EXAMPLES
8.1: Slope-deflection method

Examples outline:

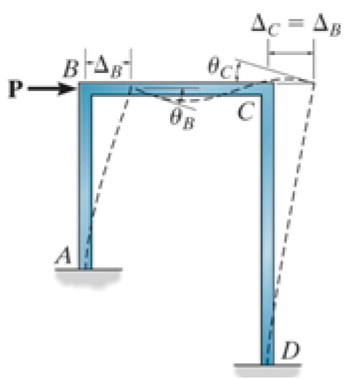
1. KI examples
2. DOF examples
3. Equation examples
4. FEM examples
5. Complete examples

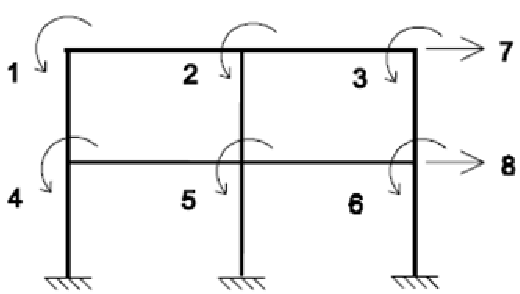
1. KI examples

Find the degree of kinematic indeterminacy:

1) Beam (assume beams are axially rigid):

2) Frames (assume columns and beams are axially rigid):

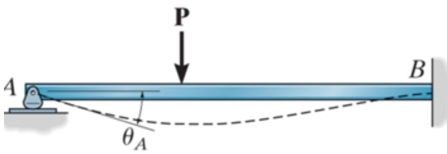


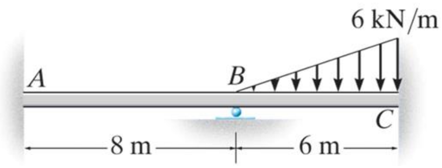


2. DOF examples – for slope deflection

Identify the:

- UDOFs (DOFs not equal to 0)
- CDOFs (DOFs=0) ... consider only slopes for the slope-deflection method

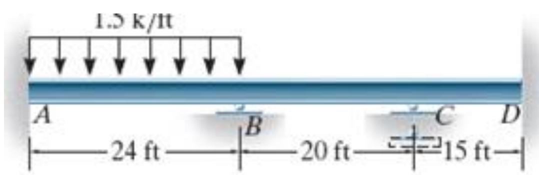


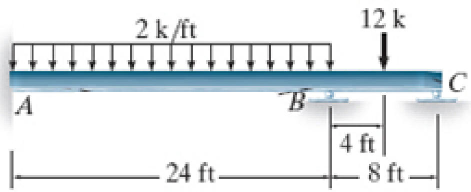


- UDOFs :
- DOFs = 0 :

UDOFs :

DOFs = 0:

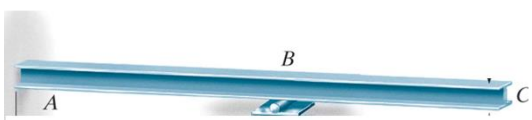




- UDOFs :
- DOFs = 0 :

UDOFs :

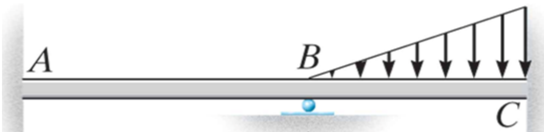
DOFs = 0:



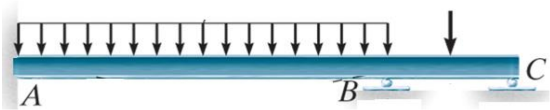
- UDOFs :
- DOFs = 0 :

3. Equations examples

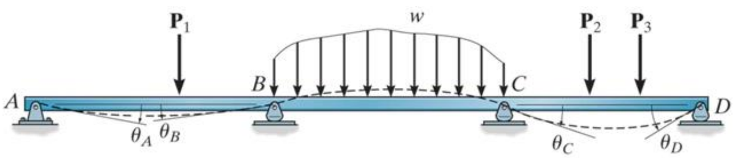
What slop-deflection equations would you use for each beam ?



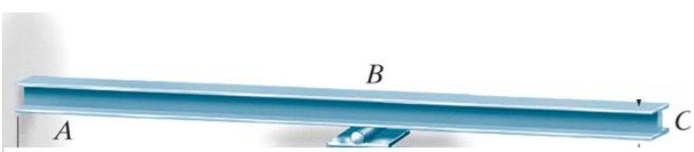
- Member AB : apply **Eq.#______** , once for _____ and once for ______.
- Member BC : apply **Eq.#______** , once for _____ and once for ______.



- Member AB : apply **Eq.#______** , once for _____ and once for ______.
- Member BC : apply **Eq.#______** , once for _____ .



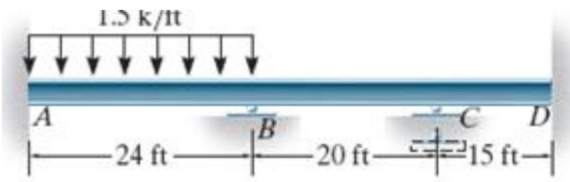
- Member AB : apply **Eq.#______** , once for _____ .
- Member BC : apply **Eq.#______** , once for _____ and once for ______.
- Member CD : apply **Eq.#______** , once for _____ .



- Member AB : apply **Eq.#______** , once for _____ and once for ______.

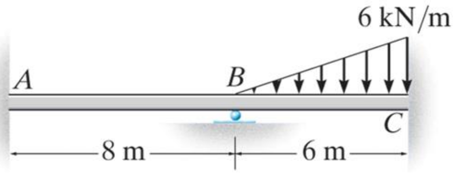
4. FEM examples

Example:



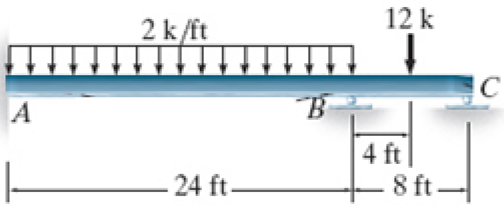
- FEM_{AB} = ____________
- FEM_{BA}= ____________
- FEM_{BC} = FEM_{CB}= ______
- FEM_{CD}= FEM_{DC}= ______

Example:



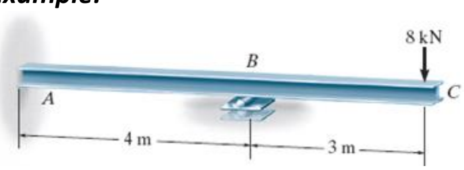
- FEM_{AB} = FEM_{BA}= ______
- FEM_{BC} = ____________
- FEM_{CB}= ____________

Example:



- FEM_{AB} = ____________
- FEM_{BA}= ____________
- FEM_{BC} = ____________
- FEM_{CB}= ____________

Example:

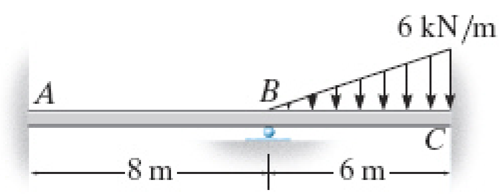


- FEM_{AB} = FEM_{BA} ____________
- FEM_{BC} = ____________
- FEM_{CB}= ____________

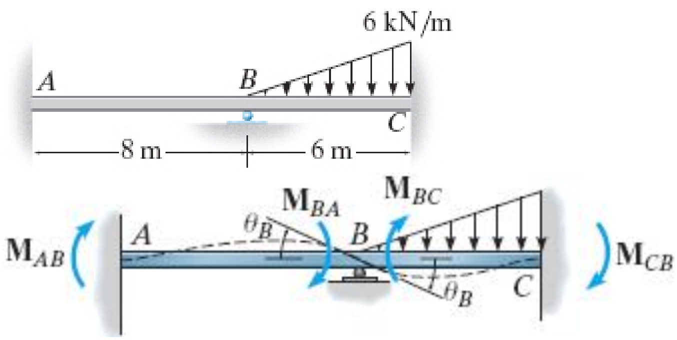
5. Complete Examples

Example #1

Draw the shear and moment diagrams for the beam shown in Fig. 11–10*a*. *EI* is constant.



Degrees of freedom



Slope-deflection equations

1) Summarize:

	Span AB	Span BC
DOFs = 0 ?		
FEMs		
Slope-def eqns		

2) Find FEMs:

$$(\text{FEM})_{BC} = -\frac{wL^2}{30} = -\frac{6(6)^2}{30} = -7.2\ \text{kN}\cdot\text{m}$$

$$(\text{FEM})_{CB} = \frac{wL^2}{20} = \frac{6(6)^2}{20} = 10.8\ \text{kN}\cdot\text{m}$$

3) Find Δ s:

Span AB:

Eq. #1:

$$M_{AB} = 2E\left(\frac{I}{8}\right)[2(0) + \theta_B - 3(0)] + 0 = \frac{EI}{4}\theta_B \qquad (1)$$

(1) → $M_{AB} = \frac{EI}{4}\theta_B$

$$M_{BA} = 2E\left(\frac{I}{8}\right)[2\theta_B + 0 - 3(0)] + 0 = \frac{EI}{2}\theta_B \qquad (2)$$

(2) → $M_{BA} = \frac{EI}{2}\theta_B$

Span BC:

Eq. #1:

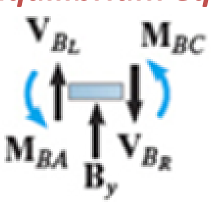
$$M_{BC} = 2E\left(\frac{I}{6}\right)[2\theta_B + 0 - 3(0)] - 7.2 = \frac{2EI}{3}\theta_B - 7.2 \qquad (3)$$

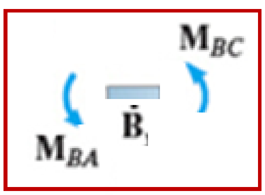
(3) → $M_{BC} = \frac{2EI}{3}\theta_B - 7.2$

$$M_{CB} = 2E\left(\frac{I}{6}\right)[2(0) + \theta_B - 3(0)] + 10.8 = \frac{EI}{3}\theta_B + 10.8 \qquad (4)$$

(4) → $M_{CB} = \frac{EI}{3}\theta_B + 10.8$

Equilibrium equations





$$\circlearrowleft + \Sigma M_B = 0; \qquad M_{BA} + M_{BC} = 0 \qquad (5)$$

Solve for displacements and moments

1) Substitute to find displacements

substitute Eqs. (2) and (3) into Eq. (5), which yields

$$\theta_B = \frac{6.17}{EI}$$

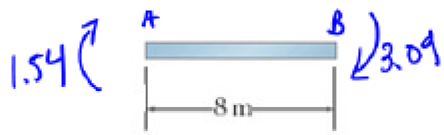
2) Plug back into slope-deflection equations to find moments

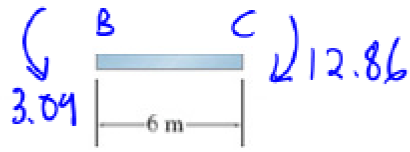
Resubstituting this value into Eqs. (1)–(4) yields

$$M_{AB} = 1.54 \text{ kN}\cdot\text{m}$$
$$M_{BA} = 3.09 \text{ kN}\cdot\text{m}$$
$$M_{BC} = -3.09 \text{ kN}\cdot\text{m}$$
$$M_{CB} = 12.86 \text{ kN}\cdot\text{m}$$

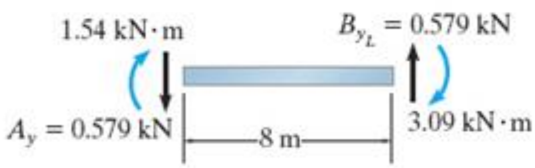
3) Determine shears/reactions using FBDs ...

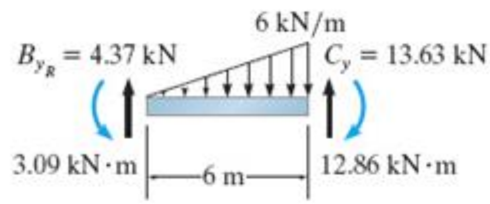
Add the moments



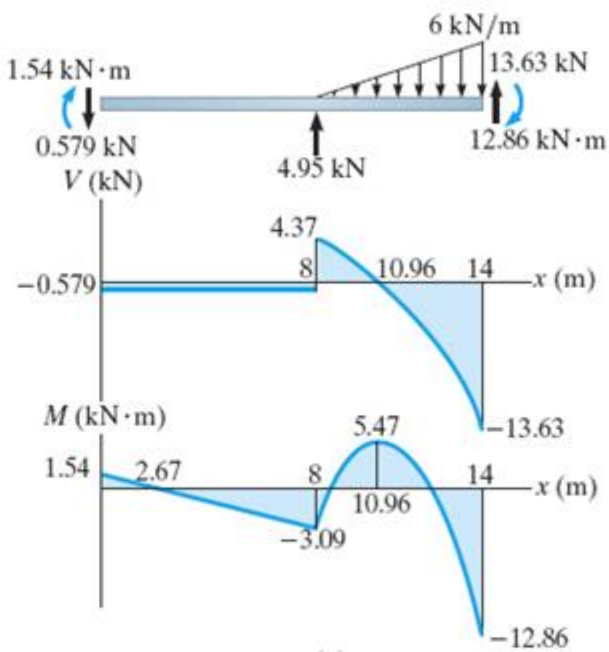


Determine shears/reactions using FBDs ...



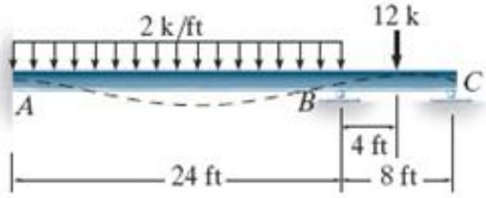


3) Draw FBD of entire structure ...

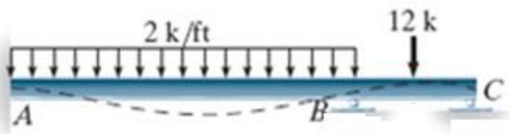


Example #2

Draw the shear and moment diagrams for the beam shown in Fig. 11–11*a*. *EI* is constant.



Degrees of freedom



Slope-deflection equations

1) Summarize:

	Span AB	Span BC
DOFs = 0 ?		
FEMs		
Slope-def eqns		

2) Find FEMs:

$$(\text{FEM})_{AB} = -\frac{wL^2}{12} = -\frac{1}{12}(2)(24)^2 = -96\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{BA} = \frac{wL^2}{12} = \frac{1}{12}(2)(24)^2 = 96\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = -\frac{3PL}{16} = -\frac{3(12)(8)}{16} = -18\ \text{k}\cdot\text{ft}$$

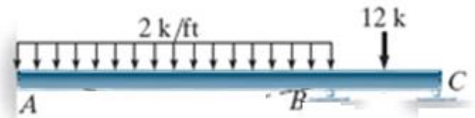
3) Find Δs:

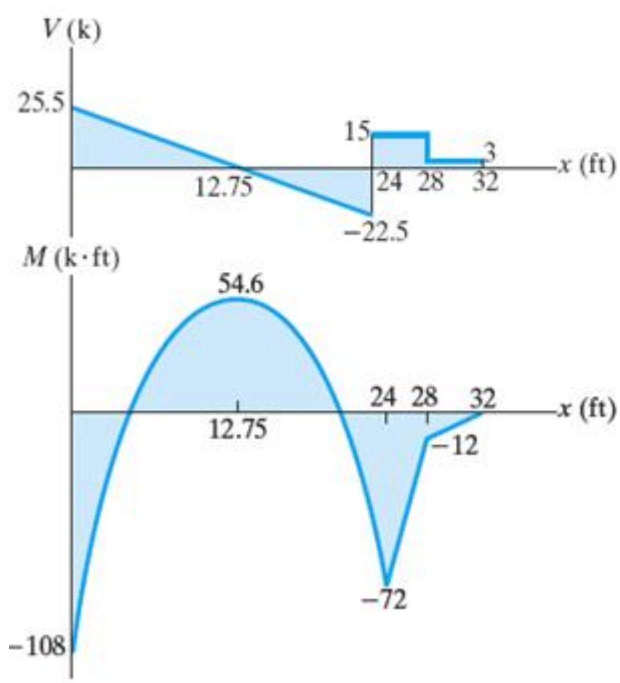
Equilibrium equations

Solve for displacements and moments

3) Determine shears/reactions using FBDs ...

3) Draw FBD of entire structure ...





Extra Examples – Hibbeler

NOTE: the following example has support settlement ... but the steps other than for finding "settlements' are the same ...

EXAMPLE 11-4

Determine the internal moments at the supports of the beam shown in Fig. 11–13*a*. The support at *C* is displaced (settles) 0.1 ft. Take $E = 29(10^3)$ ksi, $I = 1500$ in^4.

Fig. 11–13

Solution

Slope-Deflection Equations. Three spans must be considered in this problem. Equation 11–8 applies since the end supports *A* and *D* are fixed. Also, only span *AB* has FEMs.

EXAMPLE 11-4 *(Continued)*

$$(\text{FEM})_{AB} = -\frac{wL^2}{12} = -\frac{1}{12}(1.5)(24)^2 = -72.0 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BA} = \frac{wL^2}{12} = \frac{1}{12}(1.5)(24)^2 = 72.0 \text{ k}\cdot\text{ft}$$

As shown in Fig. 11–13*b*, the displacement (or settlement) of the support *C* causes ψ_{BC} to be positive, since the cord for span *BC* rotates clockwise, and ψ_{CD} to be negative, since the cord for span *CD* rotates counterclockwise. Hence,

$$\psi_{BC} = \frac{0.1 \text{ ft}}{20 \text{ ft}} = 0.005 \text{ rad} \qquad \psi_{CD} = -\frac{0.1 \text{ ft}}{15 \text{ ft}} = -0.00667 \text{ rad}$$

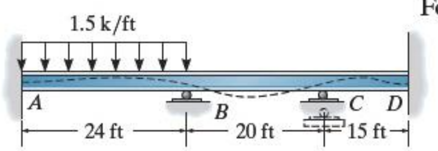
Also, expressing the units for the stiffness in feet, we have

$$k_{AB} = \frac{1500}{24(12)^4} = 0.003014 \text{ ft}^3 \qquad k_{BC} = \frac{1500}{20(12)^4} = 0.003617 \text{ ft}^3$$

$$k_{CD} = \frac{1500}{15(12)^4} = 0.004823 \text{ ft}^3$$

Noting that $\theta_A = \theta_D = 0$ since *A* and *D* are fixed supports, and applying the slope-deflection Eq. 11–8 twice to each span, we have

EXAMPLE 11-4 (Continued)



(a)

For span AB:

$$M_{AB} = 2[29(10^3)(12)^2](0.003014)[2(0) + \theta_B - 3(0)] - 72$$
$$M_{AB} = 25\,173.6\theta_B - 72 \quad (1)$$
$$M_{BA} = 2[29(10^3)(12)^2](0.003014)[2\theta_B + 0 - 3(0)] + 72$$
$$M_{BA} = 50\,347.2\theta_B + 72 \quad (2)$$

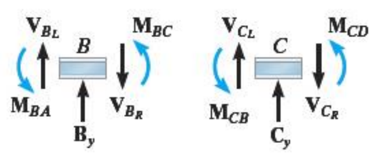
For span BC:

$$M_{BC} = 2[29(10^3)(12)^2](0.003617)[2\theta_B + \theta_C - 3(0.005)] + 0$$
$$M_{BC} = 60\,416.7\theta_B + 30\,208.3\theta_C - 453.1 \quad (3)$$
$$M_{CB} = 2[29(10^3)(12)^2](0.003617)[2\theta_C + \theta_B - 3(0.005)] + 0$$
$$M_{CB} = 60\,416.7\theta_C + 30\,208.3\theta_B - 453.1 \quad (4)$$

For span CD:

$$M_{CD} = 2[29(10^3)(12)^2](0.004823)[2\theta_C + 0 - 3(-0.00667)] + 0$$
$$M_{CD} = 80\,555.6\theta_C + 0 + 805.6 \quad (5)$$
$$M_{DC} = 2[29(10^3)(12)^2](0.004823)[2(0) + \theta_C - 3(-0.00667)] + 0$$
$$M_{DC} = 40\,277.8\theta_C + 805.6 \quad (6)$$

EXAMPLE 11-4 (Continued)



(c)

Equilibrium Equations. These six equations contain eight unknowns. Writing the moment equilibrium equations for the supports at B and C, Fig. 10–13c, we have

$$\circlearrowleft + \Sigma M_B = 0; \qquad M_{BA} + M_{BC} = 0 \quad (7)$$
$$\circlearrowleft + \Sigma M_C = 0; \qquad M_{CB} + M_{CD} = 0 \quad (8)$$

In order to solve, substitute Eqs. (2) and (3) into Eq. (7), and Eqs. (4) and (5) into Eq. (8). This yields

$$\theta_C + 3.667\theta_B = 0.01262$$
$$-\theta_C - 0.214\theta_B = 0.00250$$

Thus,

$$\theta_B = 0.00438 \text{ rad} \qquad \theta_C = -0.00344 \text{ rad}$$

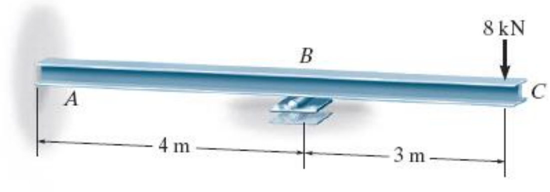
The negative value for θ_C indicates counterclockwise rotation of the tangent at C, Fig. 11–13a. Substituting these values into Eqs. (1)–(6) yields

$$M_{AB} = 38.2 \text{ k}\cdot\text{ft} \qquad \textit{Ans.}$$
$$M_{BA} = 292 \text{ k}\cdot\text{ft} \qquad \textit{Ans.}$$
$$M_{BC} = -292 \text{ k}\cdot\text{ft} \qquad \textit{Ans.}$$
$$M_{CB} = -529 \text{ k}\cdot\text{ft} \qquad \textit{Ans.}$$
$$M_{CD} = 529 \text{ k}\cdot\text{ft} \qquad \textit{Ans.}$$
$$M_{DC} = 667 \text{ k}\cdot\text{ft} \qquad \textit{Ans.}$$

NOTE: the following example has a free-end ...

EXAMPLE 11-3

Determine the moment at A and B for the beam shown in Fig. 11–12a. The support at B is displaced (settles) 80 mm. Take E = 200 GPa, $I = 5(10^6)$ mm^4.

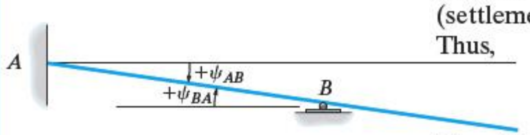


(a)

Fig. 11–12

Solution

Slope-Deflection Equations. Only one span (AB) must be considered in this problem since the moment $\mathbf{M}_{BC}$ due to the overhang can be calculated from statics. Since there is no loading on span AB, the FEMs are zero. As shown in Fig. 11–12b, the downward displacement (settlement) of B causes the cord for span AB to rotate clockwise. Thus,



(b)

$$\psi_{AB} = \psi_{BA} = \frac{0.08 \text{ m}}{4} = 0.02 \text{ rad}$$

The stiffness for AB is

$$k = \frac{I}{L} = \frac{5(10^6)\text{ mm}^4(10^{-12})\text{ m}^4/\text{mm}^4}{4 \text{ m}} = 1.25(10^{-6})\text{ m}^3$$