Lecture 10.1: Influence Lines (Part 1)

Lecture outline:

1. Introduction
2. Influence lines: Determinate Beams (Tabular method)
3. Influence lines : Determinate Beams (Muller-Bresslau method)
4. Influence lines: Determinate Trusses
5. Influence lines: Indeterminate beams
6. Influence lines: Indeterminate frames

1. Introduction

So far we have considered analysis techniques for structures that carry **static / fixed** loads. For these types of structures, we use **Shear and moment diagrams** to find the shears and moments needed for design.

If the structure is subjected to **moving** loads, the variation in shear and moment is better described using **influence lines**.

Definition:

An influence line describes:

The **variation** of:	a reaction force ... shear force **...** moment **...** deflection ...	at a **specific point ...**	.... as a concentrated unit load **moves** along the length of the member.

Applications:

The main application of influence lines is in the analysis structures where the load moves along the span (e.g. bridges, beams supporting cranes ….).

Furthermore, influence lines can be used in the analysis of buildings subjected to varying live loads (for quickly placing the live loads on the spans to produce maximum internal forces).



Influence lines vs. M & V diagrams

The difference between constructing an influence line and constructing a shear and moment diagram is:

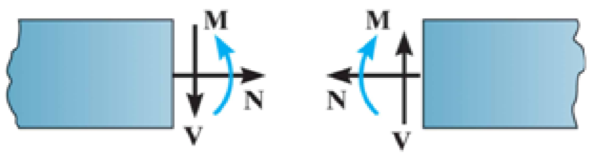
Shear / Moment diagram represents:	*Influence line represents …*
… the effect of a **fixed** load … at **all points** along a member	… the effect of a **moving** load … at **a specific point** on a member

2. Influence lines - Determinate Beams (Tabular method)

For statically determinate beams, two procedures can be used to draw influence lines:

Method #1 - Tabular method:

1. Place a unit load (1 kN) at a point “x” along the length of the member
2. Use the equations of equilibrium and method of sections to find the value of the function (reaction, shear or moment) at this specific point ... **use usual positive convention**

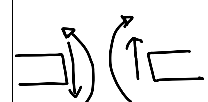


Influence line for **vertical reaction**

- at each position of the unit load the value of **R** is calculated using the **method of sections**
- For vertical reaction: reaction is **positive** (+) when it acts **upwards** (↑)

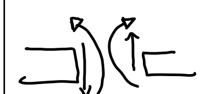
Influence line for **shear force**

- at each position of the unit load the value of **V** is calculated using the **method of sections**
- **Note:** must place unit load **just to the left** (-) and **just to the right** (+) of the location where the shear influence line is to be determined (since the shear will be discontinuous at that location)
- **positive** (+) according to the positive sign convention used for constructing shear and moment diagrams



Influence line for **moment**

- at each position of the unit load the value of **M** is calculated using the **method of sections**
- **positive** (+) according to the positive sign convention used for constructing shear and moment diagrams



3. Repeat for various values of x over the length of the beam
   - All determinate beams will have influence lines that consist of **straight** line segments
   - With some practice one should be able to minimize computations and locate the unit load only at points representing the **end points** of each line segment

4. Tabulate the values in a table

5. Plot the values of the reaction/shear/moment as a function of x along the member length

3. Influence lines - Determinate Beams (Muller-Bresslau method)

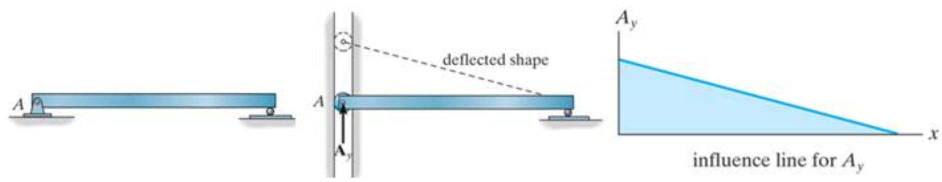
A method that can be used to rapidly draw influence lines qualitatively is the Muller-Bresslau method (developed by Heinrick Muller-Breslau in 1886).

The basic principle of the method is:

1. The influence line for a function (Reaction, Shear, Moment) is to the same scale as the <u>deflected shape</u> of the beam when the beam is acted on by the function
2. To draw the <u>deflected shape</u> the ability of the beam to resist the applied function must be **removed** so that the beam can deflect when the function is applied.

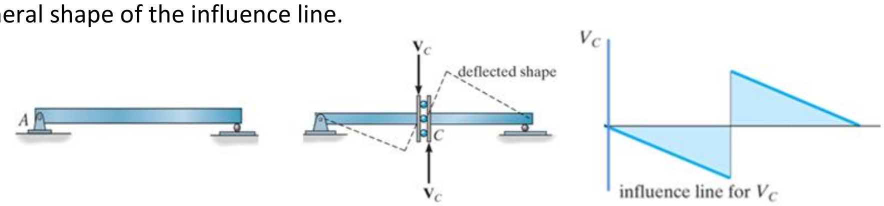
Vertical reaction

- Remove ability to resist movement in the vertical direction by using a **roller guide @ support.**
- Applying a **positive** force (upward) to the beam, the beam deflects to the dashed position which is the general shape of the influence line.



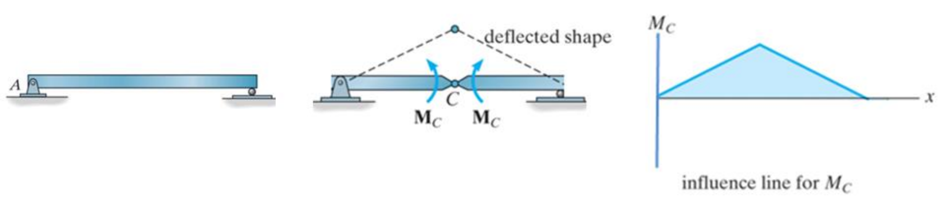
Shear

- Remove ability to resist shear by using a **sliding device** for the internal force **V**
- Applying **positive** shear to the beam, the beam deflects to the dashed position which is the general shape of the influence line.



Moment

- Remove ability to resist moment at the point by placing an **internal hinge**
- Applying **positive** moments to the beam, the beam deflects to the dashed position which is the general shape of the influence line.

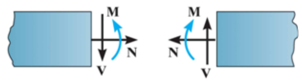


Procedure - Qualitative influence lines

1. At the point where the IL is to be determined, place a "connection" /release

- If need IL for **vertical reaction** ... use a **roller guide**
- If need IL for **shear** ... use an **sliding device**
- If need IL for a **moment** ... use an **internal hinge**

2. Place a unit ***FUNCTION*** *at the "connection/release" acting in the "positive direction"*

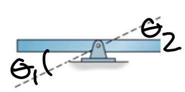
Recall the positive convention: 

, therefore:

- If the function is a **vertical reaction** use 
- If the function is **shear** use 
- If the function is a **moment** use 

3. Get qualitative "shape" of IL curve

- The deflected shape that is obtained is the "qualitative" shape of the IL curve
- Remember for **determinate** beams the IL curve consists of **only straight line segments**
- Remember certain rules for supports:

Fixed support	Pin/Roller @ end	Pin roller (interior)	Free end
Δ = 0 θ= 0	Δ = 0 Can have θ	Δ = 0 Can have θ But $\theta_1 = \theta_2$ 	Can have Δ

Procedure - Quantitative influence lines

- Follow same steps 1 to 3 and get qualitative IL
- Once you have qualitative IL, determine value at one point on IL curve
- Use similar triangles to find all other points

4. Influence lines - Determinate Trusses

Trusses are often used as the main load carrying members of bridges. Hence, for design purposes it is important to be able to construct influence lines for each of the truss members.

Note that since trusses are by definition only loaded at their joints, one simply needs to place the unit live load at successive truss joints supporting the deck and calculate the resulting member force (using the method of joints or method of sections) to obtain the ordinate of the influence line curve.

By repeating this process for successive truss joints along the deck one can construct the influence line.

The data can then be arranged in a table as "member force in member" *vs.* "joint location".

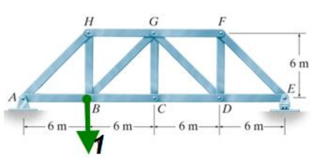
As a convention we say the member force is:

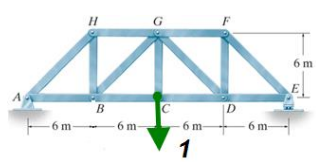
- **positive** (+) if it's in **tension**
- **negative** (–) if it's in **compression**

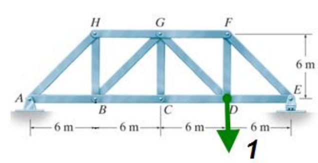
The maximum tensile force (+) developed in a truss member:

- due to a concentrated live load, *F*: F x (peak ***+ve*** influence line value)
- due to a distributed live load, *w*: *w* x (***+ve*** area under influence curve)

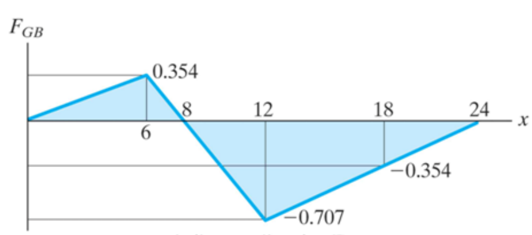
Same procedure applies for a compression force (-) except we use negative values …







x	F_{GB}
0	0
6	0.354
12	−0.707
18	−0.354
24	0



influence line for F_{GB}

Lecture 10.2: Influence Lines (Part 2)

Lecture outline:

1. Application: Determinate Beams - Maximum load effects
2. Application: Trusses - Maximum axial force
3. Application: Continuous Beams – Placing live loads
4. Side-note: Moment envelopes for continuous beams
5. Application: Truck loads

1. Application: Determinate Beams - Maximum load effects

CASE 1: *General loading case*

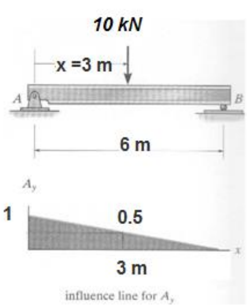
In Lecture 10.1 we saw how to find the influence lines for the **V/M** in a determinate beam.

- Once the influence line has been drawn, one can **compute** the **maximum** value of V/M using the influence line!

If one is given a loading case, one uses the following procedure:

1) *Concentrated load* (kN):

- For any concentrated load "F" acting at any position "x", the value of (R, V, M) is found by:
  - effect = ***P*** x **IL value @ load location**



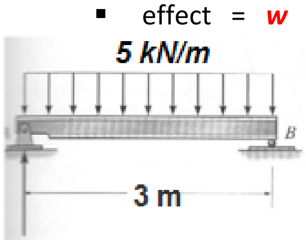
@ x = 3 m → influence = 0.5

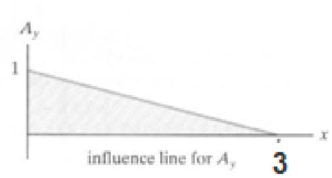
∴ A_y when load is @ x = 10 kN

A_y = 0.5 *10 = 5 kN

2) Distributed live load (kN/m):

- For a distributed load "w" acting on the entire beam, the value of (R, V, M) is :
  - effect = ***w*** x **area under entire IL curve**
- For a distributed load "w" acting on only part beam, the value of (R, V, M) is:
  - effect = ***w*** x **area of IL curve over which w is placed**





A_y = w_0 x Area

5 * {1/2 * 3 *1} = 7.5 KN

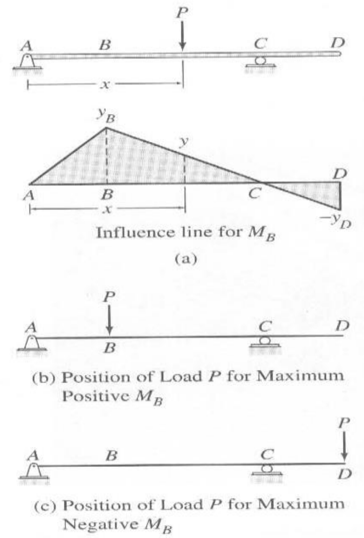
CASE 2: *Live load placement for maximum effects + computation of resulting maximum V/M:*

In Lecture 10.1 we saw how to find the influence lines for the **V/M** in a determinate beam.

- Once the influence of a function (V, M) has been constructed, one can then **position** the live loads on the beam which will produce the **maximum** value of V/M.
- One can also **compute** the **maximum** value of V/M using the influence line!

1) Concentrated live load (kN):

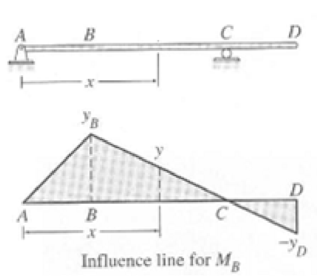
- The *maximum* influence caused by a **moving live** force "F" will be at the location "x" which corresponds to the *peak* influence line
  - for *maximum* **positive** (+) Shear / Moment, place loads over **Positive regions**
    - effect = ***P*** x **peak positive IL value**
  - for *maximum* **negative** (-) Shear / Moment, place loads over **Negative regions**
    - effect = ***P*** x **peak negative IL value**



Influence line for M_B
(a)
(b) Position of Load P for Maximum Positive M_B
(c) Position of Load P for Maximum Negative M_B

2) Distributed live load (kN/m):

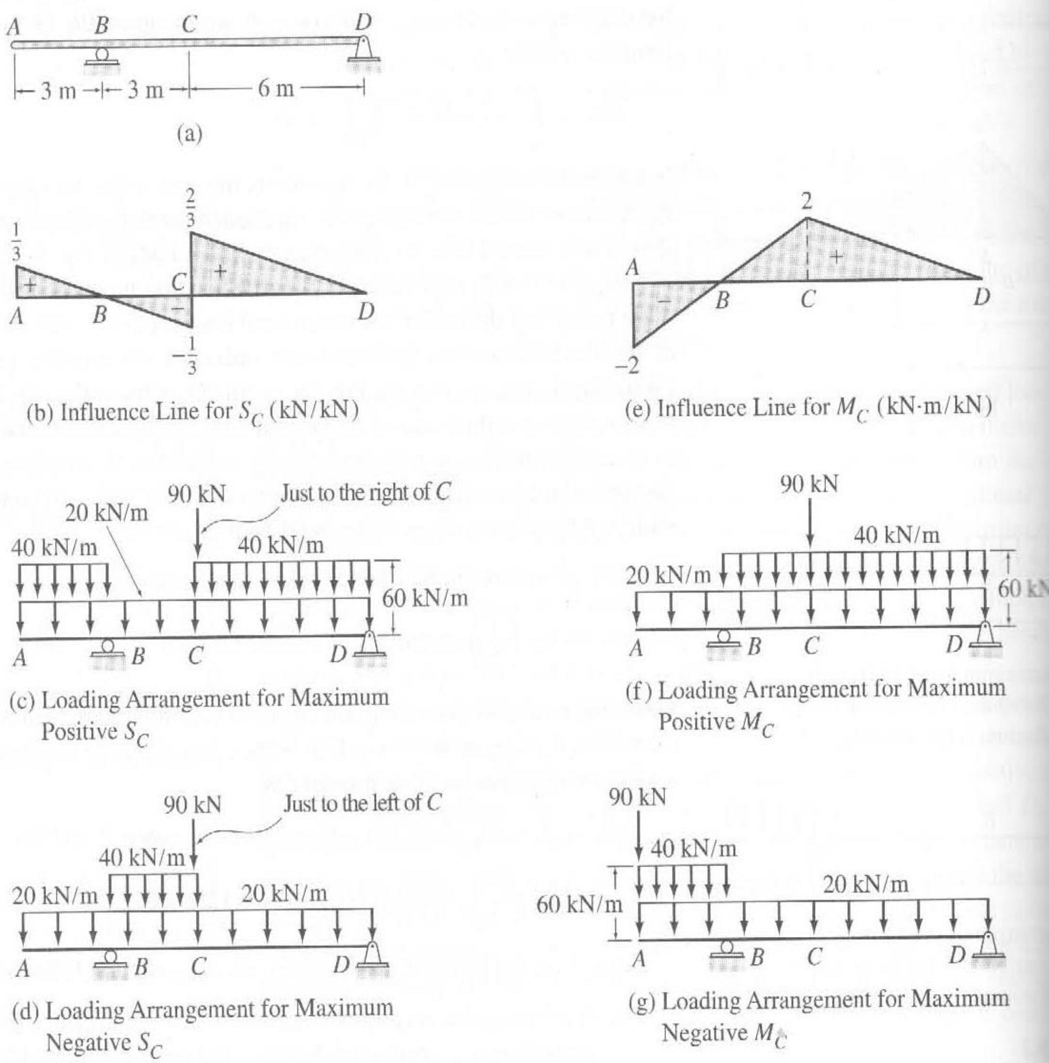
- Dead loads ... these should be placed on the entire beam
    - effect = ***w*** x **area under entire IL curve**
- Live loads:
  - for *maximum* **positive** (+) Shear / Moment, place loads over **Positive regions**
    - effect = ***w*** x **positive area under entire IL curve**
  - for *maximum* **negative** (-) Shear / Moment, place loads over **Negative regions**
    - effect = ***w*** x **negative area under entire IL curve**



Influence line for M_B

EXAMPLE 9.1 For the beam shown in Fig. 9.3(a), determine the maximum positive and negative shears and the maximum positive and negative bending moments at point C due to a concentrated live load of 90 kN, a uniformly distributed live load of 40 kN/m, and a uniformly distributed dead load of 20 kN/m.

SOLUTION *Influence Lines* The influence lines for the shear and bending moment at point C of this beam were previously constructed in Example 8.5 and are shown in Fig. 9.3(b) and (e), respectively.



(a)

(b) Influence Line for S_C (kN/kN)

(e) Influence Line for M_C (kN·m/kN)

(c) Loading Arrangement for Maximum Positive S_C

(f) Loading Arrangement for Maximum Positive M_C

(d) Loading Arrangement for Maximum Negative S_C

(g) Loading Arrangement for Maximum Negative M_C

2. Application: Trusses - Maximum axial force

In Lecture 10.1 we saw how to find influence lines for the **axial force** in truss members.
Once the influence line of a particular member has been constructed, one can then **position** the live loads on the truss which will produce the **maximum** axial force in that particular member.

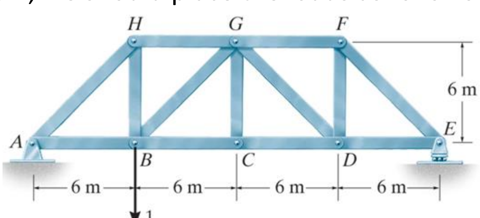
The maximum **tensile force (+)** developed in a truss member:

- due to a concentrated live load, *F*: F x PEAK POSITIVE influence line value
- due to a distributed live load, *w*: *w* x POSITIVE area under influence curve

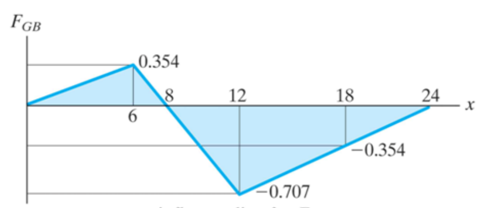
Same procedure applies for a **compression force (-)** except we use:

- due to a concentrated live load, *F*: F x PEAK NEGATIVE influence line value
- due to a distributed live load, *w*: *w* x NEGATIVE area under influence curve

Example: if we have found that the influence line diagram for member GB in the truss below is as shown below, we should place the loads as follows to produce maximum effects:

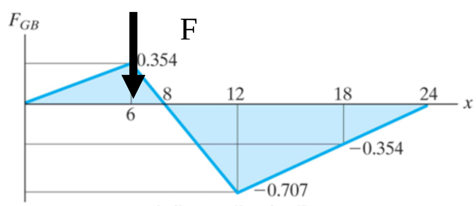


(a)



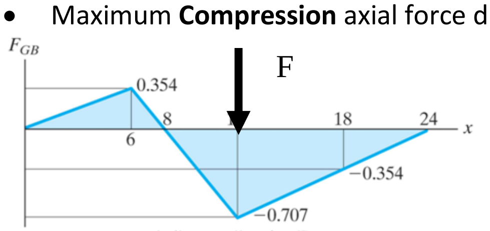
influence line for F_{GB}

- Maximum **Tension** axial force due a **concentrated live load F**:



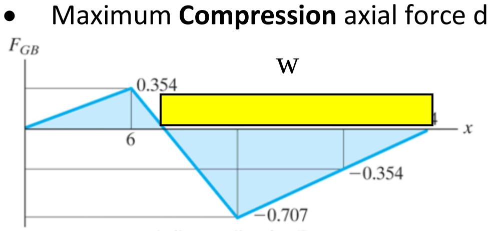
influence line for F_{GB}

- Maximum **Compression** axial force due a **concentrated live load F**:



influence line for F_{GB}

- Maximum **Compression** axial force due a **distributed live load w**:

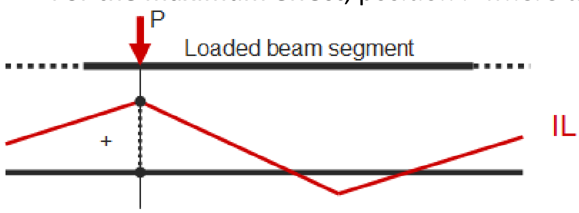


influence line for F_{GB}

5. Application: Truck loads

Recall we saw:

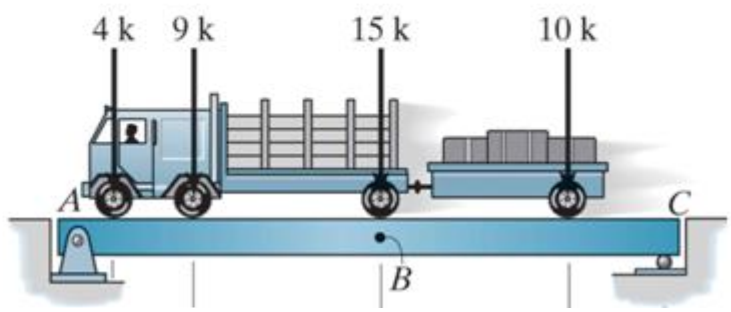
- **Effect of point load P:** function value is simply **P x ordinate of the IL**.
- **For the maximum effect**, position P where the **amplitude of the IL is maximum**

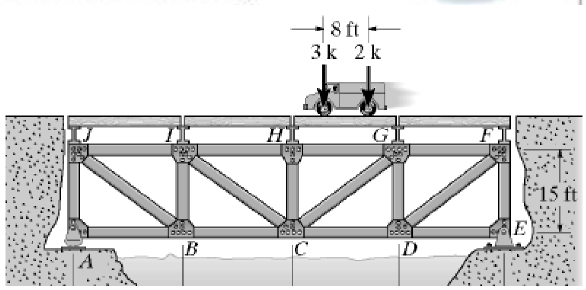


Effect of a series of point loads traveling at a fixed distance (loading train):

- In some cases, however a series of concentrated point loads must be placed on the structure
- Example: wheel loadings of a truck
- Example: wheel loadings of a train

Figure: FG06_02UN Design of this bridge girder is based on influence lines that must be considered for this train loading.

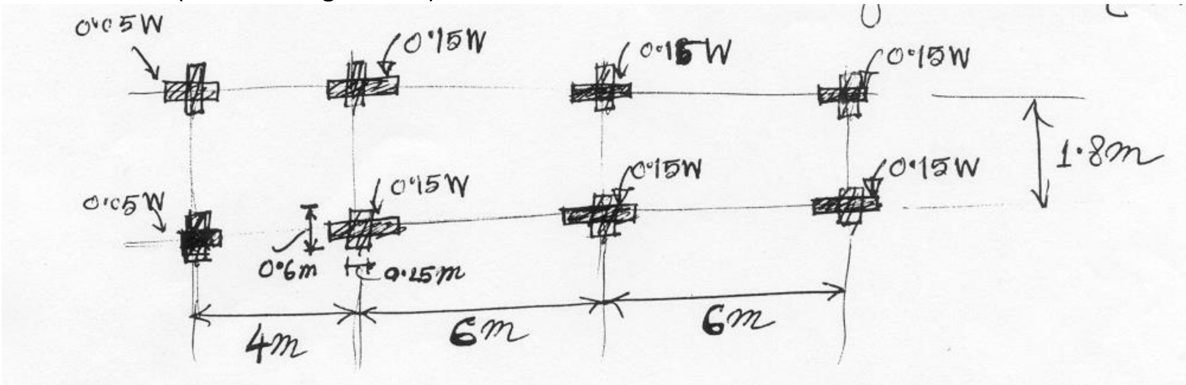




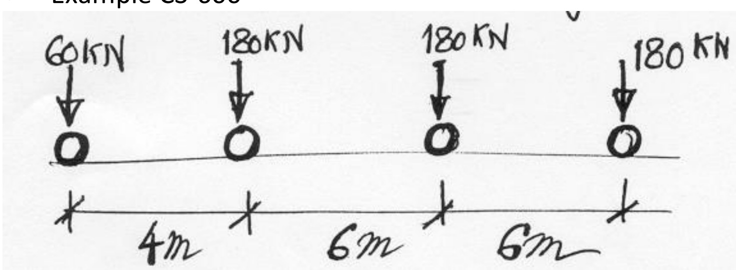
Usually in bridges, car traffic loads and pedestrian loads are considered as a uniform pressure. However for truck loads we will analyze them as a series of point loads:

Standard truck loading CAN/CSA-S6

- CS-W (W total weight in kN)



- Example CS-600



To calculate the effect of the load train at a **specific point**:

- Construct the influence line for the function as usual (i.e. using single **unit** load)
- By **trial and error**, vary the position of the loading train, always putting one of the loads at the peak of the IL of the function
- For each case, multiply each point load by the ordinate on the IL and sum the effects
- Repeat for other possible cases
- Compare and select the case that causes maximum effect at that point.

- Note: some loads may have a counteracting effect.