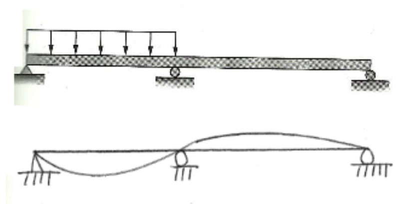
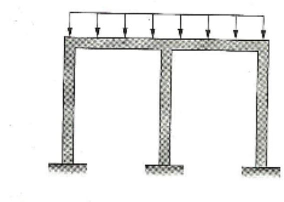
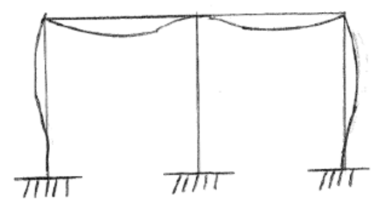
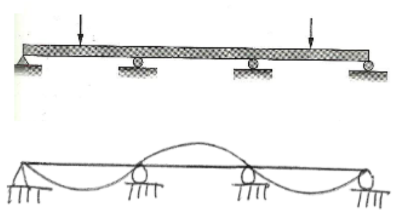
# EXAMPLES
## 3.1 Deflections
## (Drawing deflected shape)

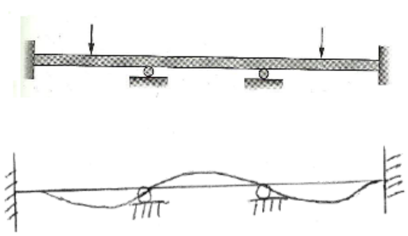
**Draw the deflected shape for the beams and frames below:**

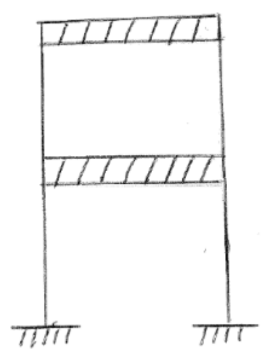


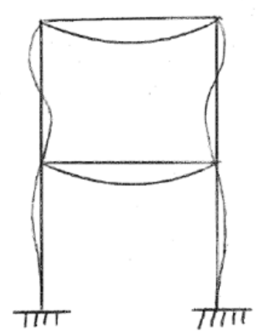


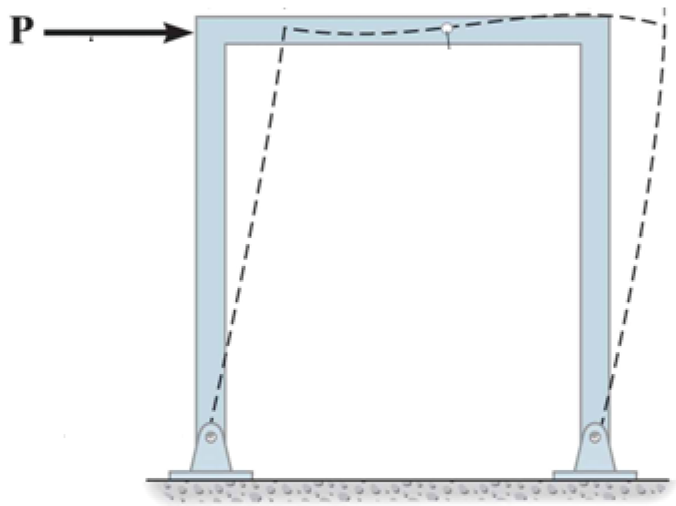








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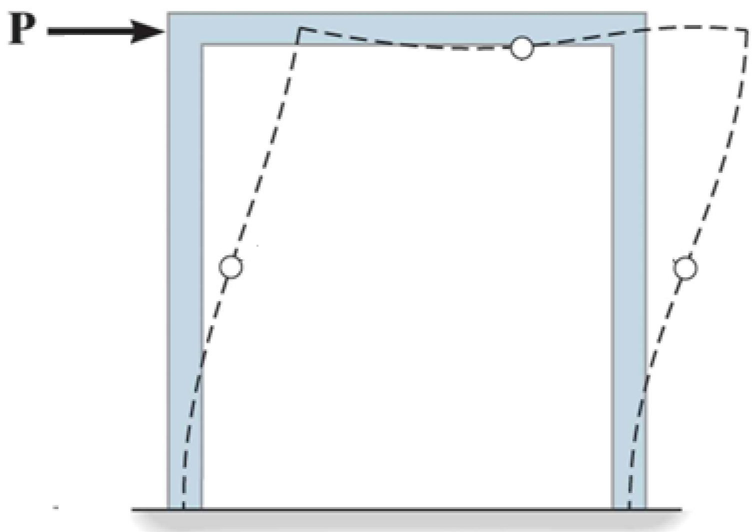
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**EXAMPLES**
**3.2A Deflections**
**(Moment-Area method)**

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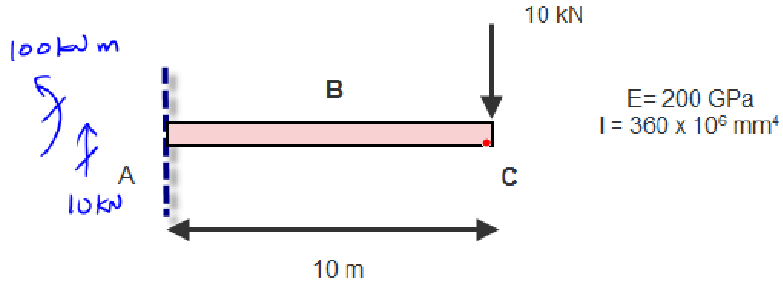
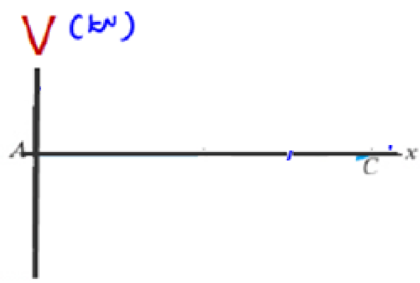
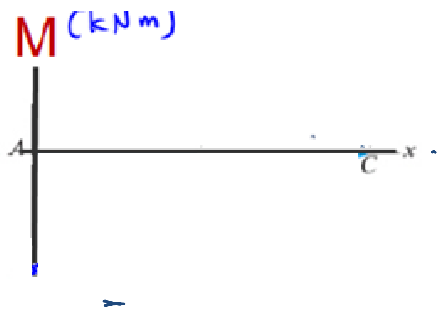
***Example 1:*** Determine the **slope** at "b" and "c" (free end) of the cantilever beam.

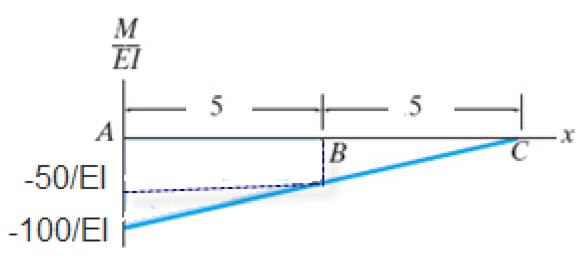



Draw shear and moment diagram ...

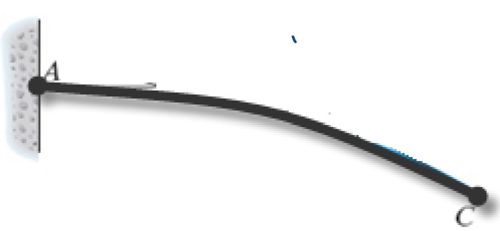


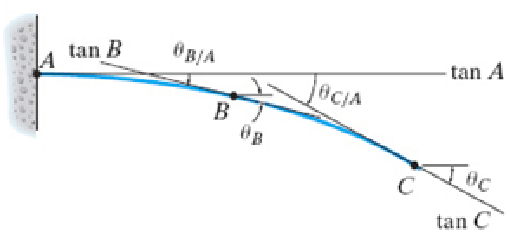




Draw M/EI diagram...

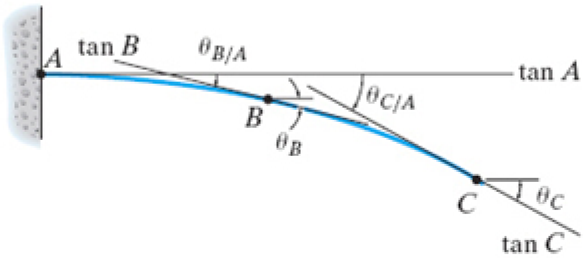


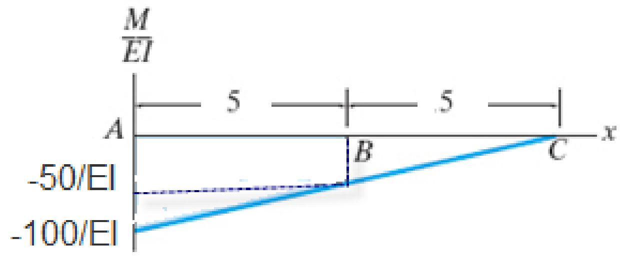

Draw an exaggerated elastic curve...



Apply theorem 1 ...




Determine the **slope at point B:**

$$\theta_B = \theta_{B/A} = -\left(\frac{50kNm}{EI}\right)(5m) - \frac{1}{2}\left(\frac{100kNm}{EI} - \frac{50kNm}{EI}\right)(5m) = -\frac{375kNm^2}{EI}$$

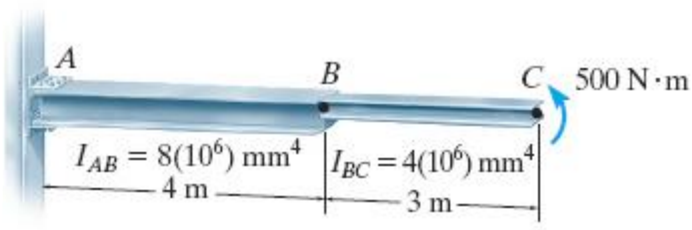
$$-\frac{375kNm^2}{[200(10^6)kN/m^2][360(10^6)(10^{-12})m^4]} = -0.00521\,rad$$

Determine the **slope at point C:**

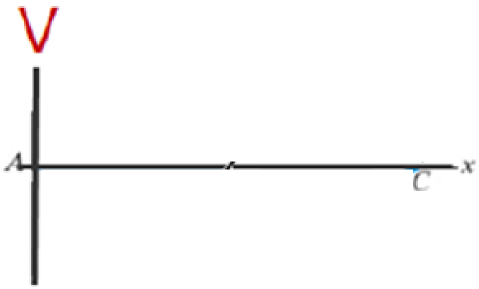
$$\theta_C = \theta_{C/A} = \frac{1}{2}\left(-\frac{100kNm}{EI}\right)(10m) = -\frac{500kNm^2}{EI}$$

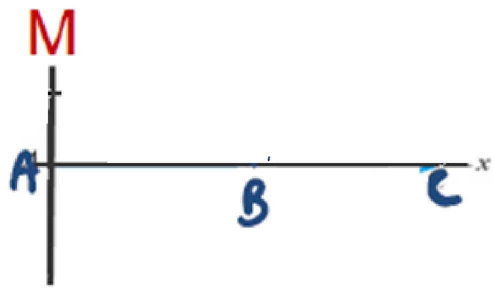
$$= \frac{-500kNm^2}{[200(10^6)kN/m^2][360(10^6)(10^{-12})m^4]} = -0.00694\,rad$$

***Example 2:*** Determine the **deflection** at points "B" and "C" of the cantilever beam. E = 200 GPa.

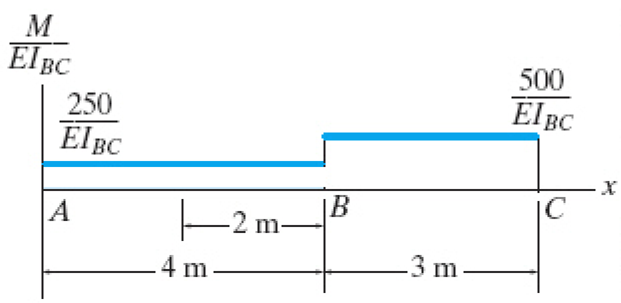



Draw shear and moment diagram ...



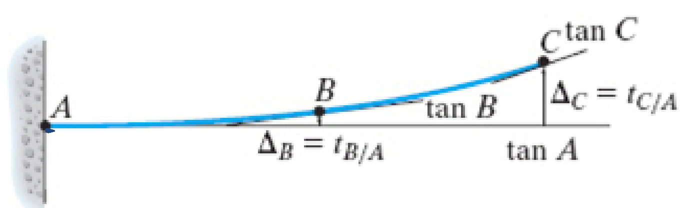




Draw M/EI diagram...




$EI_{AB} = 2EI_{BC}$

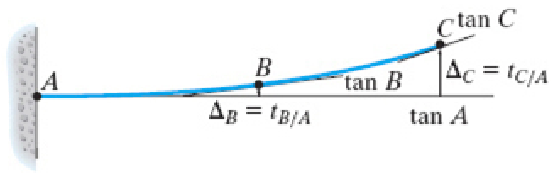
Draw an exaggerated elastic curve...

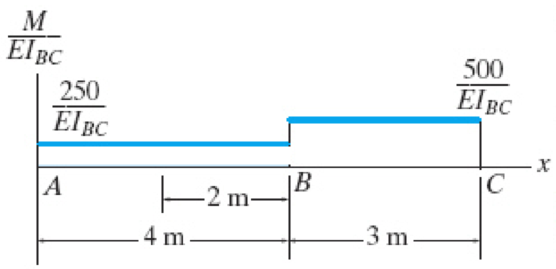



$t_{/}$ = deviation of ___ ___________
w.r.t ___ ___________

We calculate moment arm from ___

Determine the **Deflection at point B:**



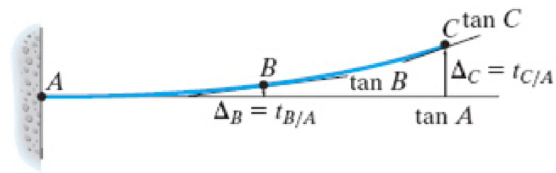



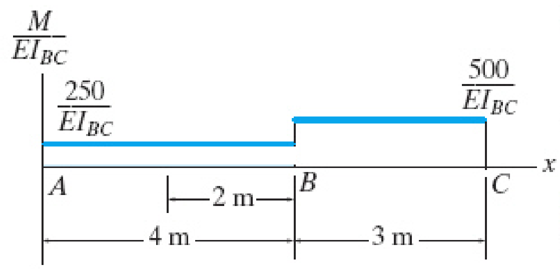

$$\Delta_B = t_{B/A} = \left[\frac{250\ \text{N}\cdot\text{m}}{EI_{BC}}(4\ \text{m})\right](2\ \text{m}) = \frac{2000\ \text{N}\cdot\text{m}^3}{EI_{BC}}$$

$$\Delta_B = \frac{2000\ \text{N}\cdot\text{m}^3}{[200(10^9)\ \text{N/m}^2][4(10^6)\ \text{mm}^4(1\ \text{m}^4/(10^3)^4\ \text{mm}^4)]}$$
$$= 0.0025\ \text{m} = 2.5\ \text{mm}.$$

Since the answer is positive it indicates that point B lies above the tangent at A (as assumed)

Determine the **Deflection at point C:**



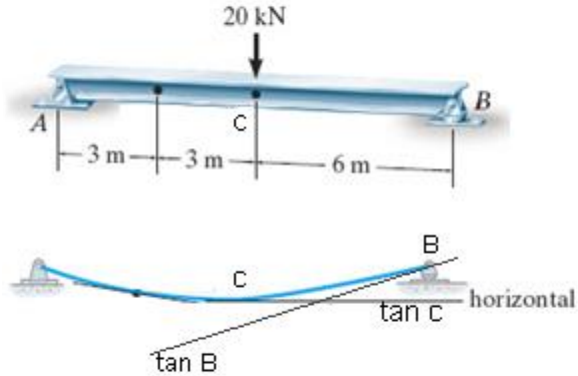

$$\Delta_C = t_{C/A} = \left[\frac{250\ \text{N}\cdot\text{m}}{EI_{BC}}(4\ \text{m})\right](5\ \text{m}) + \left[\frac{500\ \text{N}\cdot\text{m}}{EI_{BC}}(3\ \text{m})\right](1.5\ \text{m})$$
$$= \frac{7250\ \text{N}\cdot\text{m}^3}{EI_{BC}} = \frac{7250\ \text{N}\cdot\text{m}^3}{[200(10^9)\ \text{N/m}^2][4(10^6)(10^{-12})\ \text{m}^4]}$$
$$= 0.00906\ \text{m} = 9.06\ \text{mm}$$

Since the answer is positive it indicates that point C lies above the tangent at A (as assumed)

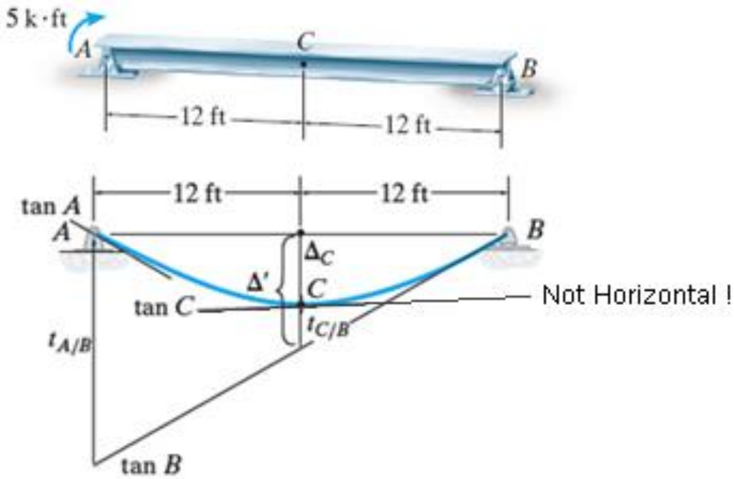
**Often the moment-area theorems will not give us the slope and deflection directly. In these cases we need to use the theorems, geometry and knowledge of similar triangles to solve the problems …**

***Some illustrative examples***

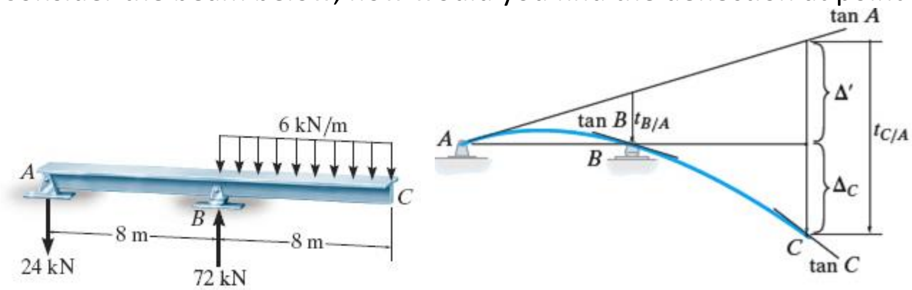
Consider the beam below, how would you find the deflection at point C ?



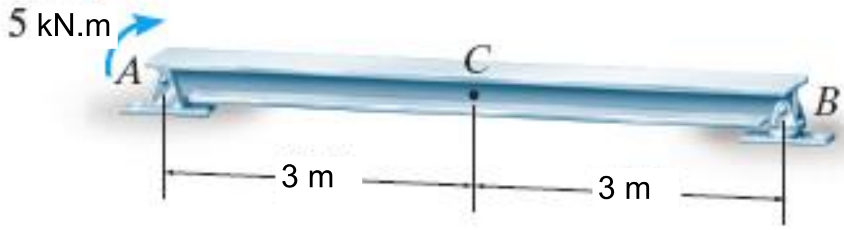

Consider the beam below, how would you find the deflection at point C ?




Consider the beam below, how would you find the deflection at point C?

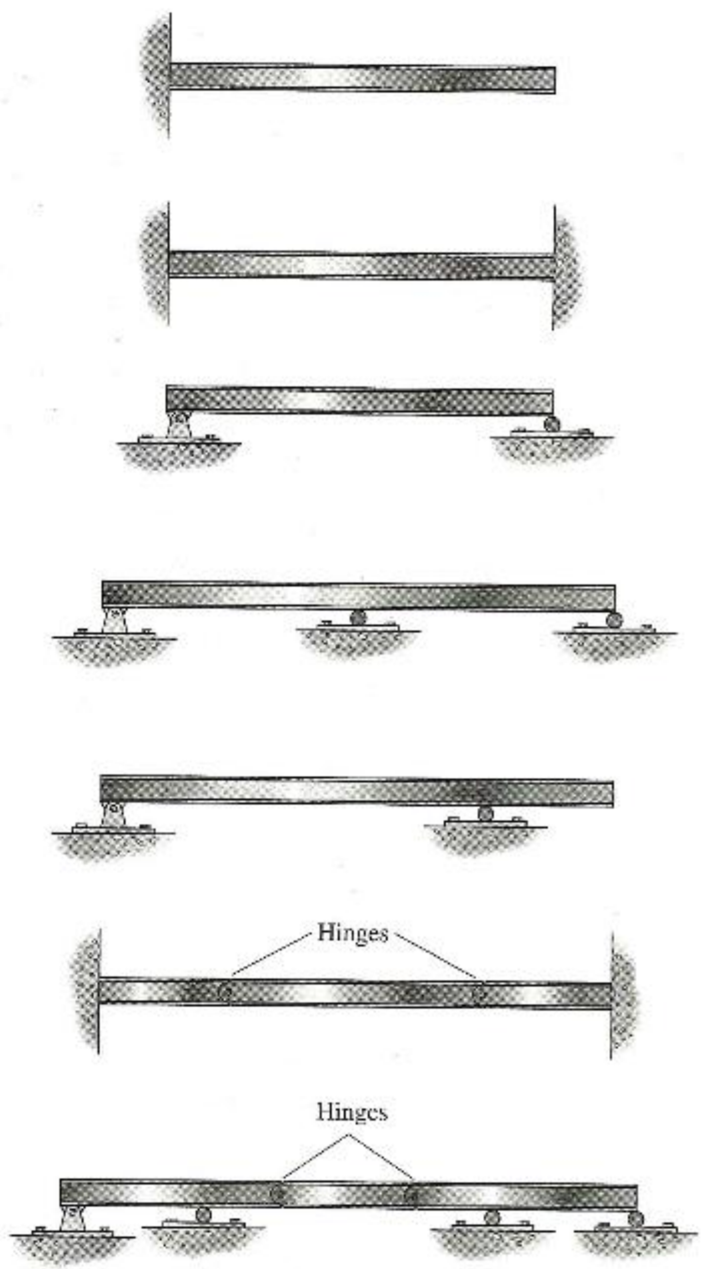


***Example 3:*** Determine the **deflection** of point C. E = 200 GPa, I= $4X10^{6}$ $mm^{4}$



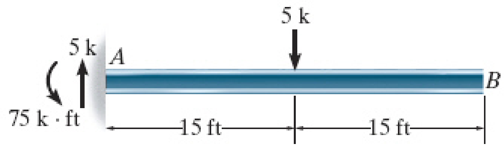
**EXAMPLES**
**3.2B Deflections**
**(Conjugate-Beam method)**

***Example 1: Draw the corresponding conjugate beams…***



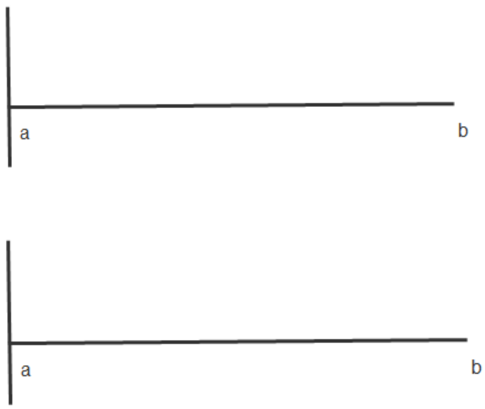
***Example 1:***

Determine the slope and deflection at point $B$ of the steel beam shown in Fig. 8–24$a$. The reactions have been computed. $E = 29(10^3)$ ksi, $I = 800 \text{ in}^4$.

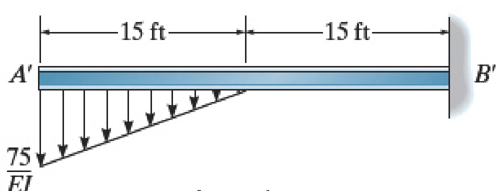



real beam
(a)

***V, M diagrams***

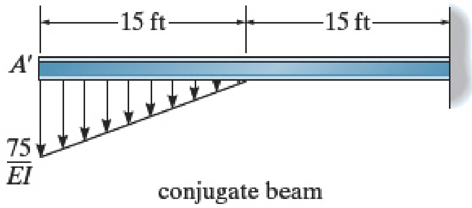



***Draw conjugate beam with appropriate supports and M/EI loading***

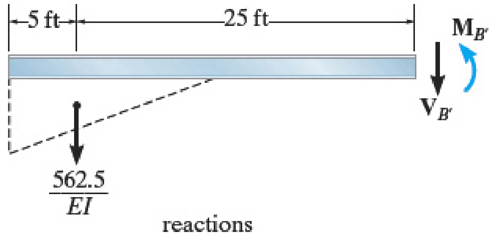



conjugate beam
(b)

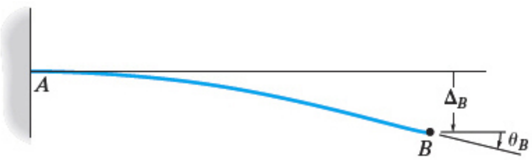
***Analyze conjugate beam***




conjugate beam
(b)




reactions




(d)

$$+\uparrow \Sigma F_y = 0; \qquad -\frac{562.5\ \text{k}\cdot\text{ft}^2}{EI} - V_{B'} = 0$$

$$\theta_B = V_{B'} = -\frac{562.5\ \text{k}\cdot\text{ft}^2}{EI}$$

$$= \frac{-562.5\ \text{k}\cdot\text{ft}^2}{29(10^3)\ \text{k/in}^2(144\ \text{in}^2/\text{ft}^2)800\ \text{in}^4(1\ \text{ft}^4/(12)^4\ \text{in}^4)}$$

$$= -0.00349\ \text{rad}$$

$$\circlearrowleft + \Sigma M_{B'} = 0; \qquad \frac{562.5\ \text{k}\cdot\text{ft}^2}{EI}(25\ \text{ft}) + M_{B'} = 0$$

$$\Delta_B = M_{B'} = -\frac{14\,062.5\ \text{k}\cdot\text{ft}^3}{EI}$$

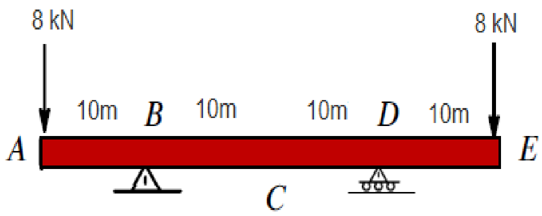
$$= \frac{-14\,062.5\ \text{k}\cdot\text{ft}^3}{29(10^3)(144)\ \text{k/ft}^2[800/(12)^4]\ \text{ft}^4}$$

$$= -0.0873\ \text{ft} = -1.05\ \text{in.}$$

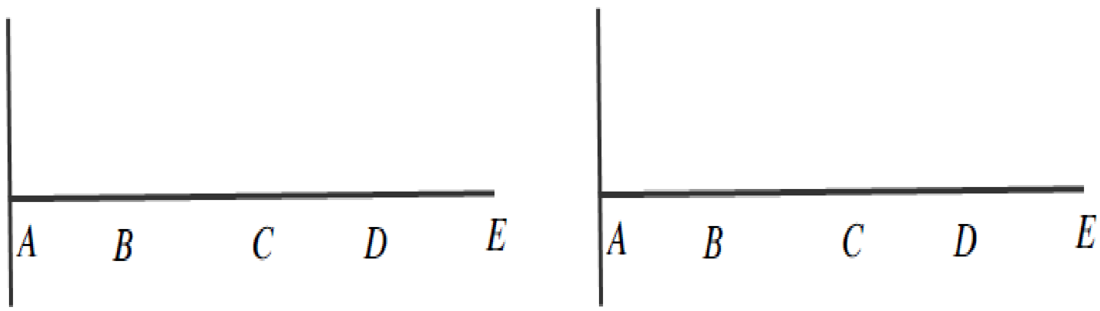
The negative signs indicate the slope of the beam is measured clockwise and the displacement is downward

*Example 2:*

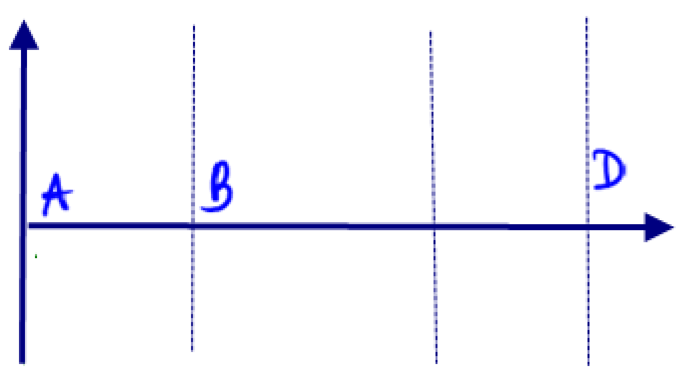
Determine the ***slope @ B*** and the ***displacement @ C*** for the beam shown below. Give your answers in terms of $EI_{AB}$ Note that: $EI_{BD} = 2\,EI_{AB}$ and $EI_{AB} = EI_{DE}$



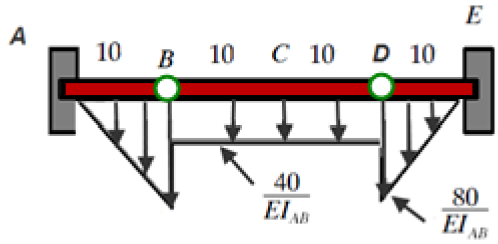

***V, M diagrams***



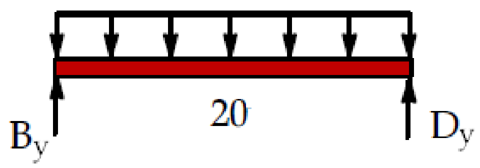

***Draw M/EI diagram :***



***Conjugate beam with appropriate loading & analyze:***




**Determine slope @ B ...** $\Theta_b = V_b'$ (shown as $B_y'$ in the figure)




**Determine deflection @ C ...** $\Delta_c = M_c'$