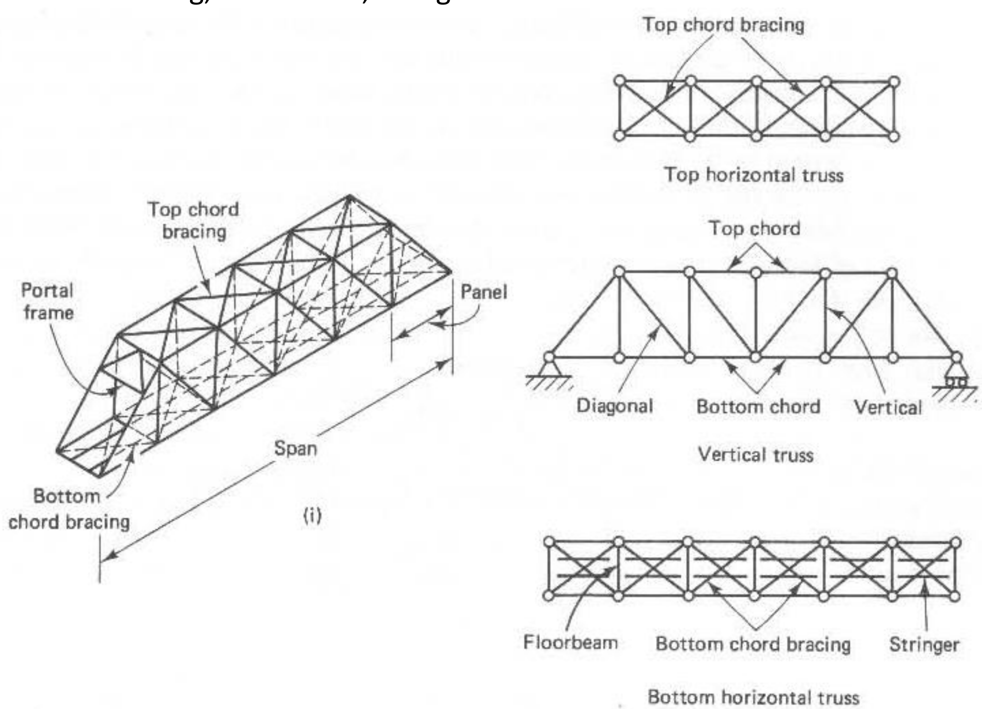
compound truss



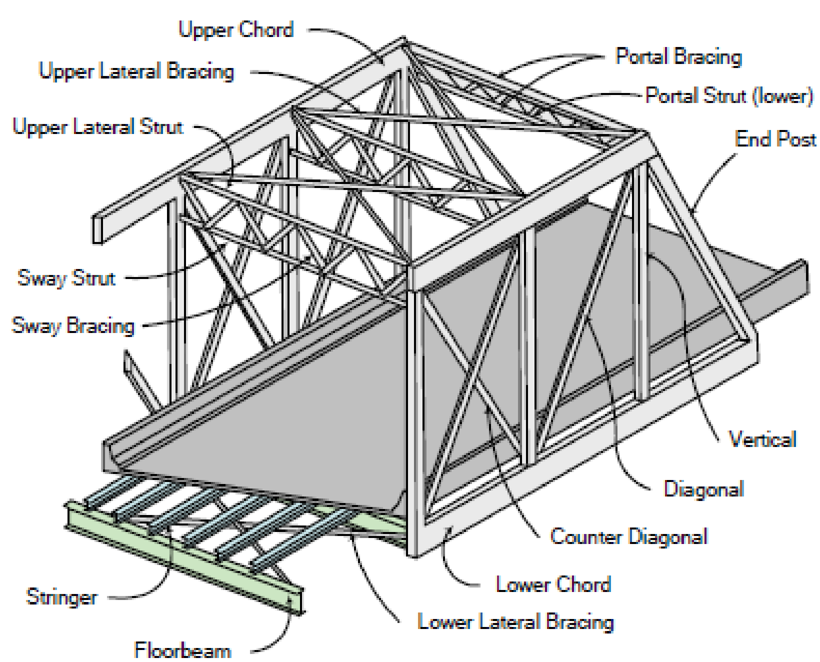
complex truss

Terminology:

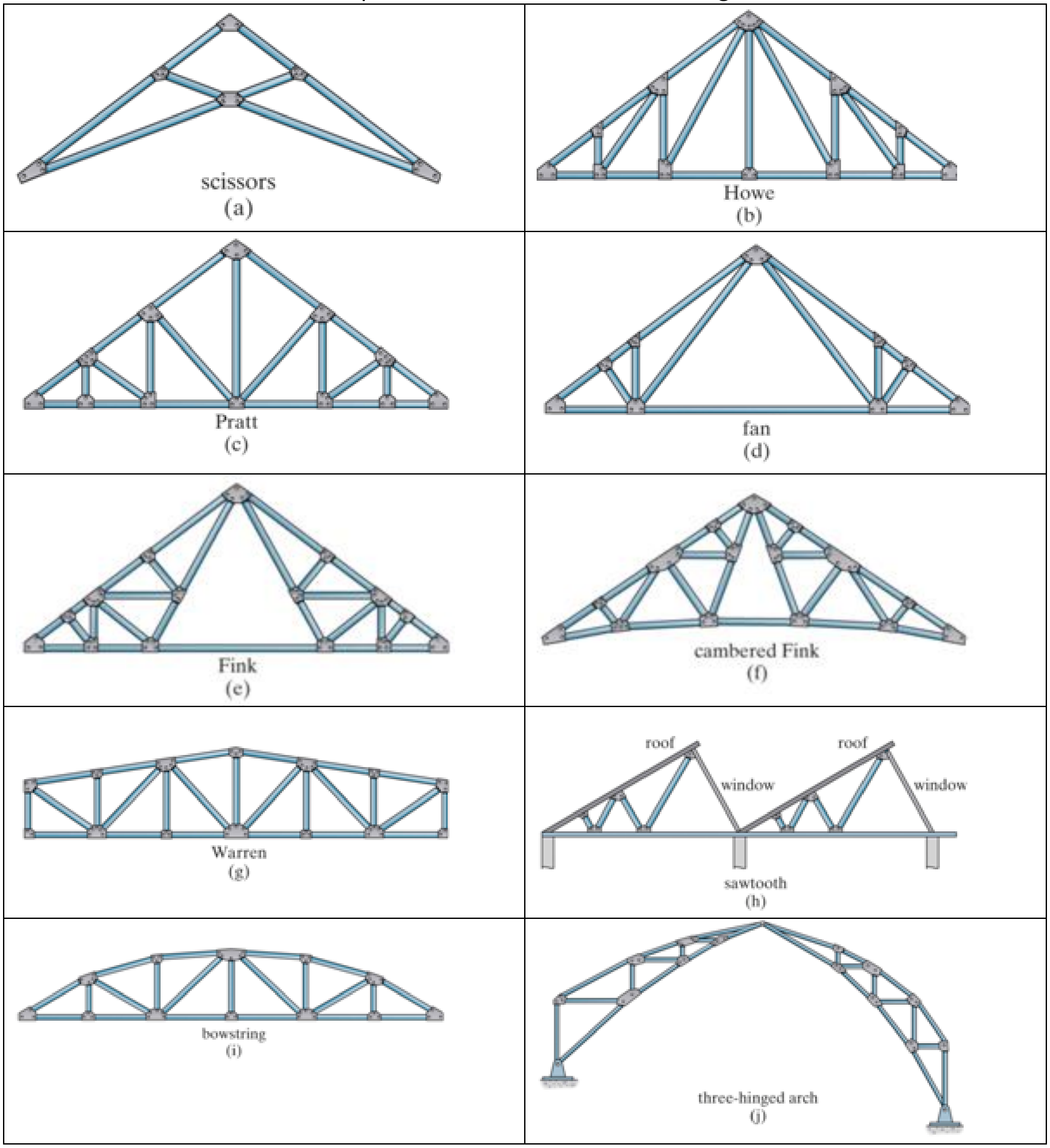
- Top chord;
- Bottom chord ;
- Diagonal ;
- Vertical.
- Other: bracing, floor beam, stringer



(Elastic analysis of structures, Kennedy, 1990)

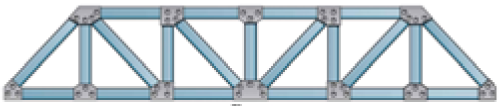
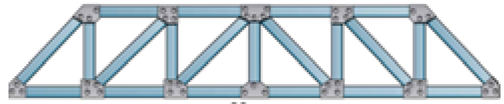
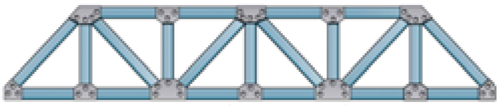
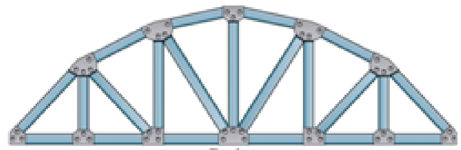
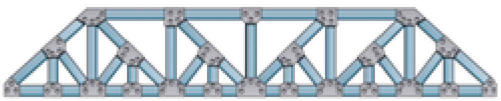
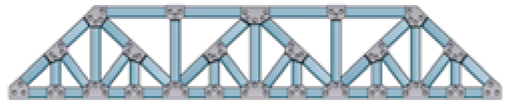
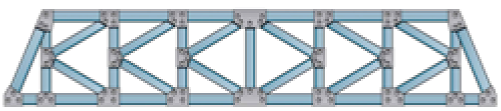


Examples of common roof truss configurations:



(Structural Analysis, Hibbeler, 2009)

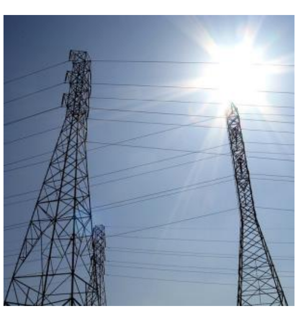
Examples of common Bridge truss configurations:

 Pratt (a)	 Howe (b)
 Warren (with verticals) (c)	 Parker (d)
 Baltimore (e)	 subdivided Warren (f)
 K-truss (g)	

(Structural Analysis, Hibbeler, 2009)

3 dimensional trusses are called "space trusses"

(Structural Analysis, Hibbeler, 2009)



Sign convention

Member in **<u>Tension</u>** will lengthen .... positive (+) force

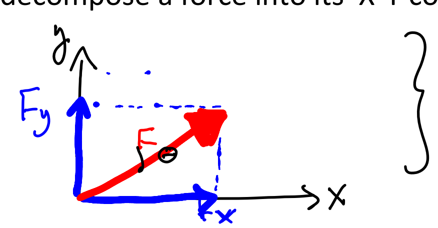
This member is acting in tension.
(At the joints the member is being stretched and is pulling back.)

Member in **<u>Compression</u>** will shorten .... negative (-) force

This member is acting in compression.
(At the joints the member is being compressed and is pushing back.)

Notes

- Since we know the direction of the force in the member
  - only unknown → magnitude of the force (i.e. C or T)
  - "two-force member"
- Recall that we can decompose a force into its X-Y components:



$$F = \sqrt{F_x^2 + F_y^2}$$

$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$

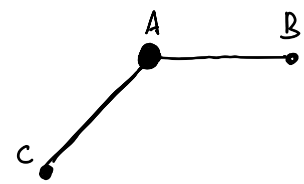
2. Zero-force members

Zero-force member:

- a member that does not develop an internal axial force due to a particular loading condition
- Truss analysis can be simplified if we identify zero-force members

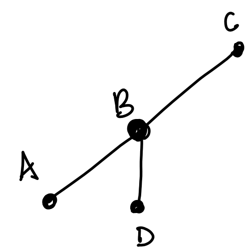
Case 1:

- 2 non-collinear members are connected at a joint...
- … and there is no external load/reaction at the joint
- … both members have N = **zero**

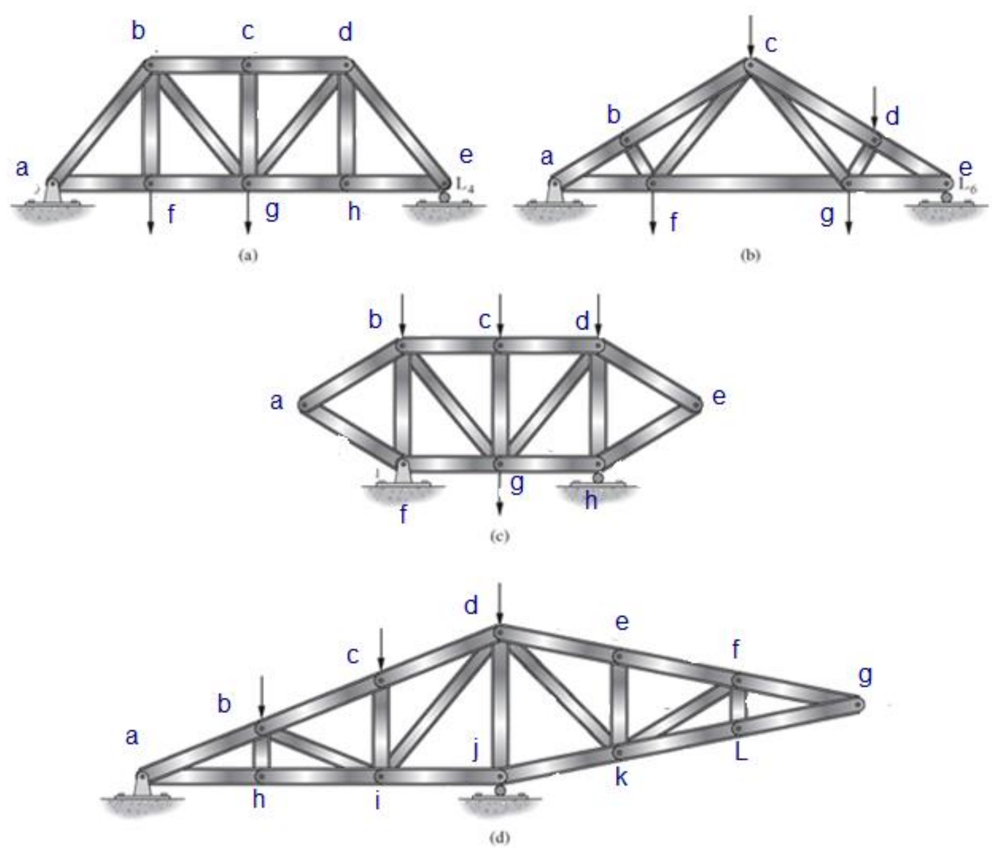


Case 2:

- 3 members
- …2 are collinear + one is not
- … and there is no external load/reaction at the joint
- … the non-collinear member has N = **zero**



Example: Identify zero-force members ….



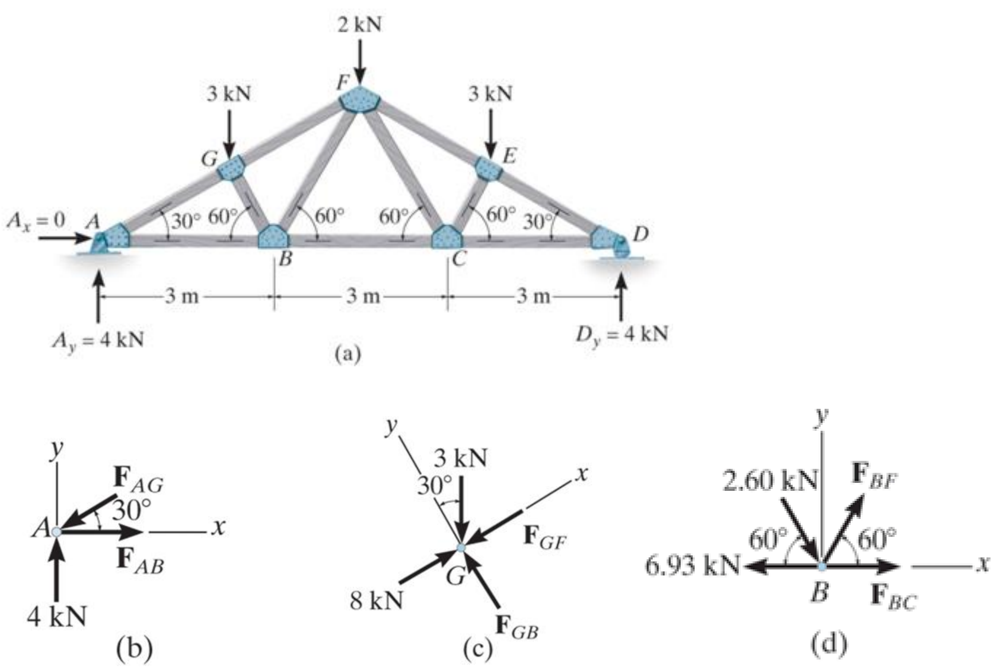
3. Method of joints

- Consists of applying equilibrium to the FBD of **joints**

- Principle #1: Equations of equilibrium must be satisfied at each joint
- Principle #2: The member forces align with the axis of each member

Procedure:

1. Is truss **determinate** ? (if yes, proceed …)
2. Draw FBD of entire truss and determine support reactions
3. Identify any zero-force members if present
4. Draw FBD of each joint:
    - Start at a joint that has 2 unknowns or less
    - Initially assume that all members are in tension (T : +)
        - If get negative value → C
5. Use equilibrium equations to determine forces ($\sum F_x = 0$, $\sum F_y = 0$)
6. Work your way from joint to joint (select joints with 2 unknowns or less)
7. repeat until all member forces are known
8. you can check your answer by taking equilibrium at a joint you have not yet analyzed



4. Method of sections

- Consists of passing an "imaginary cut" and applying equilibrium to FBD of the **section**

- Principle #1: Equations of equilibrium must be satisfied for each imaginary section
- Principle #2: The member forces align with the axis of each member

- This method is most useful if only a few member forces need to be determined

Procedure:

1. Is truss **determinate** ? (if yes, proceed …)
2. Draw FBD of entire truss … and determine support reactions (if required)
3. Identify any zero-force members if present
4. Pass a section that passes through members whose force is required
   - But no more than 3 unknown member forces
   - The section cuts the truss into 2 parts
5. Select the section that will require least amount of computational effort:
   - … draw the FBD of the section
   - Initially assume that all members are in tension (T : +)
     - If get negative value → C
6. Use equilibrium to determine forces ($\sum F_x = 0$, $\sum F_y = 0$, $\sum M = 0$)