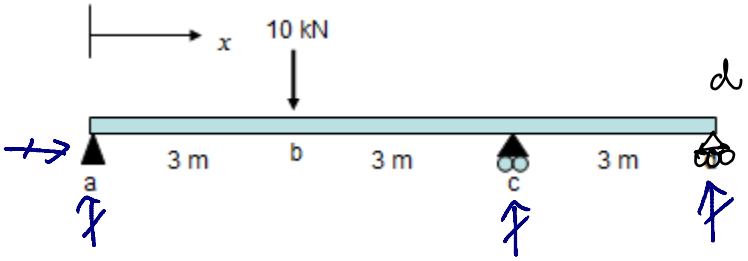
EXAMPLES
5.1: Force method (Beams & Frames)

Example-Beam

Example 1: Use the force method to determine the reactions as well as shear and moment diagrams. Use "geometric methods" and "integration tables" to compute $\int_0^L mM\,dx$



$SI = (3mrr) - (3jte_c)$

$m = 2$, $r = 4$, $j = 3$, $e_c = 0$

Check degree of indeterminacy: SI = $(3 \times 2 + 4) - (3 \times 3 + 0) = 1$

Since SI = 1 → we must identify 1 redundant (s)

… use "force method" to solve for member forces …

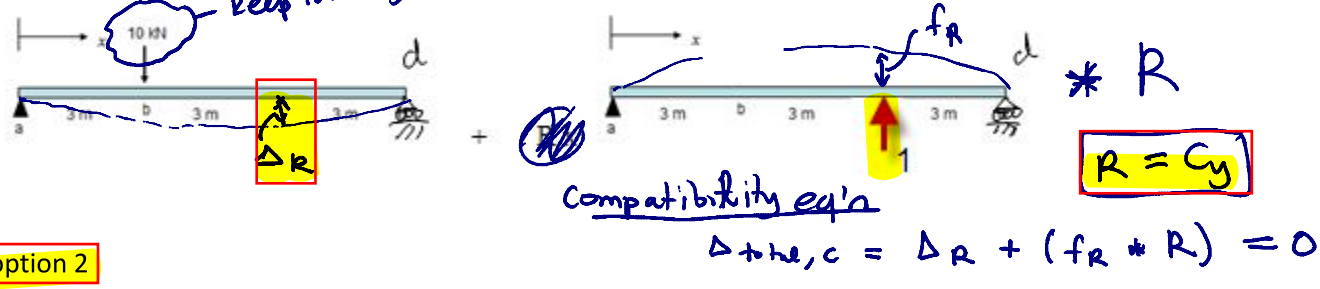
Use superposition to create a primary + redundant structure

Important: can't choose a redundant that will cause structure to be unstable

option 1

① primary — keep loading and remove reaction C_y

② Redundant structure

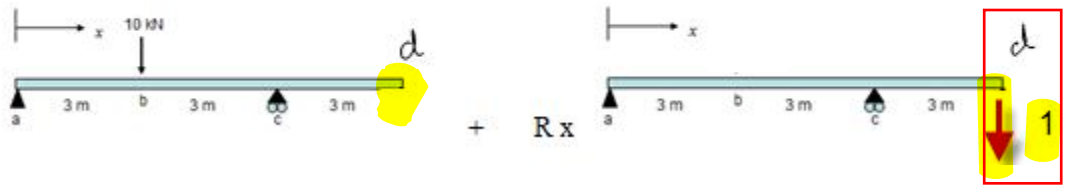


$R = C_y$

Compatibility eq'n

$\Delta_{total,c} = \Delta_R + (f_R * R) = 0$

option 2



1

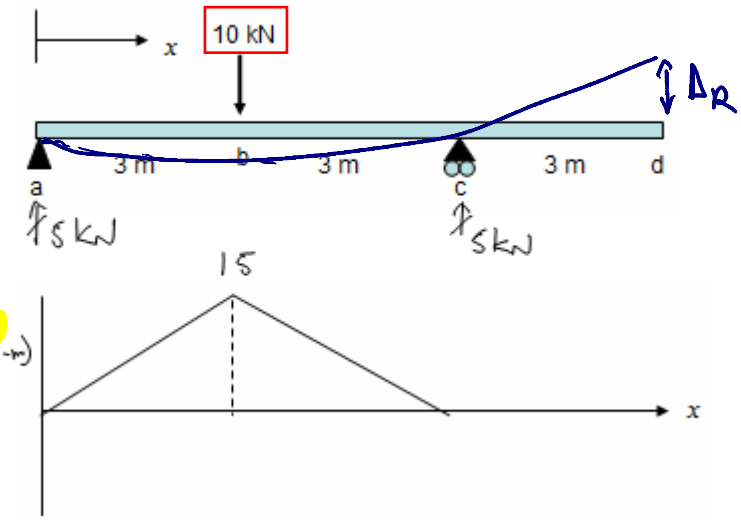
compatibility equation

$\Delta_{D,total} = \Delta_R + f_R * R$ where $R = Dy$

Let's use option #2 :

Subject determinate structure to external forces → get M_0

Primary

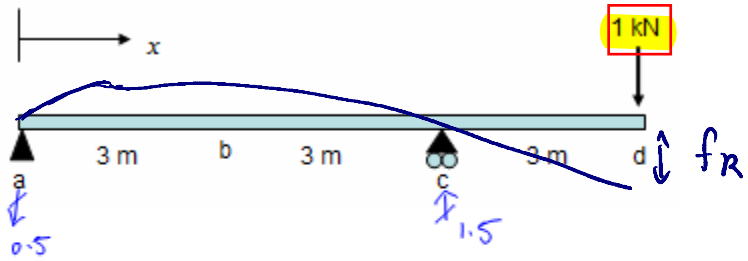


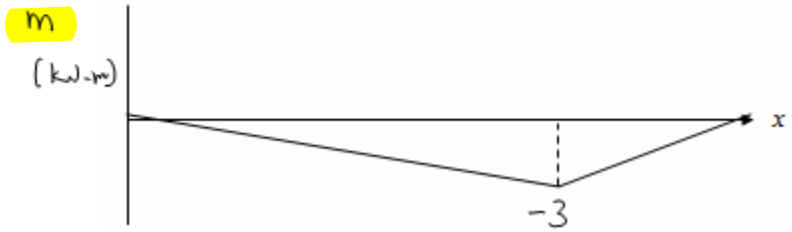
$$\Delta_R = \int \frac{M_0 m}{EI} dx$$

$f_R =$

Subject to redundant force "R" in direction of unknown reaction → get m

Redundant

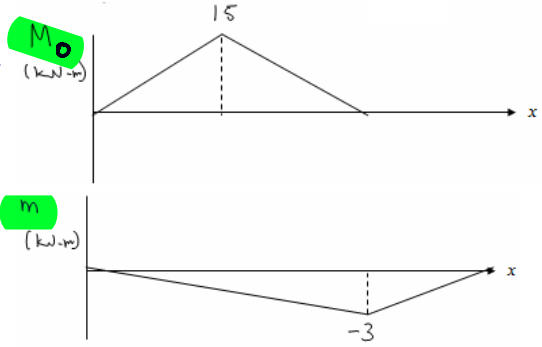




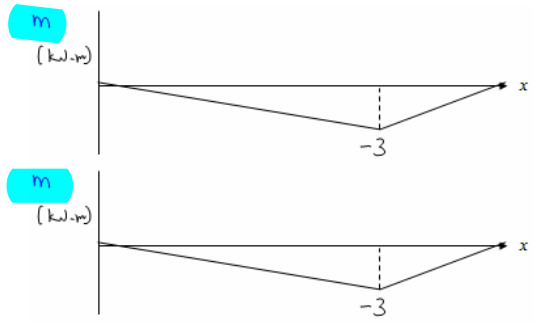
$$f_R = \int \frac{m m}{EI} dx$$

Using geometric method or integration table to compute $\int_0^L \frac{mM_o}{EI} dx$ and $\int_0^L \frac{mm}{EI} dx$

$\Delta_R = \int \frac{M_0 m}{EI} dx$



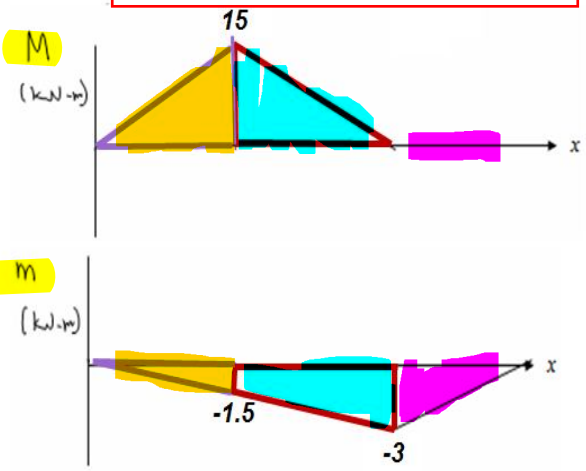
Primary



Redundant

$f_R = \int \frac{m m}{EI} dx$

Let's find $\Delta_R = \int \frac{m M_o}{EI} dx$ { M_o & m → use virtual work



① $b = 15$ $d = -15$

$\frac{1}{3} L b d$

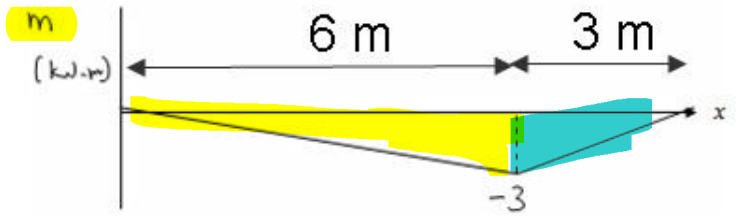
② $\frac{La}{6}(2c + d)$

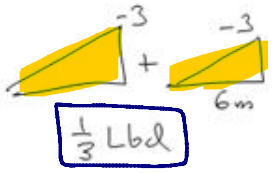
$\int_0^L mM_o dx$ — geometric — Mohr's table

Area	$(A)_i$	$(h)_i$	$(A)_i * (h)_i$	Alternative: table $\int_0^L f_1(x) * f_2(x) dx$
1	Try as an exercise			$\frac{1}{3}(3)(15)(-15) = -22.5$
2	Try as an exercise			$\frac{3 \times 15}{6}(2 * -1.5 + (-3)) = -45.0$
			Σ	**-67.5** $= \int m M_o dx$

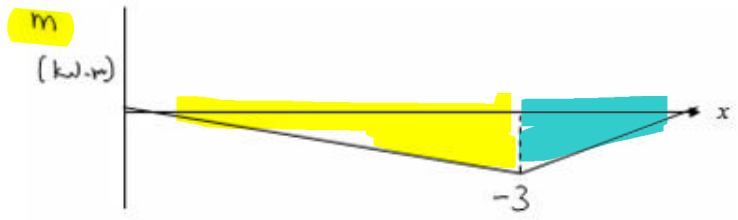
$1 \times \Delta_R = \int_0^L \frac{mM_o}{EI} dx$ → $1 * \Delta_R = -\frac{67.5}{EI}$

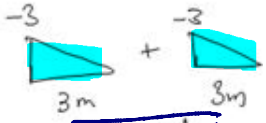
next let's determine $f_R = \int \frac{m\,m}{EI}\,dx$ { m & m → VW



① 

$\frac{1}{3}Lbd$



② 

$\frac{1}{3}Lbd$

$\int_0^L mmdx$ — geometric — Mohr's

Area	$(A)_i$	$(h)_i$	$(A)_i * (h)_i$	Alternative: table $\int_0^L f_1(x) * f_2(x)dx$
1	Try as an exercise			$\frac{1}{3}(6)(-3)(-3)$ = 18
2	Try as an exercise			$\frac{1}{3}(3)(-3)(-3)$ = 9
			Σ	**27** = $\int mm\,dx$

$1 \times f_R = \int_0^L \frac{mm}{EI}dx$ → $f_R = \frac{27}{EI}$

③ $\Delta_{total} = \Delta_R + R \times f_R$ → compatibility equation: $\Delta_{D,total} = 0 = \Delta_R + f_R * R$

$0 = -\frac{67.5}{EI} + \frac{27}{EI} * R$

$R = D_y = +2.5\ kN$

$D_y = 2.5 \downarrow$

Solve for remaining reactions and draw V and M …