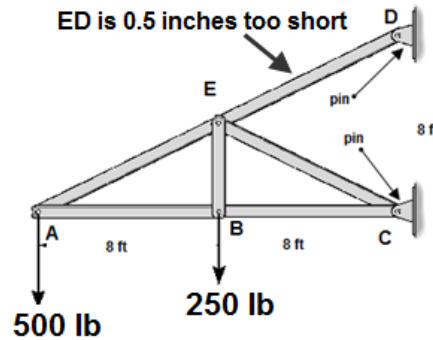


SOLUTION - A3

Problem 1

For the truss shown below determine the vertical displacement at **joint A** ...



Working in [Kips, ft] ...

Member	N (kips)	n (kips)	L (ft)	nNL	nΔL
AB	-1	-2	8	16.0	0
BC	-1	-2	8	16.0	0
AE	1.118	2.24	8.944	22.4	0
ED	1.398	2.24	8.944	28.0	2.24 * -0.5/12 = -0.0933
BE	0.250	0	4	0.0	0
CE	-0.280	0	8.944	0.0	0
			Total	82.4	-0.0933

$$\Delta_A = \left[\frac{82.4}{\left(2 * \frac{1}{12^2}\right) \times (29 \times 10^3 * 12^2)} \right] + [0] + [-0.0933]$$

$$= 0.00142 \text{ ft} - 0.0933 \text{ ft} = (-0.0921 \text{ ft}) = 1.11 \text{ in. upwards}$$

or

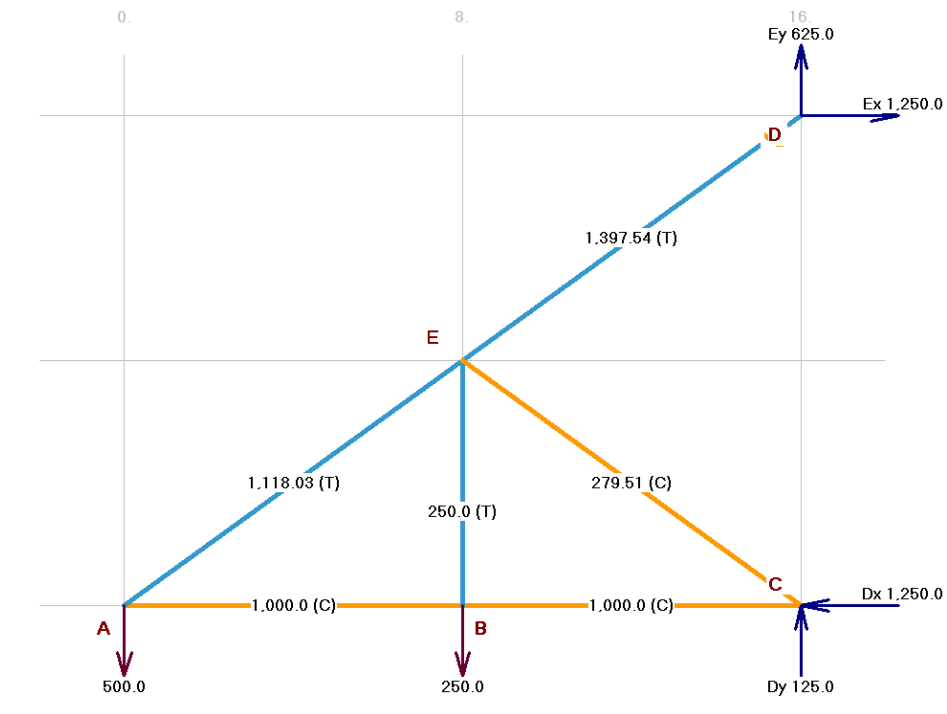
Working in [Kips, in] ...

Member	N (kips)	n (kips)	L (in)	nNL	nΔL
AB	-1	-2	8 x 12	16.0 x 12	0
BC	-1	-2	8 x 12	16.0 x 12	0
AE	1.118	2.24	8.944 x 12	22.4 x 12	0
ED	1.398	2.24	8.944 x 12	28.0 x 12	2.24 * -0.5 = -1.12
BE	0.250	0	4 x 12	0.0	0
CE	-0.280	0	8.944 x 12	0.0	0
			Total	82.4 x 12	-1.12

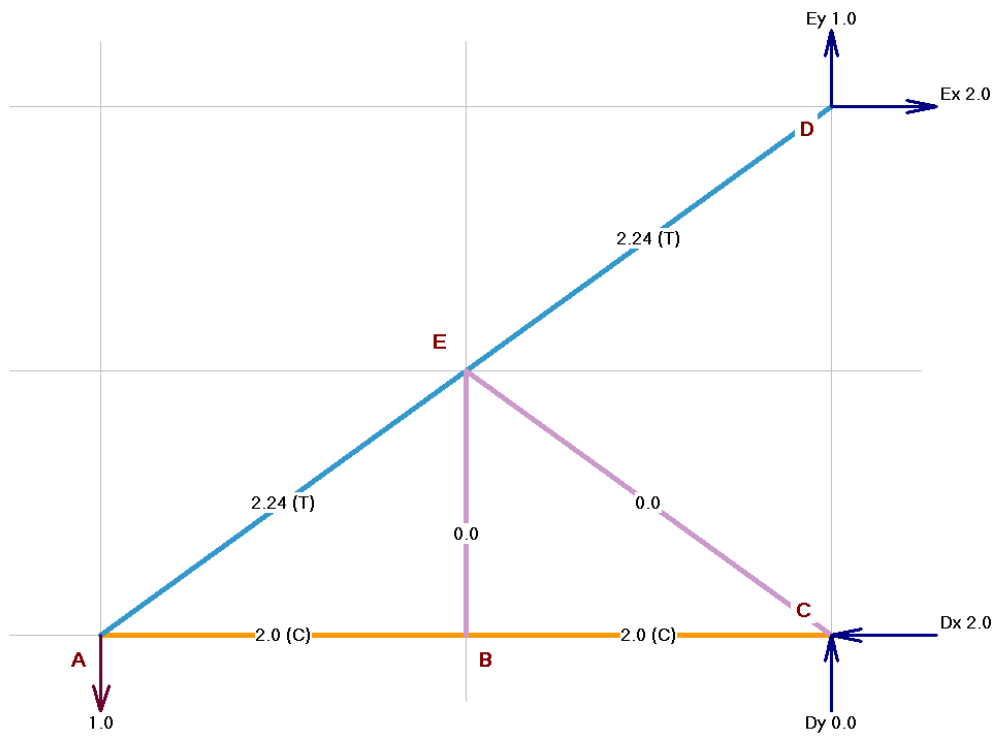
$$\Delta_A = \left[\frac{988.8}{(29 \times 10^3)(2)} \right] + [0] + [-1.12]$$

$$= 0.017 \text{ in} - 1.12 \text{ in} = 1.11 \text{ in. upwards}$$

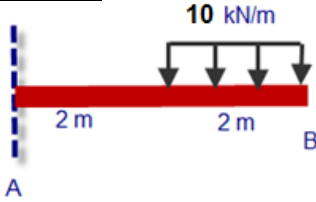
Under real loads: get "N" forces (shown in lbs but we will put in KIPs in the table)



Under virtual unit load: get "n" forces

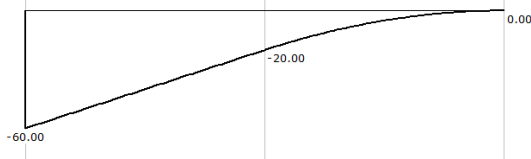


Problem 2



(a) Slope at B using direct integration

(some students might start their x from a different location so their M(x) functions will be different, you can easily check if the M function is correct by plugging x in the functions ... should give the right value on the M diagram ... and final answer should be the same)



REAL BEAM	VIRTUAL BEAM
Cut 1: taking sum of moments gives: $M1 = -60 + 20x$ $0 < x < 2$	Cut 1: taking sum of moments gives: $m1 = -1$ $0 < x < 2$
Cut 2: taking sum of moments gives: $M2 = -60 + 20x - 10(x-2)^2/2$ $2 < x < 4$	Cut 2: taking sum of moments gives: $m2 = -1$ $2 < x < 4$

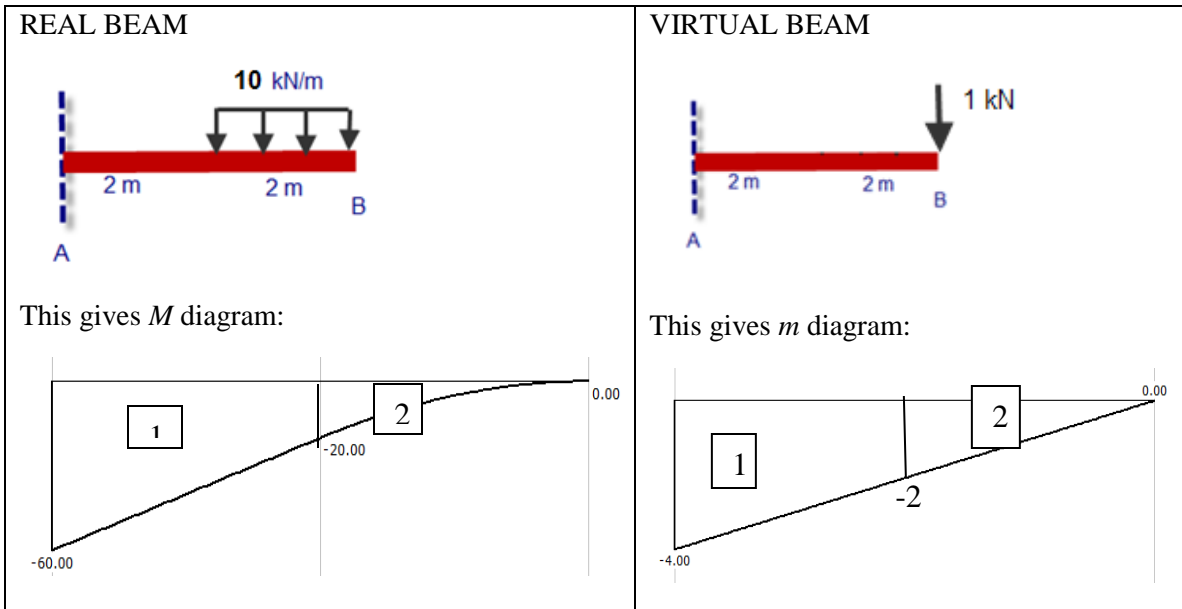
Solve virtual work equation: $\Delta = \frac{1}{EI} \int_0^2 (-1)(-60 + 20x)dx + \frac{1}{EI} \int_2^4 (-1)(-60 + 20x - 5(x-2)^2)dx$

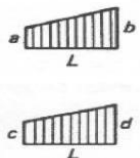
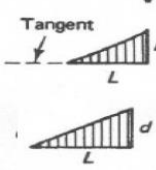
$$60x - 10x^2 \Big|_0^2 = 280$$

$$60x - 10x^2 + \frac{5}{3}(x-2)^3 \Big|_2^4 = \frac{40}{3}$$

Therefore: $\theta = \frac{93.33}{EI} = \frac{93.33}{(200 \times 10^6)(50 \times 10^{-6})} = 0.00933 \text{ rad. CLOCKWISE}$

a) Displacement at B using Mohr's table



Area	$\int_0^L mMdx$... using Mohr's table ...
1	 $\frac{L}{6} (2ac + ad + 2bd + bc)$ $\frac{2}{6} ([2 * -60 * -4] + [-60 * -2] + [2 * -20 * -2] + [-20 * -4]) = 253.33$
2	 $\frac{1}{4} Lbd$ $\frac{1}{4} (2 * -20 * -2) = 20$
Σ	273.33

Therefore:
$$\Delta = \frac{273.33}{EI} = \frac{273.33}{(200 \times 10^6)(50 \times 10^{-6})} =$$

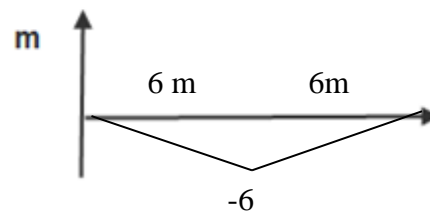
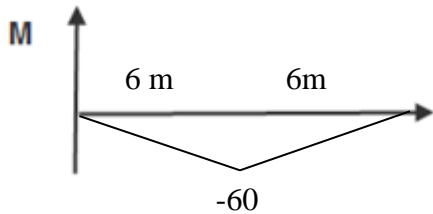
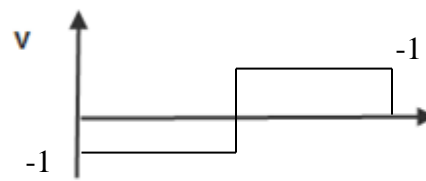
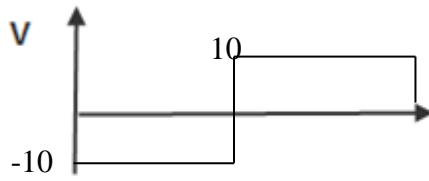
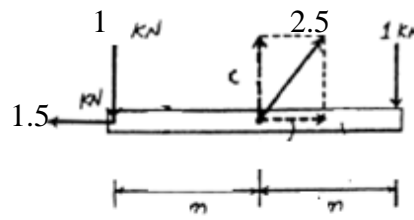
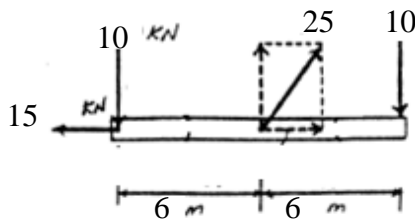
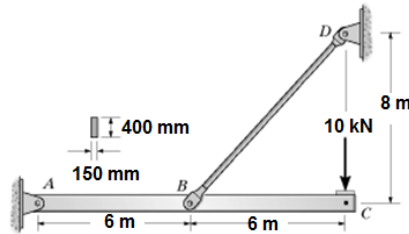
= 0.02733 m = 27.33 mm Downwards

Question 3

Beam **ABC** has a rectangular cross-section of 400 mm by 150 mm.

The rod (truss member) **DB** has a diameter of 50 mm.

Find the displacement at point C due to the loading using the method of virtual work.



For the **beam**, using Mohr's tables $\rightarrow \int mM dx = 1/3 * (6*-60*-6) + 1/3 * (6*-60*-6) = 1440$

For the **rod** $\rightarrow \sum nNL = 2.5 \times 25 \times 10 = 625$

Therefore (working in [kN, m]):

$$\Delta_C = \frac{1440}{(EI)_{AC}} + \frac{625}{(AE)_{BD}}$$

$$= \frac{1440}{\left(E * \left[\frac{1}{12} * 0.15 * 0.4^3\right]\right)} + \frac{625}{\left(\left[\frac{\pi}{4} * 0.05^2\right]E\right)} = \frac{1.8 \times 10^6}{(E)} + \frac{0.319 \times 10^6}{(E)} = \frac{2.119 \times 10^6}{(E)}$$

In the question I forgot to give E ... if we did assume a value of **E = 200 GPa**, then:

$$\Delta_C = \frac{2.119 \times 10^6}{(E)} = \frac{2.119 \times 10^6}{(200 \times 10^6)} = 0.0106 \text{ m} = 10.6 \text{ mm downwards}$$