Lecture 5.1: Force method (Beams and Frames)

Lecture outline:

1. Advantages and disadvantages of indeterminate structures
2. Important principles
3. Force method: Introduction
4. Force method – Beams/frames

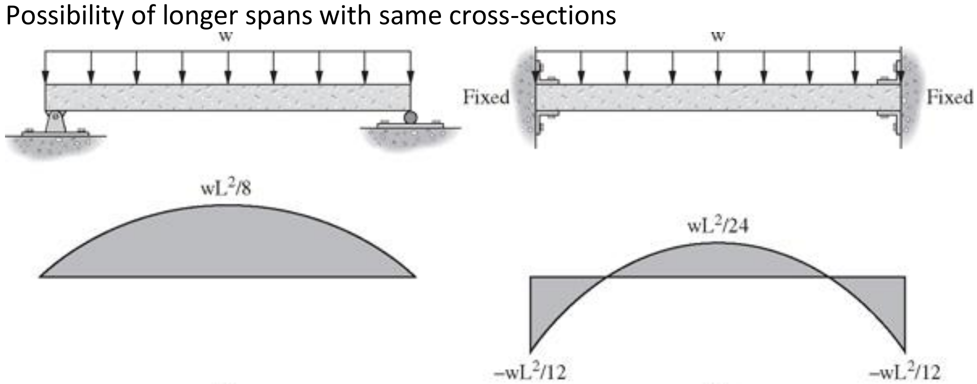
1. Advantages and disadvantages of indeterminate structures:

Advantages

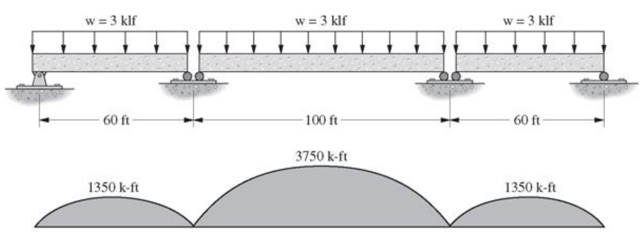
Savings in material costs ($)

Smaller moments = Smaller stresses, greater stiffness = possible savings in material costs

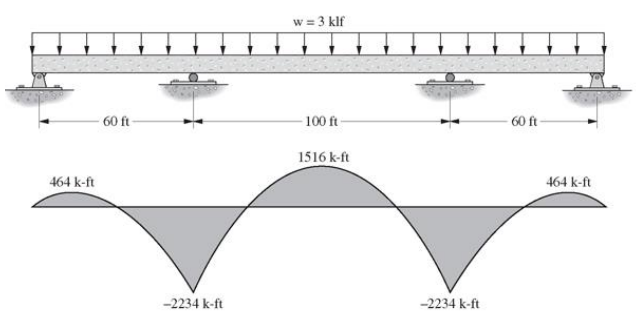
Possibility of longer spans with same cross-sections



(a) (b)



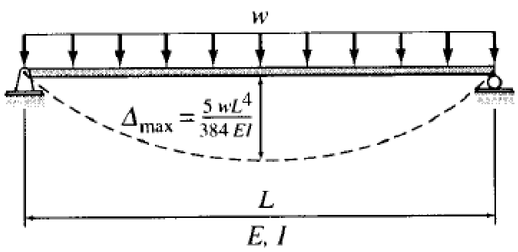
(a) Moment diagrams if three simple beams are used



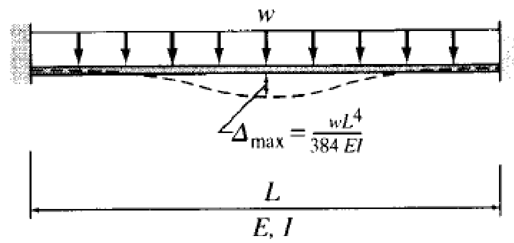
(b) Moment diagrams if one continuous beam is used

Greater rigidity/ stiffness

Indeterminate structures are typically more rigid and have smaller deflections .



Statically Determinate Beam

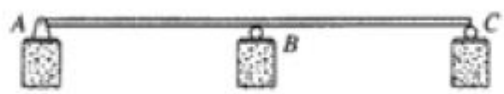


Statically Indeterminate Beam

Larger safety factor:

When indeterminate structures are over-stressed they have the ability to redistribute those stresses to less stressed areas = Ability to redistribute loads in case of faulty design

Statically Determinate Beam



Statically Indeterminate Beam

More architectural freedom

Many attractive structures such as arches are indeterminate.

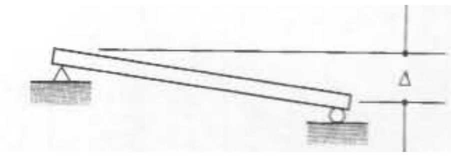
Also with computers it is now possible to analyze very highly indeterminate structures.

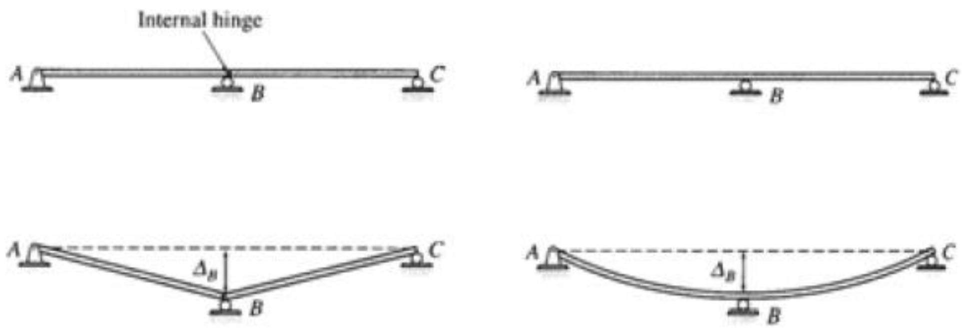
Disadvantages

Support settlements

Statically indeterminate are not desirable when there are poor soil conditions because even minor support settlements may cause significant increases in bending stresses, shear forces etc...

Must account for differential settlements:

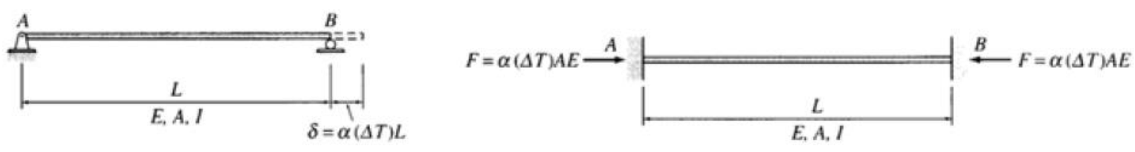




Statically Determinate Beam Statically Indeterminate Beam

Development of other stresses

Temperature variations and **fabrication errors** may cause additional stresses to develop



Statically Determinate Beam Statically Indeterminate Beam

Difficulty in analysis and design

To be able to analyze indeterminate structures we will need to know not only member dimensions but also cross-sectional and material properties.

This poses a problem because forces cannot be determinate until member sizes are known and member sizes cannot be known until forces are known... hence an iterative process is needed

2. Important principles:

There are some important principles that are at the basis of structural analysis, theses are:

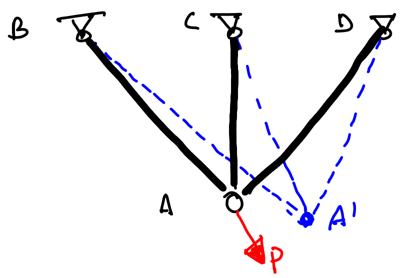
- Equilibrium
- Compatibility of deformations
- Stress-strain compatibility
- Principle of Superposition

Equilibrium:

- When a structure is subjected to loads, the loads and internal forces are in equilibrium

Compatibility:

- loads → structure goes from *undeformed shape* to *deformed shape*
- *External displacements* and *Internal deformations* have to be **compatible**:
  - 2 initially separate points remain seperate
  - Holes do not appear when the structure deforms
  - Members initially connected together remain connected
  - In other words ... the structure **"fits together"**

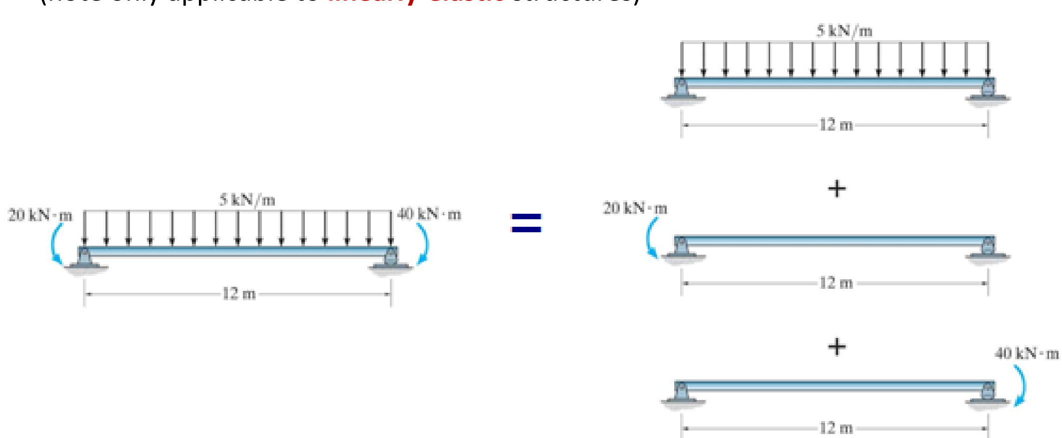


Stress-strain compatibility:

- The stresses and strains have to satisfy a $\sigma - \varepsilon$ relationship
- In this course we will deal with "linear elastic structures", meaning that the proportionality between stresses and strains is linear

Principle of Superposition:

- Another important principle that is used in structural analysis is the **principle of superposition**: For example if we have multiple loadings on a structure, rather than considering all the loads simultaneously we can instead superimpose the results …
- (note only applicable to **linearly-elastic** structures)



3. Force method: Introduction

Methods of analyzing indeterminate structures:

	Unknowns	***Equations used to find a solution***	***Coefficients*** (when method is put in matrix format)
Force based methods	Forces	Compatibility equations	Flexibility coefficients
Displacement based methods	Displacements	Equilibrium equations	Stiffness coefficients

The force method, also called the "flexibility method" is one of the two most commonly used methods to analyze indeterminate structures (the most common method being the "stiffness" method).

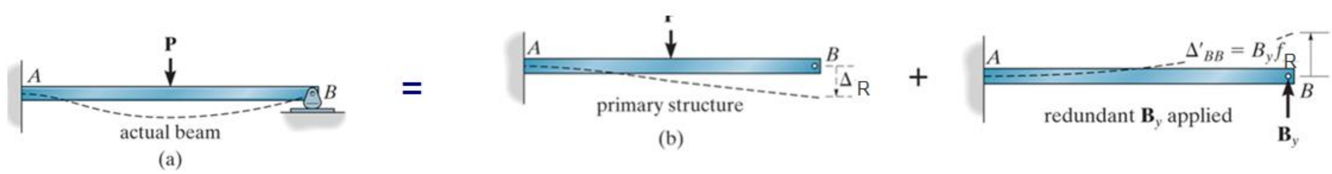
This method uses the concepts of "**Compatibility of displacement**" & the "**principle of superposition**" to solve indeterminate structures …

4. Force method - beams/frames:

The concept of the force method as applied to beams and frames is illustrated below.

Any indeterminate structure can be divided up into a primary and complementary structure.

In the case of a beam with SI=1, we can split the beam into a primary structure and a "redundant" complementary structure:



We will have the following "compatibility equation" ... $\Delta_{total} = \Delta_R + R \times f_R$

Note

- if SI = 1 → need 1 redundant
- if SI= n → we will need n redundants

Procedure: Force method - beams/frames: (for SI =1)

1. Use superposition to create a statically determinate structure and a redundant structure
   - Draw primary structure : statically determinate structure with real loads
     - get the moment diagram → M_0

   - Draw redundant structure: same structure with redundant force
     - the redundant will be one of the reactions
     - get the moment diagram → m

2. Evaluate the compatibility equation : $\Delta_{total} = \Delta_R + R \times f_R$

   (note: if unknown reaction is a moment use $\theta = \theta_R + R_\theta \times f_{R\theta}$)

   Δ_{total} = using known information about displacements at redundant location
   - could be 0 (if at support)
   - could be nonzero value (support settlement …)

   Δ_R = displacement at R in primary (statically determinate) structure
   - can solve by virtual work (or other methods)

   $$1 \times \Delta_R = \int_0^L \frac{mM_o}{EI} dx$$

   f_R = displacement at R due to a unit load at R
   - can solve by virtual work (or other methods)
   - called flexibility coefficient

   $$1 \times f_R = \int_0^L \frac{mm}{EI} dx$$

   R = force at the redundant
   (if a force reaction : R x 1 kN, if a moment reaction R x 1kN-m)

3. Solve the compatibility equation: $\Delta_{total} = \Delta_R + R \times f_R$
   - Get the redundant reaction force, R.

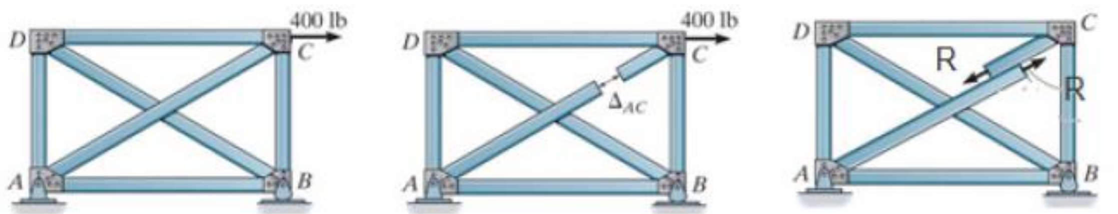
4. Solve for the remaining reactions in the structure (since you know now 1 reaction) and draw the shear moment diagrams for the actual structure

Lecture 6.1: Force method (Part 2- Trusses)

Lecture outline:
1. Force method - Trusses
2. Examples - trusses

1. Force method: trusses

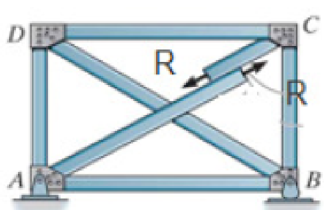
Consider the following indeterminate truss with SI = 1:



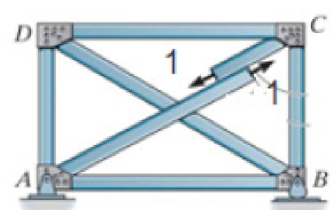
Original = **Primary Structure** + **Redundant structure**

- We can take the actual indeterminate structure and split it into a determinate **Primary** structure and **Redundant** structure by identifying a redundant force
  - **Primary structure** (with R = 0) is subject to the **real external loads**
  - **Redundant structure** (no external loads) is subjected to the **redundant force "R"**
  - Original structure = Primary structure + Redundant structure

We can simplify the analysis of the Redundant structure, using linear superposition:

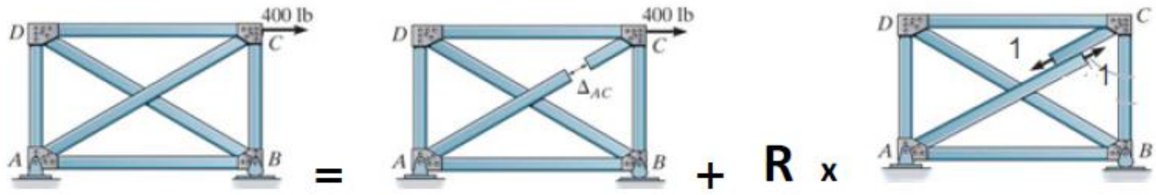


= **R** x



Redundant Structure = **Redundant (R)** x **Unit load @ location of R**

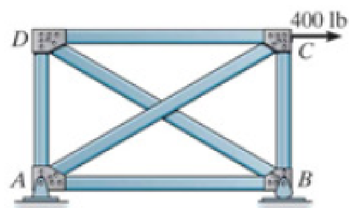
We end up with:



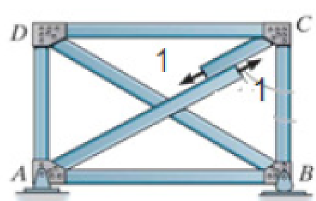
Original Structure = **Primary Structure** + **Redundant structure (Redundant x unit load @ R)**

It is noted that one can solve for the member forces in both the primary and redundant structures using statics (since both are statically determinate)

- Solve for member forces in **Primary structure** → get $\mathbf{N_{0i}}$



- Solve for member forces in **redundant structure** → get $\mathbf{n_i}$



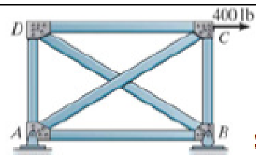
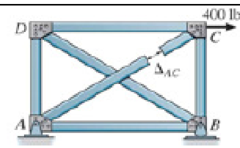
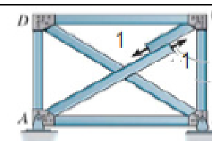
- The actual member forces are : $\mathbf{N_{0i}} + \mathbf{R} \times \mathbf{n_i}$ (total internal force in member i)

Therefore, the **only unknown** is **the redundant force, R**.

We will use the *principle of virtual work* to solve for this unknown

Using Virtual Work to Find the redundant "R"

(1) Choice of **Primary and redundant structures**

 =	 +	R x 
Original Structure =	**Primary Structure** +	**Redundant structure**

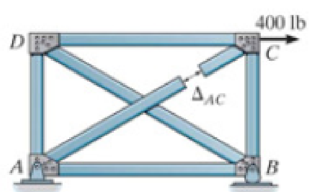
In order to solve for "R" we will come up with a compatibility equation based on VW:

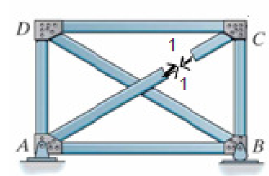
(2) In the **actual structure** the member is continuous: $\Delta_{total} = 0$!

- Compatibility equation :
  - $\boxed{\Delta_{total} = \Delta_R + R \times f_r = 0}$
  - Only unknown is "**R**".

(3a) In the **primary system**

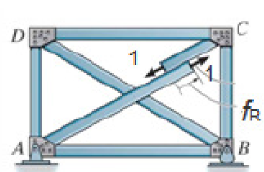
- overlap/opening @ cut = Δ_R
- using virtual work : $\boxed{1 \times \Delta_R = \sum n \frac{NL}{AE}}$

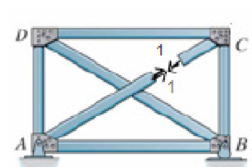




(3b) In the **redundant system**

- overlap/opening @ cut = $\Delta_x = R \times f_R$
- using virtual work : $\boxed{1 \times f_R = \sum n \frac{nL}{AE}}$
  - f_R = displacement at cut location due to **unit force** in direction of R

R x 



(4) Solve for the unknown redundant force $\Delta_{total} = \Delta_R + R \times f_R$ .... get R

(5) Once we know R we can find the unknown member forces: $N_{0i} + R \times n_i$

Procedure: Force Method (Trusses)

*** *Note: applies to structures with SI = 1* ***

1. Use superposition to create a statically determinate structure and a redundant structure
   - Draw primary structure (P_0) : statically determinate structure with real loads
   - Draw redundant structure (R x p_1): same structure with redundant force

   - Solve for member forces in P_0 → get N_{0i}
   - Solve for member forces in p_1 → get n_i

2. Write the compatibility equation:

$$\Delta_{total} = \Delta_R + \Delta_x$$

$$\Delta_{total} = \Delta_R + R \times f_R$$

3. Evaluate the compatibility equation

Δ_{total} = using known information about displacements at redundant location
   - could be 0 (if at support, if in member)
   - could be nonzero value (elongation, support settlement …)

Δ_R = displacement @ redundant location in primary structure
   - can solve by virtual work

$$1 \times \Delta_R = \sum n_i \frac{N_{0i} L}{AE}$$

f_R = displacement @ redundant location due to a unit load at @ redundant location
   - can solve by virtual work
   - called flexibility coefficient

$$1 \times f_R = \sum n_i \frac{n_i L}{AE}$$

R = force at the redundant

4. Solve the compatibility equation: $\Delta_{total} = \Delta_R + R \times f_R$
   - Get the redundant force, R.

5. Solve for the other forces in the structure (use superposition or solve directly):
   - $N_i = N_{0i} + R \times n_i$ = total internal force in member *i*.

Lecture 7.1: Force method
(Composite structures)

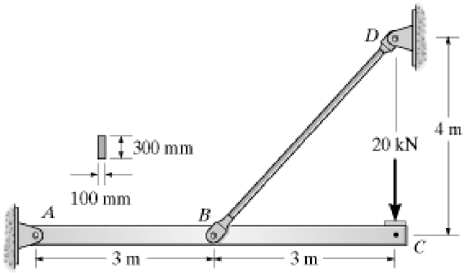
Lecture outline:

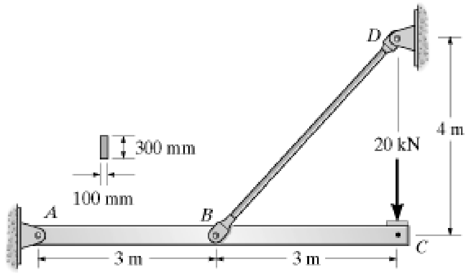
1. Determinate composite structures
2. Indeterminate composite structures

1. **Determinate** composite structures - DISPLACMENTS

Sometimes a structure may be composed of some members that are primarily subjected to axial deformations (i.e. like truss members, cables) and other members that are primarily subjected to flexural deformations (i.e. like beam and frame members).

Composite structures – How to determine DISPLACEMENTS using Virtual work





Recall:

- For trusses → $1 \cdot \Delta = \sum \frac{nNL}{AE}$
- For beams → $1 \cdot \Delta = \int_0^L \frac{mM}{EI} dx$
- For composite structures? $1 \cdot \Delta = \int_0^L \frac{mM}{EI} dx + \sum \frac{nNL}{AE}$

Step 1 ... apply REAL loads

Axial member:

- Get member force = ***N***

Beam:

- Get ***M*** diagram

Step2 ... apply VIRTUAL unit load in direction of required displacement

Axial member:

- Get member force = ***n***

Beam:

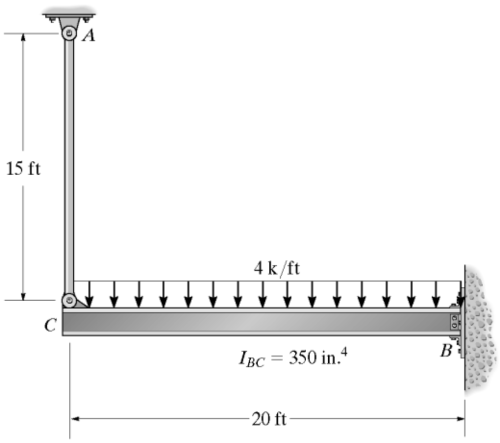
- Get ***m*** diagram

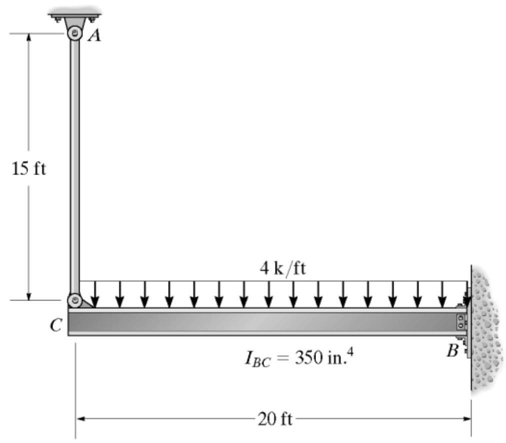
Step 3 ... apply VW equation : $1 \cdot \Delta = \int_0^L \frac{mM}{EI} dx + \sum \frac{nNL}{AE}$

2. **Indeterminate** composite structures - MEMBER FORCES

Sometimes a structure may be composed of some members that are primarily subjected to axial deformations (i.e. like truss members, cables) and other members that are primarily subjected to flexural deformations (i.e. like beam and frame members).

Composite structures – How to determine MEMEBER FORCES using Force method





Step 1 ... Select primary and redundant structure

- For example : Redundant = member force in AC = N_{AC} = **R1**

Step 2 ... analyse PRIMARY structure

Axial member:

- Get member force = ***N***N_o

Beam:

- Get ***Mo*** diagram

Step3 ... analyse REDUNDANT structure ... apply unit R1 force

Axial member:

- Get member force = n_1

Beam:

- Get m_1 diagram

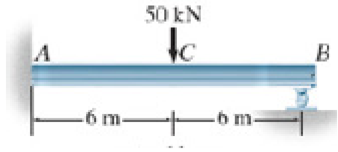
Step 4 ... use force method:

- $\Delta_{10} = \Delta_R = \int_0^L \frac{m_1 M_O}{EI} dx + \sum \frac{n_1 N_1 L}{AE}$
- $f_R = f_{11} = \int_0^L \frac{m_1 m_1}{EI} dx + \sum \frac{n_1 n_1 L}{AE}$
- $\Delta_{10} + R_1 \times f_{11} = 0$ $(\Delta \; + R_1 \times f_R = 0)$

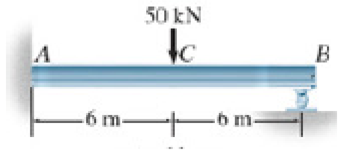
Examples of choosing redundants

IMPORTANT: Cannot choose a redundant that causes system to become unstable

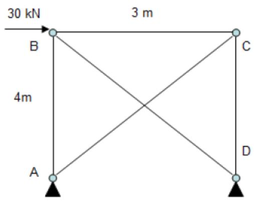
Example - Beam: Option #1, take reaction at B as redundnat



Example - Beam: Option #2, take moment reaction at A as redundant



Example- Truss: option #1 - reaction as redundant



Example - truss: option #2 - member as redundant